Tutorial letter 202/2/2018

Basic Statistics

STA1510

Semester 2

Department of Statistics

SOLUTIONS TO ASSIGNMENT 02

CHAPTER 7

QUESTION 1

Sampling distribution (proportion)

We given, $\pi = 0.31$ and $n = 300$

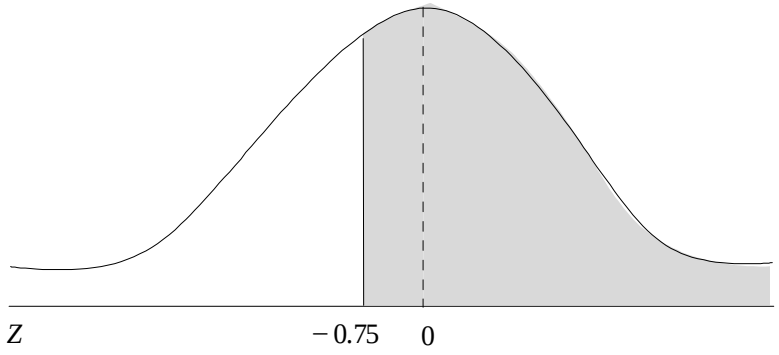
We need to find $P\,(p > 0.29)$ more than 29%

$$\sigma_p = \sqrt{\frac{\pi\,(1-\pi)}{n}} = \sqrt{\frac{0.31\,(0.69)}{300}} = 0.0267$$

$$\therefore \quad P\,(p > 0.29)$$

$$P\left(Z > \frac{0.29 - 0.31}{0.0267}\right)$$

$$P\,(Z > -0.75)$$



$$\Rightarrow \quad 0.7734$$

Option 1

QUESTION 2

Sampling distribution (proportion)

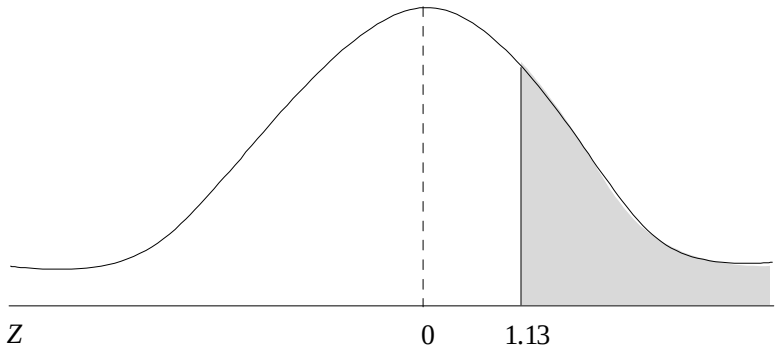
Given, $\pi = 0.12$ and $n = 150$

then $\sigma_p = \sqrt{\frac{\pi\,(1-\pi)}{n}} = \sqrt{\frac{0.12\,(0.88)}{150}} = 0.0265$

$$\therefore \quad P\,(p \geq 0.15) \quad \text{at least } 15\%$$

$$P\left(Z > \frac{p - \pi}{r_p}\right) = P\left(Z > \frac{0.15 - 0.12}{0.0265}\right)$$

$$P\,(Z > 1.13)$$



$$\Rightarrow \quad 0.1292$$

Option 3

QUESTION 3

Sampling distribution (mean)
Given, $\mu = 60, \quad n = 50, \quad \bar{X} = 57.1 \quad$ and $\quad \sigma = 10$
but

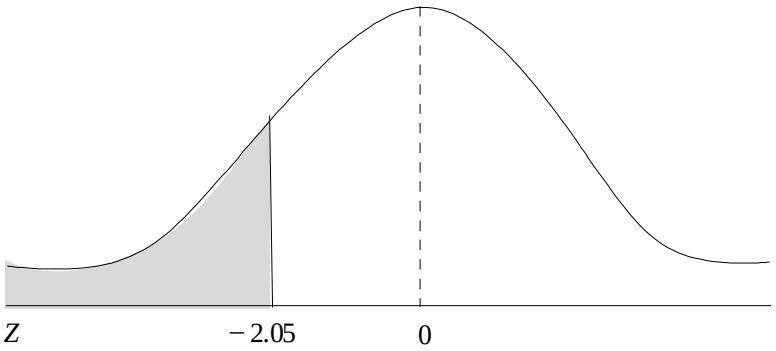
$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{10}{\sqrt{50}} = 1.4142$$

$$\therefore \quad P\left(\bar{X} \leq 57.1\right) \quad \text{at most } 57.1$$

$$P\left(Z \leq \frac{\bar{X} - \mu}{\sigma_{\bar{X}}}\right)$$

$$P\left(Z < \frac{57.1 - 60}{1.4142}\right)$$

$$P\left(Z < -2.05\right)$$



$$\Rightarrow \quad 0.0202$$

Option 2

CHAPTER 8

QUESTION 4

Confidence interval (proportion)
Given : $n = 100, X = 80 \quad$ then $p = \frac{X}{n} = \frac{80}{100} = 0.8$
99% CI, $p \pm Z_{\frac{\alpha}{2}}\sqrt{\frac{P(1-p)}{n}}$ then $p = 0.80, \quad Z_{\frac{\alpha}{2}} = Z_{0.005} = 2.58$

$$\sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{(0.8)(0.2)}{100}} = 0.04$$

$$\therefore p \pm Z_{\frac{\alpha}{2}}\sqrt{\frac{p(1-p)}{n}} \Rightarrow 0.8 \pm (2.58)(0.04)$$

0.8 ± 0.1032
[0.6968; 0.9032]

Option 5

QUESTION 5

Confidence interval (mean, σ known)
Given : $\bar{X} = 160, \quad \sigma = 45, \quad n = 36$
95% CI for μ, $\bar{X} \pm Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$

$\therefore \quad \bar{X} = 160, \quad Z_{\frac{\alpha}{2}} = Z_{0.05} = 1.645, \quad \frac{\sigma}{\sqrt{n}} = \frac{45}{\sqrt{36}} = 7.5$

$\therefore \quad \bar{X} \pm Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} \Rightarrow 160 \pm (1.645)(7.5)$
160 ± 12.3375
$[147.66; \ 172.34]$

Option 5

QUESTION 6

Confidence interval (proportion)
Given : $n = 1000, \quad X = 410$ then $p = \frac{X}{n} = 0.41$
90% CI for π, $p \pm Z_{\frac{\alpha}{2}} \sqrt{\frac{p(1-p)}{n}}$ with
$p = 0.41$
$Z_{\frac{\alpha}{2}} = Z_{0.05} = 1.645$
$\sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.41(0.59)}{1.000}} = 0.0156$
$\therefore \quad p \pm Z_{\frac{\alpha}{2}} \sqrt{\frac{p(1-p)}{n}} \Rightarrow 0.41 \pm (1.645)(0.0156)$
$0.41 \pm 0.0.0257$
$[0.3843; \ 0.4357]$

Option 5

CHAPTER 9

QUESTION 7

Hypothesis test (proportion)
Given:
$H_0 : \pi = 0.40 \quad \alpha = 0.05$ and
$H_1 : \pi > 0.40 \quad p = 0.45, \ n = 49$
One tailed (right) test.
$Z_\alpha = Z_{0.05} = 1.645$
Reject H_0 if the Z_{stat} is > 1.645

Option 3

QUESTION 8

Hypothesis test (proportion)

$$\begin{aligned} Z_{\text{stat}} &= \frac{p-\pi}{\sqrt{\frac{\pi\,(1-\pi)}{n}}} \\ &= \frac{0.45-0.40}{0.07} \\ &= 0.7143 \simeq 0.71 \end{aligned}$$

Option 1

QUESTION 9

Hypothesis test (mean, σ known)
Given : $H_0 : \mu = 50$ $H_1 : \mu \neq 50$ two tailed test
$n = 64$ $\bar{X} = 53.5$ and $\sigma = 10$
$\alpha = 1\%$
$Z_{\frac{\alpha}{2}} = Z_{0.005} = \pm 2.58$
Reject H_0 if the Z_{stat} is < -2.58 or > 2.58

Option 3

QUESTION 10

Hypothesis test (mean, p–value)
Given,
$H_0 : \mu = 50$
$H_1 : \mu \neq 50$ two–tailed test
$Z_{stat} = 2.80$

$$\begin{aligned} p\text{–value} &= P\,(Z > 2.80) \times 2 \\ &= 0.0026 \times 2 \\ &= 0.0052 \quad \text{or} \quad 0.52\% \end{aligned}$$

Option 2

CHAPTER 11

QUESTION 11

χ^2 distribution
Option 5 is the incorrect statement as the χ^2 curve is positively skewed.

Option 5

QUESTION 12

χ^2 test of independence
χ^2 test statistic or χ^2_{stat} is given by

$$\chi^2_{\text{stat}} = \sum \frac{(f_0 - f_e)^2}{f_e}$$

$$f_e = \frac{\text{row total} \times \text{column total}}{n}$$

	Beat	Youth	Grow	Live	Total
G	21.84	14.82	21.06	20.28	78
B	34.16	23.18	32.94	31.72	122
Total	56	38	54	52	200

$$\begin{aligned}\chi^2_{\text{stat}} &= 0.6752 + 0.5366 + 0.0534 + 2.9388 + \\ &\quad 0.4317 + 0.3431 + 0.0341 + 1.8789 \\ &= 6.8918\end{aligned}$$

Option 3

CHAPTER 12

QUESTION 13

Simple linear regression

The coefficient of determination is given by: $r^2 = \frac{SSR}{SST}$ or just using the statistics mode of your calculator to get $r = 0.9798$.
Option 1

QUESTION 14

Simple regression
Using the calculator or formula, $b_0 = 12.0354$ and $b_1 = 1.416$
$\therefore \quad \hat{Y}_i = 12.0354 + 1.416X_i$

Option 4

QUESTION 15

Simple regression
Given that $X = \text{R}38.00$

$$\begin{aligned}\hat{Y} &= 12.0354 + 1.416\ (38) \\ &= 12.0354 + 53.808 \\ &= 65.84 \quad \text{approximate profit of R}65.84\end{aligned}$$

Option 5