

1. The probability that a certain machine will produce a defective item is 0.25. If a random sample of 6 items is taken from the output of this machine, what is the probability that there will be at least 5 defective items in the sample?

Answer:

Let X be the no. of defective items
and $X \sim \text{Bin}(6, 0.25)$

$$P(X \geq 5) = P(X=5) + P(X=6)$$

$$= \binom{6}{5} 0.25^5 0.75^{6-5} + \binom{6}{6} (0.25)^6 (0.75)^{6-6}$$

$$= 6 (0.25)^5 (0.75) + (0.25)^6$$

$$= 6 \times (0.25)^5 \times (0.75) + 0.25^6$$

$$= \dots$$

2. Consider 8 digits (1, 2, 3, 4, 5, 6, 7, 8)

with equal probability $\Rightarrow P(X=1) = P(X=2) = \dots = P(X=8) = \frac{1}{8}$
 $\Rightarrow P(X) = \frac{1}{8}$

Event $A = (1, 3, 5, 7)$

Event $B = (5, 6, 7, 8)$

Event $C = (1, 2)$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$(A \cap B) \Rightarrow$ nois that appear in both A and B
 $\Rightarrow (5, 7)$

$$\Rightarrow P(A \cap B) = \frac{2}{8}$$

$$\Rightarrow P(B) = \frac{4}{8}$$

$$\Rightarrow P(A|B) = \frac{\frac{2}{8}}{\frac{4}{8}} = \frac{2}{4} = \frac{1}{2} = 0.5$$