



Christopher C. Tisdell

$$a^2 + b^2 = c^2$$

Christopher C. Tisdell

Introduction to Complex Numbers:

YouTube Workbook

Introduction to Complex Numbers: *YouTube* Workbook

1st edition

© 2015 Christopher C. Tisdell & bookboon.com

ISBN 978-87-403-1110-5

Contents

	How to use this workbook	8
	About the author	9
	Acknowledgments	10
1	What is a complex number?	11
1.1	Video 1: Complex numbers are AWESOME	11
2	Basic operations involving complex numbers	15
2.1	Video 2: How to add/subtract two complex numbers	15
2.2	Video 3: How to multiply a real number with a complex number	16
2.3	Video 4: How to multiply complex numbers together	17
2.4	Video 5: How to divide complex numbers	19
2.5	Video 6: Complex numbers: Quadratic formula	21



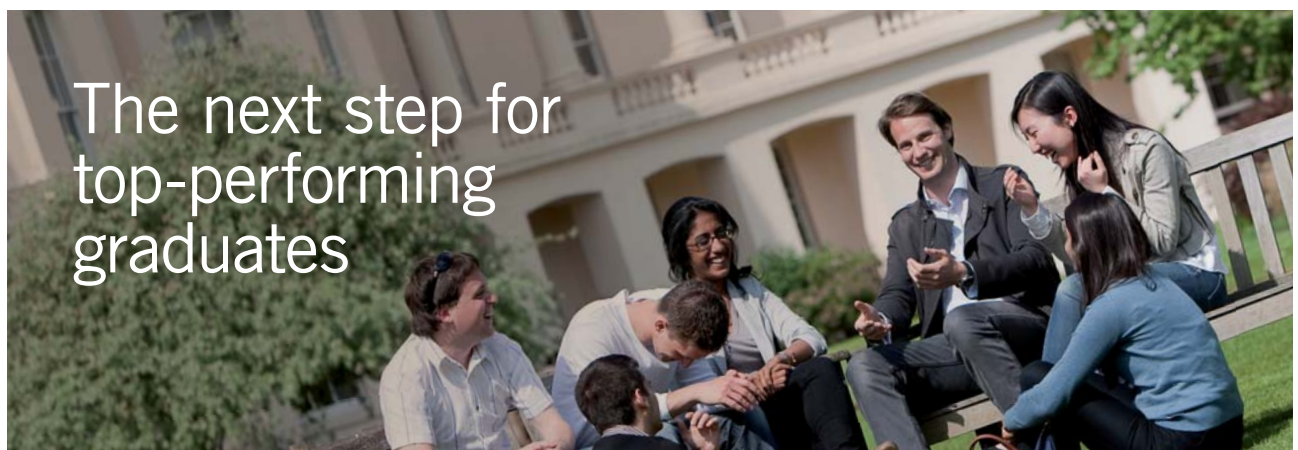
Giving
"turning the page"
a new meaning

Download free e-books on the go
Turn To Bookboon.com

 **GAUTRAIN**
FOR PEOPLE ON THE MOVE

bookboon.com

3	What is the complex conjugate?	22
3.1	Video 7: What is the complex conjugate?	22
3.2	Video 8: Calculations with the complex conjugate	25
3.3	Video 9: How to show a number is purely imaginary	27
3.4	Video 10: How to prove the real part of a complex number is zero	28
3.5	Video 11: Complex conjugate and linear systems	29
3.6	Video 12: When are the squares of z and its conjugate equal?	30
3.7	Video 13: Conjugate of products is product of conjugates	31
3.8	Video 14: Why complex solutions appear in conjugate pairs	32
4	How big are complex numbers?	33
4.1	Video 15: How big are complex numbers?	33
4.2	Video 16: Modulus of a product is the product of moduli	35
4.3	Video 17: Square roots of complex numbers	36
4.4	Video 18: Quadratic equations with complex coefficients	37
4.5	Video 19: Show real part of complex number is zero	38
5	Polar trig form	39
5.1	Video 20: Polar trig form of complex number	39



Masters in Management



Designed for high-achieving graduates across all disciplines, London Business School's Masters in Management provides specific and tangible foundations for a successful career in business.

This 12-month, full-time programme is a business qualification with impact. In 2010, our MiM employment rate was 95% within 3 months of graduation*; the majority of graduates choosing to work in consulting or financial services.

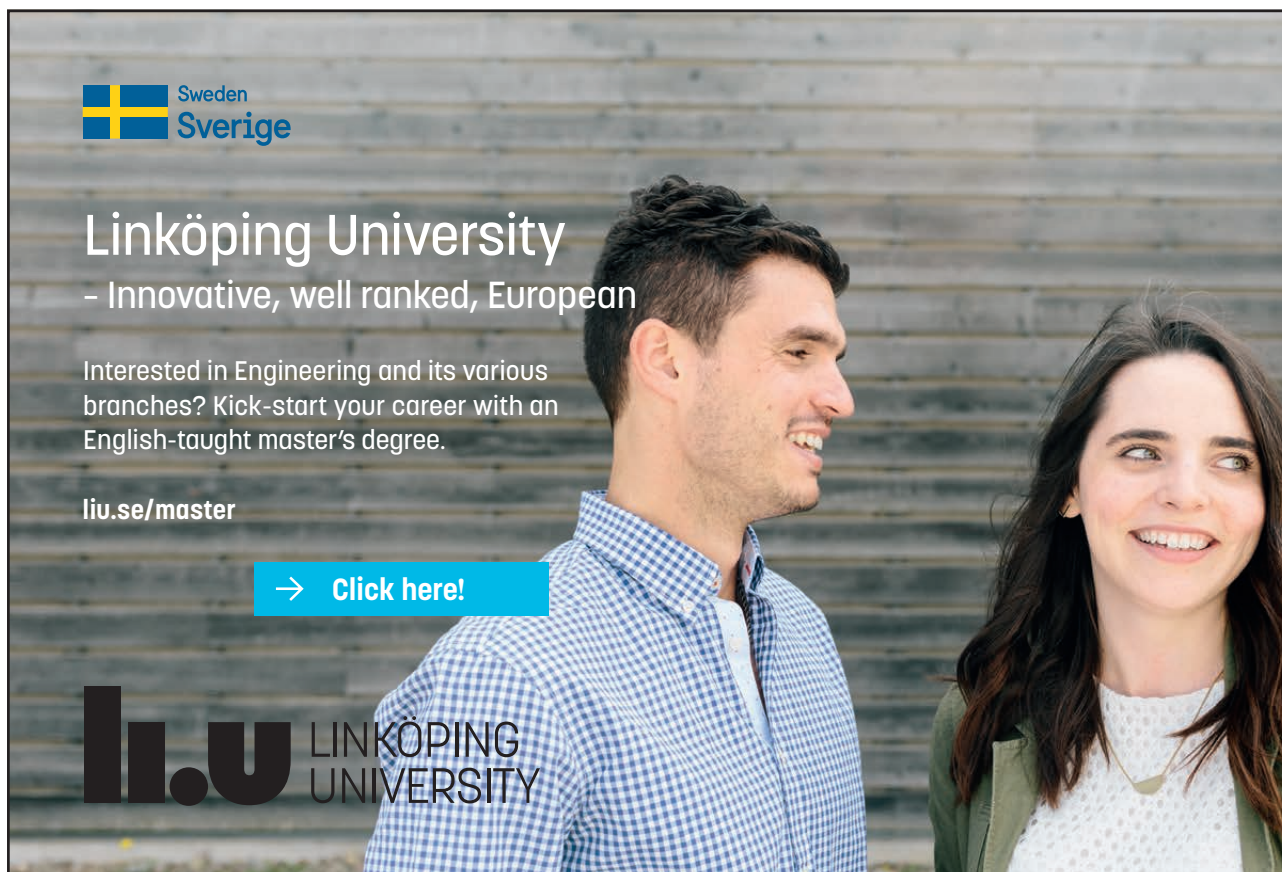
As well as a renowned qualification from a world-class business school, you also gain access to the School's network of more than 34,000 global alumni – a community that offers support and opportunities throughout your career.

For more information visit www.london.edu/mm, email mim@london.edu or give us a call on +44 (0)20 7000 7573.

* Figures taken from London Business School's Masters in Management 2010 employment report



6	Polar exponential form	41
6.1	Video 21: Polar exponential form of a complex number	41
6.2	Revision Video 22: Intro to complex numbers + basic operations	43
6.3	Revision Video 23: Complex numbers and calculations	44
6.4	Video 24: Powers of complex numbers via polar forms	45
7	Powers of complex numbers	46
7.1	Video 25: Powers of complex numbers	46
7.2	Video 26: What is the power of a complex number?	47
7.3	Video 27: Roots of complex numbers	48
7.4	Video 28: Complex numbers solutions to polynomial equations	49
7.5	Video 29: Complex numbers and $\tan(\pi/12)$	50
7.6	Video 30: Euler's formula: A cool proof	51
8	De Moivre's formula	52
8.1	Video 31: De Moivre's formula: A cool proof	52
8.2	Video 32: Trig identities from De Moivre's theorem	53
8.3	Video 33: Trig identities: De Moivre's formula	54



Sweden
Sverige

Linköping University
– Innovative, well ranked, European

Interested in Engineering and its various branches? Kick-start your career with an English-taught master's degree.

liu.se/master

→ Click here!

li.u LINKÖPING UNIVERSITY

9	Connecting sin, cos with e	55
9.1	Video 34: Trig identities and Euler's formula	55
9.2	Video 35: Trig identities from Euler's formula	57
9.3	Video 36: How to prove trig identities WITHOUT trig!	58
9.4	Revision Video 37: Complex numbers + trig identities	59
10	Regions in the complex plane	60
10.1	Video 38: How to determine regions in the complex plane	60
10.2	Video 39: Circular sector in the complex plane	63
10.3	Video 40: Circle in the complex plane	64
10.4	Video 41: How to sketch regions in the complex plane	65
11	Complex polynomials	66
11.1	Video 42: How to factor complex polynomials	66
11.2	Video 43: Factorizing complex polynomials	68
11.3	Video 44: Factor polynomials into linear parts	69
11.4	Video 45: Complex linear factors	70
	Bibliography	71

**Get a higher mark
on your course
assignment!**

Get feedback & advice from experts in your subject
area. Find out how to improve the quality of your work!

Get Started



Go to www.helpmyassignment.co.uk for more info



How to use this workbook

This workbook is designed to be used in conjunction with the author's free online video tutorials. Inside this workbook each chapter is divided into learning modules (subsections), each having its own dedicated video tutorial.

View the online video via the hyperlink located at the top of the page of each learning module, with workbook and paper or tablet at the ready. Or click on the *Introduction to Complex Numbers* playlist where all the videos for the workbook are located in chronological order:

Introduction to Complex Numbers

www.youtube.com/playlist?list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP
www.tinyurl.com/ComplexNumbersYT.

While watching each video, ll in the spaces provided after each example in the workbook and annotate to the associated text.

You can also access the above via the author's YouTube channel

[Dr Chris Tisdell's YouTube Channel](http://www.youtube.com/DrChrisTisdell)
<http://www.youtube.com/DrChrisTisdell>

There has been an explosion in books that blend text with video since the author's pioneering work *Engineering Mathematics: YouTube Workbook* [46]. The current text takes innovation in learning to a new level, with:

- the video presentations herein streamed live online, giving the classes a live, dynamic and fun feeling;
- each video featuring closed captions, providing each learner with the ability to watch, read or listen to each video presentation.

About the author

Dr Chris Tisdell is Associate Dean (Education), Faculty of Science at UNSW Australia who has inspired millions of learners through his passion for mathematics and his innovative online approach to maths education. He is best-known for creating YouTube university-level maths videos, which have attracted millions of downloads. This has made his virtual classroom the top-ranked learning and teaching website across Australian universities on the education hub YouTube EDU.

His free online etextbook, *Engineering Mathematics: YouTube Workbook*, is one of the most popular mathematical books of its kind, with more than 1 million downloads in over 200 countries. A champion of free and flexible education, he is driven by a desire to ensure that anyone, anywhere at any time, has equal access to the mathematical skills that are critical for careers in science, engineering and technology.

Vision, leadership and management skills underpins his experience in educational change. In 2008 he dared to dream of educational experiences that featured personalized and scalable learning. His early leadership on enabling technologies such as: lecture capture; open educational resources; MOOCs; learning analytics; and gamification, has significantly influenced and positively changed L&T strategies at the institutional level.

He is a recognized leader in the online learning space at national and institutional levels, winning education awards and positively transforming learning and teaching.

As an Associate Dean (Education) at UNSW Australia he has been responsible for leading, managing and operationalising educational change at-scale, including inspiring positive transformation within 7,000 7,000 science students, 400 academic staff, 300+ courses and scores of programs within UNSW Science.

Chris has collaborated with industry and policy-makers, championed educational thought-leadership in the media and constantly draws on the feedback of key stakeholders worldwide to advance learning and teaching.

Acknowledgments

I'm grateful to the following, who admirably transcribed audio to text for each video to create closed captions and helped me proofread drafts of the manuscript. **Thank you:**

Anubhav Ashish; Johann Blanco; Sean Cossins; Jonathan Kim Sing; Madeleine Kyng; Jeffry Lay; Harris Phan; Anthony Tran; Koha Tran; Ines Vallely; Velushomaz; Wilson Yuan.

I would also like to express my thanks to the Bookboon team for their support.

1 What is a complex number?

1.1 Video 1: Complex numbers are AWESOME

1.1.1 Where are we going?

[View this lesson on YouTube](#) [1]

- We will learn about a new kind of number known as a “complex number”.
- We will discover the basic properties of complex numbers and investigate some of their mathematical applications.

Complex numbers rest on the idea of the “imaginary unit” i , which is dened via

$$i = \sqrt{-1}$$

with i satisfying the equation

$$i^2 = -1.$$

Even though the thought of i may seem crazy, we will see that is a really useful idea.

1.1.2 Why are complex numbers AWESOME?

There are at least two reasons why complex numbers are AWESOME:-

1. their real-world applications;
2. their ability to SIMPLIFY mathematics.

For example, i arises in the solutions

$$x(t) = e^{i\sqrt{k/m} t} \text{ and } x(t) = e^{-i\sqrt{k/m} t}.$$

to a basic spring-mass differential equation

$$m \frac{d^2 x}{dt^2} + kx = 0$$

where: $x = x(t)$ is the position of the mass at time t ; $m > 0$ is the mass; and $k > 0$ is the stiffness of the spring.

Also, i appears in Fourier transform techniques, which are important for solving partial differential equations from science and engineering.

Complex numbers are AWESOME because they provide a SIMPLER framework from which we can view and do mathematics.

As a result, applying methods involving complex numbers can simplify calculations, removing a lot of the boring and tedious parts of mathematical work.

For example, complex numbers provides a quick alternative to integration by parts for something like

$$\int e^{-t} \cos t \, dt$$

and gives easy ways of constructing trig formulae, for example

$$\begin{aligned} \sin(x + y) &= \sin x \cos y + \cos x \sin y \\ \cos 2\theta &= \cos^2 \theta - \sin^2 \theta \end{aligned}$$

so you might never have to remember another trig formula ever again!

1.1.3 What is a complex number?

Here are some examples of complex numbers:

$$\begin{array}{ll} 3 + 2i, & -7 + 3i, \\ 6 - i, & 2i, \\ -1 - 4i, & -2 - 2i. \end{array}$$

Important idea (What is a complex number? (Cartesian form)).

The Cartesian form of a complex number z is

$$x + yi \quad \text{or} \quad x + iy$$

where x and y are both real numbers and i is known as the imaginary unit $i = \sqrt{-1}$ and satisfies $i^2 = -1$. The number x is called the “real part of z ”; while y is called the “imaginary part of z ”.

YOU WOULD NEVER ALLOW YOUR CHILD TO DO THIS

SO WHY LET THEM DO THIS?

It's a scary thought. If you have a crash, the impact is much the same as falling from the roof. And should your child be flung from the car during a crash, possibly worse. It's worth remembering the next time you set off in your car.

CHEKICOAST
SAVE A LIFE

www.sanral.co.za

ROAD OF ACTION FOR ROAD SAFETY

20 YEARS OF FREEDOM

THE SOUTH AFRICAN NATIONAL ROADS AGENCY SOC LTD

1.1.4 How to graphically represent complex numbers?

Complex numbers can be represented in the "complex plane" via what is known as an Argand diagram, which features:

- a “real” (horizontal) axis;
- an “imaginary” (vertical) axis.

2 Basic operations involving complex numbers

2.1 Video 2: How to add/subtract two complex numbers

[View this lesson on YouTube](#) [3]

To add/subtract two complex numbers just add/subtract their corresponding components.

Example.

If $z = 1 + 3i$ and $w = 2 + i$ then

$$\begin{aligned}z + w &= (1 + 3i) + (2 + i) \\&= (1 + 2) + (3i + i) \\&= 3 + 4i\end{aligned}$$

and

$$\begin{aligned}z - w &= (1 + 3i) - (2 + i) \\&= (1 - 2) + (3i - i) \\&= -1 + 2i.\end{aligned}$$

A geometric interpretation of addition is seen through a simple parallelogram or triangle law.



The advertisement features a large central image of a smiling teacher leaning over a laptop to assist two young students, a boy and a girl. To the right, there are two smaller circular inset images: one showing three children looking at a tablet together, and another showing a child working on a laptop. The background is a vibrant yellow with orange and white abstract swirls. In the top left corner is the 'e-learning for kids' logo, which consists of a colorful grid of squares. A green oval callout on the right contains three bullet points. At the bottom, a paragraph of text provides information about the foundation, and a green arrow points towards a call to action.

e-learning for kids

- The number 1 MOOC for Primary Education
- Free Digital Learning for Children 5-12
- 15 Million Children Reached

About e-Learning for Kids Established in 2004, e-Learning for Kids is a global nonprofit foundation dedicated to fun and free learning on the Internet for children ages 5 - 12 with courses in math, science, language arts, computers, health and environmental skills. Since 2005, more than 15 million children in over 190 countries have benefitted from eLessons provided by EFK! An all-volunteer staff consists of education and e-learning experts and business professionals from around the world committed to making difference. eLearning for Kids is actively seeking funding, volunteers, sponsors and courseware developers; get involved! For more information, please visit www.e-learningforkids.org.



2.2 Video 3: How to multiply a real number with a complex number

[View this lesson on YouTube](#) [3]

Multiplication of a real number with a complex number involves multiplying each component in a natural distributive fashion.

Example.

If $z = 2 + 3i$ then

$$\begin{aligned}2z &= 2(2 + 3i) \\&= (2 * 2) + (2 * 3i) \\&= 4 + 6i\end{aligned}$$

and

$$\begin{aligned}-4z &= -4(2 + 3i) \\&= (-4 * 2) + (-4 * 3i) \\&= -8 - 12i.\end{aligned}$$

A geometric interpretation of (scalar) multiplication is seen through a stretching principle.

2.3 Video 4: How to multiply complex numbers together

[View this lesson on YouTube](#) [4]

Multiplication of two complex numbers involves natural distribution (and remembering $i^2 = -1$).

Example.

If $z = 2 + i$ and $w = 1 + i$ then

$$\begin{aligned}zw &= (2 + i)(1 + i) \\&= (2 * 1 + i * i) + (2 * i + i * 1) \\&= (2 - 1) + 3i \\&= 1 + 3i.\end{aligned}$$

The geometric interpretation of multiplication is seen through rotation and stretching/compression.

Teach with the Best. Learn with the Best.

Agilent offers a wide variety of affordable, industry-leading electronic test equipment as well as knowledge-rich, on-line resources—for professors and students.

We have 100's of comprehensive web-based teaching tools, lab experiments, application notes, brochures, DVDs/CDs, posters, and more.



See what Agilent can do for you.
www.agilent.com/find/EDUstudents
www.agilent.com/find/EDUeducators

© Agilent Technologies, Inc. 2012

u.s. 1-800-829-4444 canada: 1-877-894-4414

Anticipate — Accelerate — Achieve



Agilent Technologies



Click on the ad to read more

2.3.1 What is the geometric explanation of multiplication?

Example.

Let us consider $z = 2i$ and $w = 1 + i$ in the complex plane.

If we compute the distances from z and w to the origin (using Pythagoras) then we see that

$$|z| = 2, \quad |w| = \sqrt{2}.$$

Now consider the line segments joining z and w to the origin. If we compute the angles θ_1, θ_2 to the positive real axis (using trig) with $-\pi < \theta_k \leq \pi$ then we see

$$\theta_1 = \pi/2, \quad \theta_2 = \pi/4.$$

Now consider $zw = -2 + 2i$. We have

$$|zw| = 2\sqrt{2}, \quad \theta_3 = 3\pi/4.$$

We thus see that $|zw| = |z| |w|$ and $\theta_3 = \theta_1 + \theta_2$.

2.4 Video 5: How to divide complex numbers

[View this lesson on YouTube](#) [5]

2.4.1 How to divide by a complex number

Division of two complex numbers involves multiplying through by a “factor of one” that turns the denominator into a real number. To do this, we use the “conjugate” of the denominator.

Example.

If $z = 2 + i$ and $w = 3 + 2i$ then

$$\begin{aligned}\frac{z}{w} &= \frac{2 + i}{3 + 2i} \\ &= \frac{2 + i}{3 + 2i} * \frac{3 - 2i}{3 - 2i} \\ &= \frac{(6 - 2i^2) + (3i - 4i)}{(9 - 4i^2) + (6i - 6i)} \\ &= \frac{8 - i}{13} = \frac{8}{13} - i\frac{1}{13}.\end{aligned}$$

Observe that the denominator is now real and we can (say) easily plot the complex number z/w .

If we interpret division as a kind of multiplication, then the geometric interpretation of division can also be seen through rotation/stretching.

2.4.2 Basic operations with complex numbers

Example.

If $z = -2 + 3i$ then calculate z^2 .

Consider

$$\begin{aligned} z^2 &= (-2 + 3i) * (-2 + 3i) \\ &= (4 + 9i^2) - 6i - 6i \\ &= -5 - 12i. \end{aligned}$$

Independent learning exercise: plot z and z^2 . Can you see a relationship between their lengths to the origin?



Discover the truth at www.deloitte.ca/careers

Deloitte.

© Deloitte & Touche LLP and affiliated entities.



Click on the ad to read more

2.5 Video 6: Complex numbers: Quadratic formula

Applying the quadratic formula for complex solutions

[View this lesson on YouTube](#) [6]

Example.

Solve the quadratic equation

$$13z^2 - 6z + 1 = 0,$$

writing the solutions in the Cartesian form $x + yi$.

3 What is the complex conjugate?

3.1 Video 7: What is the complex conjugate?

[View this lesson on YouTube](#) [7]

As we saw when performing division of complex numbers, an idea called the conjugate was applied to simplify the denominator. Let us look at this idea a bit further.

Important idea (Complex conjugate).

For a complex number $z = x + yi$ we define and denote the “complex conjugate of z ” by

$$\bar{z} = x - yi.$$

If $z = 3 + i$ then $\bar{z} = 3 - i$. If $w = 1 - 2i$ then $\bar{w} = 1 + 2i$. If $u = -1 - i$ then $\bar{u} = -1 + i$.

For any point z in the complex plane, we can geometrically determine $\bar{z}$ by reflecting the position of z through the real axis.

3.1.1 What are the properties of the conjugate?

Important idea (Conjugate properties).

Let $z = a + bi$ and $w = c + di$. Some basic properties of the conjugate are:-

$$z\bar{z} = (a + bi)(a - bi) = a^2 + b^2, \text{ real and non\{neg number};$$

$$\bar{\bar{z}} = z;$$

$$\overline{z + w} = \bar{z} + \bar{w} = (a + c) - (b + d)i;$$

$$\overline{z - w} = \bar{z} - \bar{w} = (a - c) + (d - b)i;$$

$$\overline{zw} = \bar{z}\bar{w};$$

$$\overline{z/w} = \bar{z}/\bar{w};$$

$$\overline{z^n} = \bar{z}^n;$$

$$\frac{z + \bar{z}}{2} = a = \Re(z);$$

$$\frac{z - \bar{z}}{2} = b = \Im(z).$$



The “what-do-you-call-it-again?” for mechanical engineering.

Sometimes an ordinary dictionary just isn't enough. The online Glossary from item provides accurate translations for technical terminology – and full definitions in German and English.

www.item24.de/en/mechanical-engineering-glossary

Or get the app  

item

Voltage measurement
Ritter's method of dissection LED
Identification systems
Offset moment Continuous casting Real power
Band brake Resistance
Z-diode
Wire drawing IGBT surface metrology
Gear metrology I-beam

3.1.2 Basic operations with the conjugate

Example.

If $z = -2 + 3i$ then calculate the following: a) $\bar{z}$; b) $z + \bar{z}$.

By definition,

$$\bar{z} = -2 - 3i.$$

Also,

$$\begin{aligned} z + \bar{z} &= (-2 + 3i) + (-2 - 3i) \\ &= -4 + 0i \\ &= 4. \end{aligned}$$

3.2 Video 8: Calculations with the complex conjugate

[View this lesson on YouTube](#) [8]

Example.

If $z = 4 - 3i$ and $w = 1 + 4i$ then calculate the following in Cartesian form $x + yi$:

- a) $25/z$; b) $iw(\bar{z} - 4)$

3.2.1 Simplifying complex numbers with the conjugate

Example.

Simplify

$$\frac{2 - 7i}{3 - i}$$

into the Cartesian form $x + yi$.

We multiply by a factor of one that involves the conjugate of the denominator, namely

$$\begin{aligned}\frac{2 - 7i}{3 - i} &= \frac{2 - 7i}{3 - i} * \frac{3 + i}{3 + i} \\ &= \frac{(6 - 7i^2) + 2i - 21i}{(9 - i^2) + 3i - 3i} \\ &= 13/10 - 19i/10.\end{aligned}$$



Giving
"turning the page"
a new meaning

Download free e-books on the go
Turn To Bookboon.com

 **GAUTRAIN**
FOR PEOPLE ON THE MOVE

bookboon.com



3.3 Video 9: How to show a number is purely imaginary

3.3.1 Using the conjugate to show a number is purely imaginary

[View this lesson on YouTube](#) [9]

Example.

Let

$$\Im\left(\frac{z+i}{z-i}\right) = 0$$

with $z \neq i$. Show $\Re(z) = 0$.

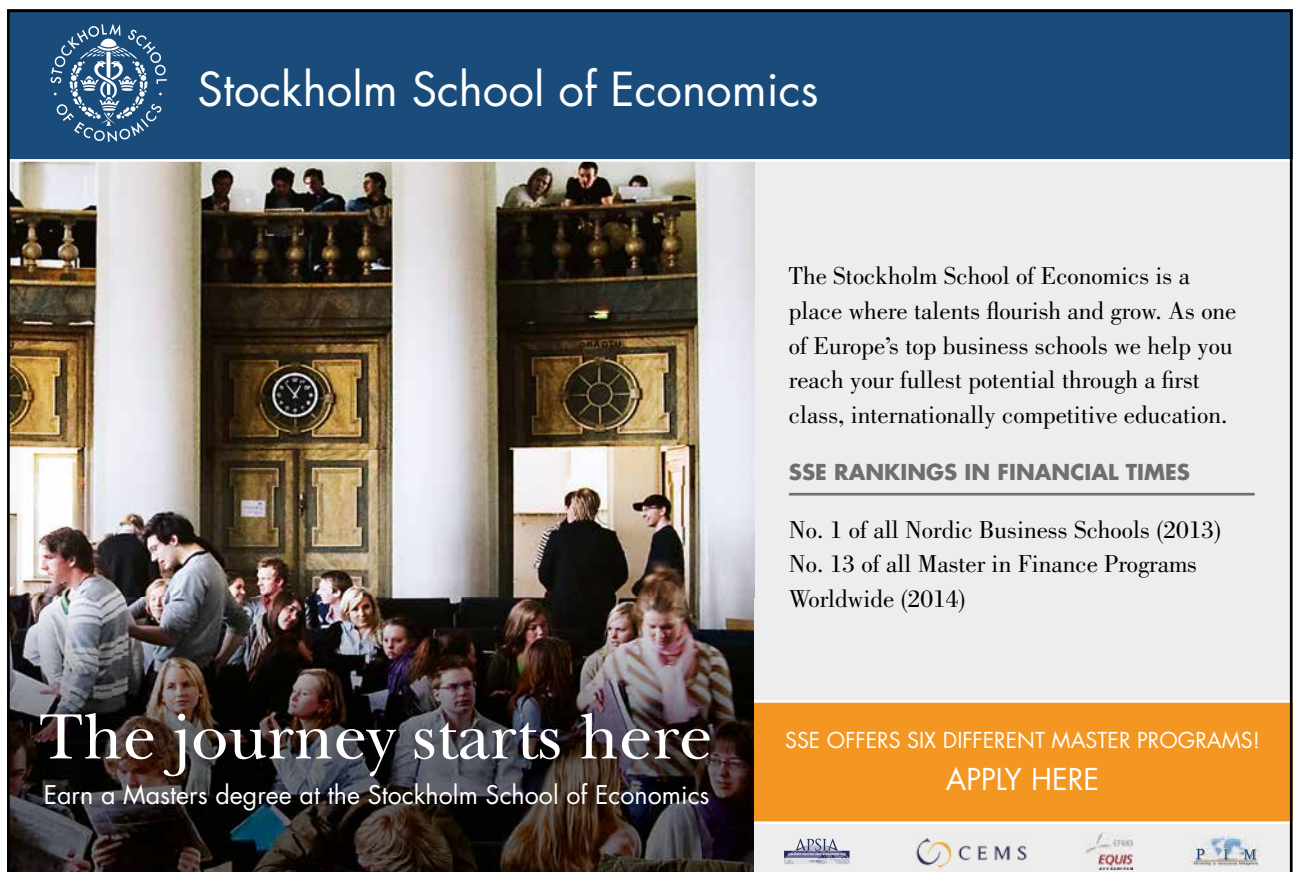
3.4 Video 10: How to prove the real part of a complex number is zero

[View this lesson on YouTube](#) [10]

Example.

Let $z \in \mathbb{C}$ with $|z| = 1$. Show

$$\Re\left(\frac{z-1}{z+1}\right) = 0.$$



The advertisement for the Stockholm School of Economics features a blue header with the school's logo and name. Below this is a large photograph of a historic building interior with students. The text 'The journey starts here' is overlaid on the photo, followed by 'Earn a Masters degree at the Stockholm School of Economics'. To the right, a text box describes the school as a top business school and lists its rankings. An orange banner at the bottom right encourages applying for master programs. Accreditation logos for APSIA, CEMS, EQUIS, and AACSB are at the bottom.

Stockholm School of Economics

The Stockholm School of Economics is a place where talents flourish and grow. As one of Europe's top business schools we help you reach your fullest potential through a first class, internationally competitive education.

SSE RANKINGS IN FINANCIAL TIMES

No. 1 of all Nordic Business Schools (2013)
No. 13 of all Master in Finance Programs Worldwide (2014)

**SSE OFFERS SIX DIFFERENT MASTER PROGRAMS!
APPLY HERE**

APSIA CEMS EQUIS AACSB



3.5 Video 11: Complex conjugate and linear systems

3.5.1 Solving systems of equations with the conjugate

[View this lesson on YouTube](#) [11]

Example.

Solve the following system for complex numbers z and w :

$$2z + 3w = 1 + 5i,$$

$$3\bar{z} - \bar{w} = 4 + 3i.$$

3.6 Video 12: When are the squares of z and its conjugate equal?

3.6.1 Showing real or imag parts are zero via the conjugate

[View this lesson on YouTube](#) [12]

Example.

Prove the following: For all $z \in \mathbb{C}$ we have

$$z^2 = \bar{z}^2$$

if and only if

$$\Re(z) = 0 \quad \text{or} \quad \Im(z) = 0.$$

3.7 Video 13: Conjugate of products is product of conjugates

[View this lesson on YouTube](#) [13]

Example.

Prove, for all complex numbers z and w :

$$\overline{zw} = \bar{z} \bar{w}.$$

3.8 Video 14: Why complex solutions appear in conjugate pairs

[View this lesson on YouTube](#) [14]

Example.

Let $z = \alpha + \beta i$ satisfy

$$ax^2 + bx + c = 0.$$

Show that $\bar{z}$ is also a solution.

4 How big are complex numbers?

4.1 Video 15: How big are complex numbers?

[View this lesson on YouTube](#) [15]

To measure how “big” certain complex numbers are, we introduce a way of measuring their size, known as the modulus or the magnitude.

Important idea (Modulus/magnitude of a complex number).

For a complex number $z = x + yi$ we define the modulus or magnitude of z by

$$|z| := \sqrt{x^2 + y^2}.$$

Geometrically, $|z|$ represents the length r of the line segment connecting z to the origin.

The advertisement features a central white box with the text "It's only an opportunity if you act on it" and the URL "IKEA.SE/STUDENT". Surrounding this box are several circular buttons with various messages: "Reduce re-use recycle", "WORK WITH US", "to gether ness", "save water. Shower together", "everyone deserves good design", and a button with a red lamp. The Ikea logo is in the bottom left corner. A small copyright notice "© Inter IKEA Systems B.V. 2009" is on the right side.

4.1.1 Properties of the modulus/magnitude

Important idea.

Let $z = a + bi$ and $w = c + di$. Some basic properties of the modulus are:-

$$|z| = \sqrt{a^2 + b^2} \geq 0;$$

$$|z| = 0 \quad \text{iff} \quad z = 0;$$

$$|z^2| = |z|^2;$$

$$|z + w| \leq |z| + |w|;$$

$$|\alpha z| = |\alpha||z| \text{ where } \alpha \text{ is a real number};$$

$$|zw| = |z||w|;$$

$$z\bar{z} = |z|^2.$$

Example.

If $z = 7 + i$ and $w = 3 - i$ then calculate:

$$|z + iw|.$$

Example.

If $w = 1 + 4i$ then calculate the following in Cartesian form $x + yi$:

$$|w + 2|.$$

We have

$$\begin{aligned} |w + 2| &= |3 + 4i| \\ &= \sqrt{3^2 + 4^2} \\ &= 5. \end{aligned}$$

4.2 Video 16: Modulus of a product is the product of moduli

[View this lesson on YouTube](#) [16]

Example.

Prove, for all complex numbers z and w :

$$|zw| = |z| |w|.$$

YOUR CHANCE
TO CHANGE
THE WORLD

Here at Ericsson we have a deep rooted belief that the innovations we make on a daily basis can have a profound effect on making the world a better place for people, business and society. Join us.

In Germany we are especially looking for graduates as Integration Engineers for

- Radio Access and IP Networks
- IMS and IPTV

We are looking forward to getting your application! To apply and for all current job openings please visit our web page: www.ericsson.com/careers

ericsson.
com



4.3 Video 17: Square roots of complex numbers

[View this lesson on YouTube](#) [17]

Example.

Solve

$$z^2 = (x + yi)^2 = -24 - 10i$$

for $z \in \mathbb{C}$ by computing the real numbers x and y . Hence write down the square roots of $-24 - 10i$.

4.4 Video 18: Quadratic equations with complex coefficients

4.4.1 Square roots of complex numbers

[View this lesson on YouTube](#) [18]

Example.

i) Solve

$$z^2 = (x + yi)^2 = 15 + 8i$$

for $z \in \mathbb{C}$ by computing x and y which are assumed to be integers.

Hence write down the square roots of $15 + 8i$.

ii) Hence solve, in $x + yi$ form,

$$z^2 - (2 + 3i)z - 5 + i = 0.$$

4.5 Video 19: Show real part of complex number is zero

[View this lesson on YouTube](#) [19]

Example.

Let $z \in \mathbb{C}$ with $z \neq i$. If $|z| = 1$ then show

$$\Re\left(\frac{z+i}{z-i}\right) = 0.$$

I joined MITAS because
I wanted **real responsibility**

The Graduate Programme
for Engineers and Geoscientists
www.discovermitas.com



Month 16

I was a construction
supervisor in
the North Sea
advising and
helping foremen
solve problems

Real work
International opportunities
Three work placements



 **MAERSK**



Click on the ad to read more

5 Polar trig form

5.1 Video 20: Polar trig form of complex number

[View this lesson on YouTube](#) [20]

Instead of the Cartesian $x + yi$ form, sometimes it is convenient to express complex numbers in other equivalent forms.

Using trigonometry in the complex plane we see that we can express any (non-zero) complex number z in the form

$$z = r(\cos \theta + i \sin \theta)$$

where r is the distance to the origin and θ is the angle to the pos. real axis.

Important idea (Formulae for polar trig form).

For $z = x + yi$ a polar trig form is $z = r(\cos \theta + i \sin \theta)$ where:

$$r = \sqrt{x^2 + y^2} = |z|;$$

$$x = r \cos \theta, \quad y = r \sin \theta, \quad \tan \theta = y/x.$$

We denote the angle θ by $\arg(z)$ and call $\arg(z)$ “an argument of z ”.

Because $\cos \theta = \cos(\theta + 2k\pi)$ and $\sin \theta = \sin(\theta + 2k\pi)$ for all integers k , the angle θ associated with a complex number is not unique.

For example, if $z = 1 + i$ then we may represent z in polar trig form via

$$z = \sqrt{2}(\cos(\pi/4) + i \sin(\pi/4))$$

and

$$z = \sqrt{2}(\cos(9\pi/4) + i \sin(9\pi/4)).$$

Thus, $\theta = \arg(z)$ is not uniquely determined by z .

To provide some definiteness, we define what is known as the principal argument of z .

Important idea ($\arg(z)$ versus $\text{Arg}(z)$).

For any complex number $z = x + yi$ with $\theta = \arg(z)$ we can always choose an integer k such that $-\pi < \arg(z) - 2k\pi \leq \pi$. We denote this special angle by $\text{Arg}(z)$ and call $\text{Arg}(z)$ “the principal argument of z ”.

SIMPLY CLEVER

ŠKODA



We will turn your CV into
an opportunity of a lifetime



Do you like cars? Would you like to be a part of a successful brand?
We will appreciate and reward both your enthusiasm and talent.
Send us your CV. You will be surprised where it can take you.

Send us your CV on
www.employerforlife.com



Click on the ad to read more

6 Polar exponential form

6.1 Video 21: Polar exponential form of a complex number

[View this lesson on YouTube](#) [21]

Instead of the Cartesian form $z = x + yi$ or the polar trig form $z = r(\cos \theta + i \sin \theta)$ sometimes it is convenient for multiplication and solving polynomials to express complex numbers in yet another equivalent form

$$z = re^{i\theta}.$$

Important idea (Formula for polar exponential form $z = re^{i\theta}$).

For $z = x + yi$ a polar exponential form is $z = re^{i\theta}$ where:

$$r = \sqrt{x^2 + y^2} \text{ and } \tan \theta = y/x.$$

If we combine the polar exponential form with the polar trig form then we obtain a special identity called “Euler’s formula”

$$e^{i\theta} = \cos \theta + i \sin \theta$$

and if $\theta = \pi$ then we obtain the famous formula

$$e^{\pi i} = -1.$$

Because $\cos \theta = \cos(\theta + 2k\pi)$ and $\sin \theta = \sin(\theta + 2k\pi)$ for all integers k , the angle θ associated with a complex number is not unique.

For example, if $z = 1 + i$ then we may represent z in polar trig and polar exp. form via

$$z = \sqrt{2}(\cos(\pi/4) + i \sin(\pi/4)) = \sqrt{2}e^{i\pi/4}$$

and

$$z = \sqrt{2}(\cos(9\pi/4) + i \sin(9\pi/4)) = \sqrt{2}e^{i9\pi/4}.$$

Thus, $\theta = \arg(z)$ is not uniquely determined by z .

To provide some definiteness, we define what is known as the principal argument of z .

Important idea ($\arg(z)$ versus $\text{Arg}(z)$).

For any complex number $z = x + yi$ with $\theta = \arg(z)$ we can always choose an integer k such that $-\pi < \arg(z) - 2k\pi \leq \pi$. We denote this special angle by $\text{Arg}(z)$ and call it “the principal argument of z ”.

6.2 Revision Video 22: Intro to complex numbers + basic operations

[View this lesson on YouTube](#) [22]

Example.

Let $z := 2e^{i\pi/6}$. Calculate: z^3 ; z^{-1} ; and $-3z$. In addition, plot your calculated complex numbers on the same Argand diagram.

Budget-Friendly. Knowledge-Rich.

The Agilent InfiniiVision X-Series and 1000 Series offer affordable oscilloscopes for your labs. Plus resources such as lab guides, experiments, and more, to help enrich your curriculum and make your job easier.



Scan for free
Agilent iPhone
Apps or visit
qrs.ly/po20pli



See what Agilent can do for you.
www.agilent.com/find/EducationKit

© Agilent Technologies, Inc. 2012

u.s. 1-800-829-4444 canada: 1-877-894-4414

Anticipate — Accelerate — Achieve



Agilent Technologies



Click on the ad to read more

6.3 Revision Video 23: Complex numbers and calculations

[View this lesson on YouTube](#) [23]

Example.

Define the complex numbers z and w by $z := 2 - 5i$ and $w = 1 + 2i$. Calculate:

$$\frac{1 + 7i}{w}; \quad 4\bar{z}w; \quad \text{Arg}(w - 3i).$$

6.4 Video 24: Powers of complex numbers via polar forms

6.4.1 Calculations with the polar exponential form

[View this lesson on YouTube](#) [24]

Example.

If $z = 2e^{5\pi i/6}$ then compute z^2 , $1/z$ and $\Im(z)$. Plot z , z^2 and $1/z$ in the same complex plane.

7 Powers of complex numbers

7.1 Video 25: Powers of complex numbers

[View this lesson on YouTube](#) [25]

Example.

Powers of complex numbers If $z = -1 + i\sqrt{3}$ then:

- a) Calculate a polar exponential form of z ;
- b) Hence determine $\text{Arg}(z^{23})$ and write z^{23} in Cartesian form.

With us you can
shape the future.
Every single day.

For more information go to:
www.eon-career.com

Your energy shapes the future.

e-on



7.2 Video 26: What is the power of a complex number?

[View this lesson on YouTube](#) [26]

Example.

Suppose $z = 1 + i$, $w = 1 - i\sqrt{3}$. If

$$q := z^6/w^5$$

then:

- a) Calculate $|q|$;
- b) Determine $\text{Arg}(q)$.

7.3 Video 27: Roots of complex numbers

[View this lesson on YouTube](#) [27]

Example.

Solve

$$z^5 = 16(1 - i\sqrt{3})$$

leaving your answers in simplified polar exponential form.

7.4 Video 28: Complex numbers solutions to polynomial equations

[View this lesson on YouTube](#) [28]

Example.

Determine all of the (complex) fourth roots of $8(-1 + \sqrt{3}i)$. You may leave your answer in polar form.



Giving
"turning the page"
a new meaning

Download free e-books on the go
Turn To Bookboon.com

bookboon.com

GAUTRAIN
FOR PEOPLE ON THE MOVE

7.5 Video 29: Complex numbers and $\tan(\pi/12)$

[View this lesson on YouTube](#) [29]

Example.

If $z = -2 + 2i$ and $w = -1 - i\sqrt{3}$ then:

- a) Compute zw in Cartesian form;
- b) Rewrite z and w in polar exponential form and thus calculate zw in polar exponential form;
- c) Hence determine a precise value for $\tan(\pi/12)$.

7.6 Video 30: Euler's formula: A cool proof

[View this lesson on YouTube](#) [30]

Important idea (Euler's formula).

We prove

$$e^{i\theta} = \cos \theta + i \sin \theta.$$

Let $f(\theta) := \cos \theta + i \sin \theta$. Thus, $f(0) = 1$. Differentiating f we obtain

$$\begin{aligned} f'(\theta) &= -\sin \theta + i \cos \theta \\ &= i^2 \sin \theta + i \cos \theta \\ &= i(\cos \theta + i \sin \theta) \\ &= if(\theta). \end{aligned}$$

We have formed a differential equation/initial value problem. Note that $g(\theta) := e^{i\theta}$ also satisfies the IVP. By uniqueness of solutions, $f \equiv g$, that is,

$$e^{i\theta} = \cos \theta + i \sin \theta.$$

This also means that the polar exponential form $re^{i\theta}$ is an accurate representation of any complex number z .

8 De Moivre's formula

8.1 Video 31: De Moivre's formula: A cool proof

[View this lesson on YouTube](#) [31]

De Moivre's formula is useful for simplifying computations involving powers of complex numbers.

Important idea (De Moivre's formula).

For each integer n and all real θ we have

$$(\cos \theta + i \sin \theta)^n = (\cos n\theta + i \sin n\theta).$$

The proof utilizes Euler's formula

$$e^{i\theta} = \cos \theta + i \sin \theta.$$

We have,

$$\begin{aligned} (\cos \theta + i \sin \theta)^n &= (e^{i\theta})^n \\ &= e^{in\theta} \\ &= (\cos n\theta + i \sin n\theta) \end{aligned}$$

and thus we have proven the result.

8.2 Video 32: Trig identities from De Moivre's theorem

[View this lesson on YouTube](#) [32]

Example.

Write $\cos 5\theta$ in terms of $\cos \theta$ by applying De Moivre's theorem.



Nido

Luxurious accommodation

Central zone 1 & 2 locations

Meet hundreds of international students

BOOK NOW and get a £100 voucher from voucherexpress

Nido Student Living - London

Visit www.NidoStudentLiving.com/Bookboon for more info.

+44 (0)20 3102 1060



8.3 Video 33: Trig identities: De Moivre's formula

[View this lesson on YouTube](#) [33]

Example.

Write $\sin 4\theta$ in terms of $\cos \theta$ and $\sin 4\theta$ by applying De Moivre's theorem. Hence, write $\sin 4\theta \cos \theta$ as a function of $\sin 4\theta$.

9 Connecting sin, cos with e

9.1 Video 34: Trig identities and Euler's formula

[View this lesson on YouTube](#) [34]

9.1.1 More connections between $\sin \theta$, $\cos \theta$, $e^{i\theta}$

Euler's formula

$$e^{i\theta} = \cos \theta + i \sin \theta$$

can be manipulated to obtain the following identities

Important idea (Trig functions in terms of exponentials).

$$\cos \theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$$
$$\sin \theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}.$$

For example, consider

$$e^{-i\theta} = \cos(-\theta) + i \sin(-\theta) = \cos \theta - i \sin \theta$$

and so $e^{i\theta} + e^{-i\theta} = 2 \cos \theta$, which rearranges to the first identity.

9.1.2 Trig identities from Euler's formula

Example.

Apply the identity

$$\sin \theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$$

to express $\sin^4 \theta$ in terms of $\cos \theta, \cos 2\theta, \dots$.



Brain power

By 2020, wind could provide one-tenth of our planet's electricity needs. Already today, SKF's innovative know-how is crucial to running a large proportion of the world's wind turbines.

Up to 25 % of the generating costs relate to maintenance. These can be reduced dramatically thanks to our systems for on-line condition monitoring and automatic lubrication. We help make it more economical to create cleaner, cheaper energy out of thin air.

By sharing our experience, expertise, and creativity, industries can boost performance beyond expectations. Therefore we need the best employees who can meet this challenge!

The Power of Knowledge Engineering

Plug into The Power of Knowledge Engineering.
Visit us at www.skf.com/knowledge

SKF

9.2 Video 35: Trig identities from Euler's formula

[View this lesson on YouTube](#) [35]

Example.

Apply the identity

$$\sin \theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$$

to express $\sin^5 \theta$ in terms of $\sin \theta$, $\sin 2\theta$, $\dots$.

9.3 Video 36: How to prove trig identities WITHOUT trig!

[View this lesson on YouTube](#) [36]

Example.

Prove

$$\sin(x + y) = \sin x \cos y + \cos x \sin y.$$

9.4 Revision Video 37: Complex numbers + trig identities

[View this lesson on YouTube](#) [37]

The problem for this video is similar to Video 35.



Working at SimCorp means making a difference. At SimCorp, you help create the tools that shape the global financial industry of tomorrow. SimCorp provides integrated software solutions that can turn investment management companies into winners. With SimCorp, you make the most of your ambitions, realising your full potential in a challenging, empowering and stimulating work environment.

Are you among the best qualified in finance, economics, computer science or mathematics?

Find your next challenge at www.simcorp.com/careers

Mitigate risk | Reduce cost | Enable growth
simcorp.com



"When I joined SimCorp, I was very impressed with the introduction programme offered to me."

Meet Lars and other employees at simcorp.com/meetouremployees

 **SimCorp**



10 Regions in the complex plane

10.1 Video 38: How to determine regions in the complex plane

[View this lesson on YouTube](#) [38]

10.1.1 Regions in the complex plane

We can use equations or inequalities to represent regions within two-dimensional space.

With a bit of care, we can also represent regions in the complex plane via similar techniques.

We know that the modulus $|z|$ of any complex number z is the length of the line segment joining z to the origin. Thus, the set

$$\{z \in \mathbb{C} : |z| < 3\}$$

is the set of all complex numbers, whose distance to the origin is less than three units. This is an open disc, centred at the origin, with radius three.

Similarly, the set

$$\{z \in \mathbb{C} : |z - (2 + i)| < 3\}$$

is the set of all complex numbers, whose distance to $2 + i$ is less than three units. This is an open disc, centred at the $2 + i$, with radius three.

Similarly, the set

$$\{z \in \mathbb{C} : |z - i| = 3\}$$

is the set of all complex numbers, whose distance to i is exactly three units. This is a circle, centered at the i , with radius three.

The set

$$\{z \in \mathbb{C} : |z - 2| = |z - 4|\}$$

is the set of all complex numbers, whose distance to 2 and 4 are equal. This is a vertical line, passing through 3.

Also

$$\{z \in \mathbb{C} : 0 \leq \text{Arg}(z) \leq \pi/2\}$$

is the set of all complex numbers, whose principal argument is between zero and $\pi/2$. This is all those points that lie in the first quadrant, covered by a quarter-turn in the anticlockwise direction about the origin.

10.1.2 Regions in the complex plane

Example.

Determine and sketch the set of points satisfying

$$\{z \in \mathbb{C} : |z + 4| = 2|z - i|\}.$$



What do you want to do?

No matter what you want out of your future career, an employer with a broad range of operations in a load of countries will always be the ticket. Working within the Volvo Group means more than 100,000 friends and colleagues in more than 185 countries all over the world. We offer graduates great career opportunities – check out the Career section at our web site www.volvogroup.com. We look forward to getting to know you!

VOLVO
AB Volvo (publ)
www.volvogroup.com

VOLVO TRUCKS | RENAULT TRUCKS | MACK TRUCKS | VOLVO BUSES | VOLVO CONSTRUCTION EQUIPMENT | VOLVO PENTA | VOLVO AERO | VOLVO IT
VOLVO FINANCIAL SERVICES | VOLVO 3P | VOLVO POWERTRAIN | VOLVO PARTS | VOLVO TECHNOLOGY | VOLVO LOGISTICS | BUSINESS AREA ASIA

10.2 Video 39: Circular sector in the complex plane

10.2.1 Regions in the complex plane

[View this lesson on YouTube](#) [39]

Example.

Determine and sketch the set of points satisfying

$$|z - 1 - i| < 3, \quad 0 < \operatorname{Arg}(z) < \pi/4.$$

10.3 Video 40: Circle in the complex plane

10.3.1 Regions in the complex plane

[View this lesson on YouTube](#) [40]

Example.

Determine and sketch the set of points satisfying

$$|z + 3| = 2|z - 6i|.$$

10.4 Video 41: How to sketch regions in the complex plane

[View this lesson on YouTube](#) [41]

Example.

Sketch the region in the complex plane dened by all those complex numbers z such that

$$|z - 2i| < 1, \quad \text{and} \quad 0 < \text{Arg}(z - 2i) \leq \frac{3\pi}{4}.$$



"I studied English for 16 years but...
...I finally learned to speak it in just six lessons"

Jane, Chinese architect

ENGLISH OUT THERE

Click to hear me talking before and after my unique course download

11 Complex polynomials

11.1 Video 42: How to factor complex polynomials

[View this lesson on YouTube](#) [42]

Important idea.

The basic theory for complex polynomials of degree n

$$p(z) := a_n z^n + a_{n-1} z^{n-1} + \cdots + a_1 z + a_0$$

may be summarized as follows:-

- Every polynomial $p(z)$ of degree n has at least one root over $\mathbb{C}$. That is, there is at least one α such that $p(\alpha) = 0$.
- The roots of complex polynomials with **real** coefficients appear in conjugate pairs.
- If $p(\alpha) = 0$ for some number α then $(z - \alpha)$ is a factor of $p(z)$.
- Every polynomial of degree n can be factored into n linear parts. That is

$$p(z) = a_n(z - \alpha_1)(z - \alpha_2) \cdots (z - \alpha_n)$$

where the α_i are the roots of $p(z)$.

11.1.1 Complex polynomials with real coefficients

Example.

- a) Solve $p(z) := z^6 + 64 = 0$.
- b) Hence factorize $p(z)$ into linear factors.

11.2 Video 43: Factorizing complex polynomials

11.2.1 Complex polynomials with real coefficients

[View this lesson on YouTube](#) [43]

Example.

If $p(z) := 2z^4 - 5z^3 + 5z^2 - 20z - 12$ then:

- a) Show $p(2i) = 0$;
- b) Illustrate that $z^2 + 4$ is a factor of $p(z)$ (without division) and also find the other quadratic factor;
- c) Thus, factorize $p(z)$ into quadratic factors.

This e-book
is made with
SetaPDF



PDF components for PHP developers

www.setasign.com



11.3 Video 44: Factor polynomials into linear parts

11.3.1 Complex polynomials with real coefficients

[View this lesson on YouTube](#) [44]

Example.

- a) Solve $p(z) := z^7 + 3^7 = 0$.
- b) Hence factorize $p(z)$ into linear factors.

11.4 Video 45: Complex linear factors

11.4.1 Complex polynomials with real coefficients

[View this lesson on YouTube](#) [45]

Example.

If $p(z) := z^5 + 4z^3 - 8z^2 - 32$ then:

- a) Show $p(2i) = 0$;
- b) Illustrate that $z^2 + 4$ is a factor of $p(z)$ (without division) and also find the other quadratic factor;
- c) Thus, factorize $p(z)$ into complex linear factors.

Bibliography

1. Tisdell, Chris. Complex numbers are AWESOME. Streamed live on 02/04/2014 and accessed on 14/08/2014. Available on Dr Chris Tisdell's YouTube channel, <https://www.youtube.com/watch?v=YdBALaKYCO4&index=1&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP>
2. Tisdell, Chris. How to add and subtract complex numbers. Streamed live on 03/04/2014 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, <https://www.youtube.com/watch?v=nj3qJY4QO6U&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP&index=2>
3. Tisdell, Chris. Scalar multiply a complex number. Streamed live on 03/04/2014 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, <https://www.youtube.com/watch?v=MNQPu6BQ9Ok&index=3&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP>
4. Tisdell, Chris. How to multiply complex numbers. Streamed live on 03/04/2014 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, <https://www.youtube.com/watch?v=Kt11OMjXC6I&index=4&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP>
5. Tisdell, Chris. How to divide complex numbers. Streamed live on 03/04/2014 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, https://www.youtube.com/watch?v=fa7DVp_oNFE&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP&index=5

gaiteye®
Challenge the way we run

**EXPERIENCE THE POWER OF
FULL ENGAGEMENT...**

.....

**RUN FASTER.
RUN LONGER..
RUN EASIER...**

**READ MORE & PRE-ORDER TODAY
WWW.GAITEYE.COM**

6. Tisdell, Chris. Complex numbers: Quadratic formula. Streamed live on 19/04/2014 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, <https://www.youtube.com/watch?v=iNzVgErnf5w&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP&index=6>
7. Tisdell, Chris. What is the complex conjugate? Streamed live on 19/04/2014 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, <https://www.youtube.com/watch?v=C8LzaBikty8&index=7&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP>
8. Tisdell, Chris. Calculations with the complex conjugate. Streamed live on 19/04/2014 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, <https://www.youtube.com/watch?v=WlqTBPp7sRM&index=8&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP>
9. Tisdell, Chris. How to show a number is purely imaginary. Streamed live on 19/04/2014 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, https://www.youtube.com/watch?v=75D__m6q5JM&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP&index=9
10. Tisdell, Chris. Complex numbers: example of how to prove the real part of a complex number is zero. Streamed live on 25/11/2008 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, <https://www.youtube.com/watch?v=QWbLhUZ6bag&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP&index=10>
11. Tisdell, Chris. Complex conjugates and linear systems. Streamed live on 19/04/2014 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, <https://www.youtube.com/watch?v=0s8XntqBrkc&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP&index=11>
12. Tisdell, Chris. When are the squares of z and its conjugate equal? Streamed live on 19/04/2014 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, <https://www.youtube.com/watch?v=U7d0NgvctMk&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP&index=12>
13. Tisdell, Chris. Conjugate of products is product of conjugates. Streamed live on 20/04/2014 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, https://www.youtube.com/watch?v=hKe4s_6B0Qs&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP&index=13
14. Tisdell, Chris. Why complex solutions appear in conjugate pairs. Uploaded on 16/04/2014 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, <https://www.youtube.com/watch?v=XkWz76dxkkI&index=14&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP>
15. Tisdell, Chris. How big are complex numbers? Streamed live on 20/04/2014 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, <https://www.youtube.com/watch?v=NyPGV066MCM&index=15&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP>

16. Tisdell, Chris. Modulus of a product is the product of moduli. Streamed live on 20/04/2014 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, <https://www.youtube.com/watch?v=siePZ8yJFJU&index=16&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP>
17. Tisdell, Chris. Square roots of complex numbers. Streamed live on 20/04/2014 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, <https://www.youtube.com/watch?v=HQ3lqtRSo-k&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP&index=17>
18. Tisdell, Chris. Quadratic equations with complex coecients. Streamed live on 20/04/2014 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, <https://www.youtube.com/watch?v=PQi-LrSWoUM&index=18&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP>
19. Tisdell, Chris. Show real part of a complex number is zero. Streamed live on 21/04/2014 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, <https://www.youtube.com/watch?v=i8z5fDHm0JY&index=19&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP>
20. Tisdell, Chris. Polar trig form of a complex number. Streamed live on 21/04/2014 and accessed on 14/08/2014. Available on Dr Chris Tisdell's YouTube channel, <https://www.youtube.com/watch?v=B7jT9AHJrDo&index=20&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP>
21. Tisdell, Chris. Polar exponential form of a complex number. Streamed live on 21/04/2014 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, <https://www.youtube.com/watch?v=2ryt4n5WDnU&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP&index=21>
22. Tisdell, Chris. Intro to complex numbers + basic operations. Uploaded on 08/09/2010 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, <https://www.youtube.com/watch?v=QeMSqlrgQYg&index=22&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP>
23. Tisdell, Chris. Complex numbers and calculations. Uploaded on 06/09/2010 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, <https://www.youtube.com/watch?v=0JYIh8Goblg&index=23&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP>
24. Tisdell, Chris. Powers of complex numbers via polar forms. Streamed live on 22/04/2014 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, <https://www.youtube.com/watch?v=FtXPMSHBKgc&index=24&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP>
25. Tisdell, Chris. Powers of complex numbers. Streamed live on 22/04/2014 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, https://www.youtube.com/watch?v=P_sFeTtnQPs&index=25&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP

26. Tisdell, Chris. What is the power of a complex number? Streamed live on 22/04/2014 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, <https://www.youtube.com/watch?v=nZPn74GC3KM&index=26&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP>
27. Tisdell, Chris. Roots of complex numbers. Streamed live on 22/04/2014 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, <https://www.youtube.com/watch?v=RmUazwwRqso&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP&index=27>
28. Tisdell, Chris. Complex number solutions to polynomial equations. Uploaded on 08/09/2010 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, <https://www.youtube.com/watch?v=Y4btmS-uHWI&index=28&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP>
29. Tisdell, Chris. Complex numbers and $\tan(\pi/12)$ Streamed live on 22/04/2014 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, <https://www.youtube.com/watch?v=N5gRg2whooM&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP&index=29>
30. Tisdell, Chris. Euler's formula: a cool proof. Streamed live on 02/12/2014 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, <https://www.youtube.com/watch?v=stOZL05Nvyk&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP&index=30>
31. Tisdell, Chris. De Moivre's formula: a COOL proof. Streamed live on 23/04/2014 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, https://www.youtube.com/watch?v=NjYZS_XYLEQ&index=31&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP



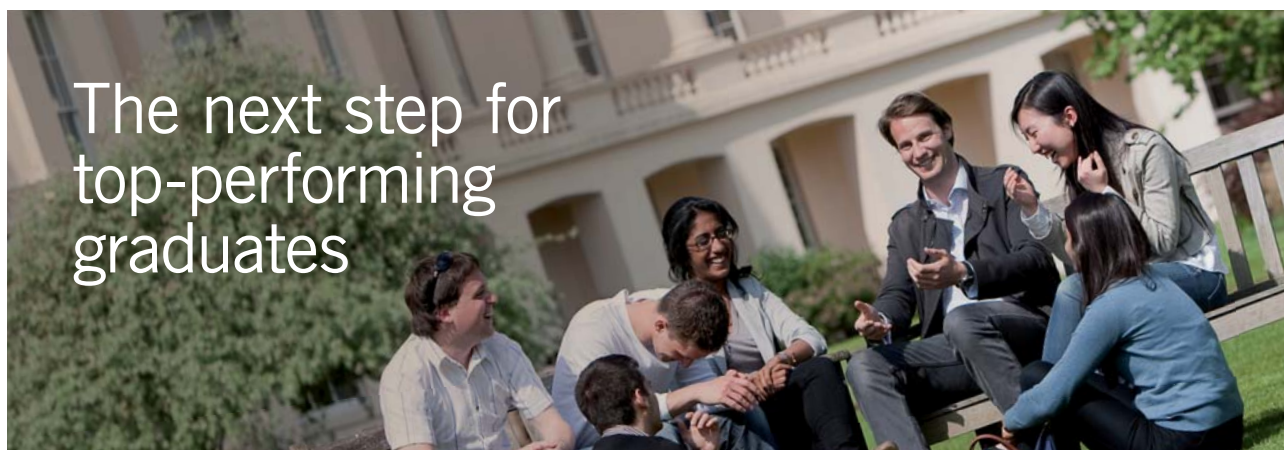
Giving
"turning the page"
a new meaning

Download free e-books on the go
Turn To Bookboon.com

 **bookboon.com**

32. Tisdell, Chris. Application of De Moivre's theorem. Streamed live on 23/04/2014 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, <https://www.youtube.com/watch?v=JjECulRsKr8&index=32&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP>
33. Tisdell, Chris. Trig identities: De Moivre's formula. Streamed live on 23/04/2014 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, <https://www.youtube.com/watch?v=uAj1zb1p0gg&index=33&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP>
34. Tisdell, Chris. Trig identities and Euler's formula. Streamed live on 23/04/2014 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, <https://www.youtube.com/watch?v=Bd22Y6NvKZk&index=34&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP>
35. Tisdell, Chris. Euler's formula and trig identities. Streamed live on 23/04/2015 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, <https://www.youtube.com/watch?v=NSYYWhUpeqs&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP&index=35>
36. Tisdell, Chris. How to prove trig identities WITHOUT trig. Streamed live on 11/12/2013 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, <https://www.youtube.com/watch?v=RGnvGjFfjBs&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP&index=36>
37. Tisdell, Chris. Complex numbers + trig identities. Uploaded on 08/09/2010 and accessed on 14/08/2014. Available on Dr Chris Tisdell's YouTube channel, <https://www.youtube.com/watch?v=CNmK48GOCuc&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP&index=37>
38. Tisdell, Chris. How to determine regions in the complex plane. Streamed live on 26/04/2015 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, https://www.youtube.com/watch?v=0vjsF_n-DBs&index=38&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP
39. Tisdell, Chris. Circular sector in the complex plane. Streamed live on 26/04/2014 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, https://www.youtube.com/watch?v=_2Z3qbhf8c&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP&index=39
40. Tisdell, Chris. Circle in the complex plane. Streamed live on 26/04/2014 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, <https://www.youtube.com/watch?v=sLkdqTg1-1c&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP&index=40>
41. Tisdell, Chris. How to sketch regions in the complex plane. Uploaded on 08/09/2010 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, <https://www.youtube.com/watch?v=8gtnZ5xSLuE&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP&index=41>
42. Tisdell, Chris. How to factor complex polynomials. Streamed live on 01/05/2014 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, <https://www.youtube.com/watch?v=UG3TtIPTVZE&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP&index=42>

43. Tisdell, Chris. Factorizing complex polynomials. Streamed live on 01/05/2014 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, https://www.youtube.com/watch?v=r_h_10ovGU0&index=43&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP
44. Tisdell, Chris. Factor polynomials into linear parts. Streamed live on 02/05/2014 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, <https://www.youtube.com/watch?v=ebrLfGRLfBc&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP&index=44>
45. Tisdell, Chris. Complex linear factors of polynomials. Streamed live on 02/05/2014 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, <https://www.youtube.com/watch?v=9r1MSXG4ENw&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP&index=45>
46. Tisdell, Chris. Engineering mathematics YouTube workbook playlist <http://www.youtube.com/playlist?list=PL13760D87FA88691D>, accessed on 1/11/2011 at DrChrisTisdell's YouTube Channel <http://www.youtube.com/DrChrisTisdell>.



Masters in Management



Designed for high-achieving graduates across all disciplines, London Business School's Masters in Management provides specific and tangible foundations for a successful career in business.

This 12-month, full-time programme is a business qualification with impact. In 2010, our MiM employment rate was 95% within 3 months of graduation*; the majority of graduates choosing to work in consulting or financial services.

As well as a renowned qualification from a world-class business school, you also gain access to the School's network of more than 34,000 global alumni – a community that offers support and opportunities throughout your career.

For more information visit www.london.edu/mm, email mim@london.edu or give us a call on +44 (0)20 7000 7573.

* Figures taken from London Business School's Masters in Management 2010 employment report

