Module 3

SELF-TEST 1

1.

$$\begin{vmatrix} (2-\lambda) & 1 & 1 \\ 1 & (3-\lambda) & 2 \\ -1 & 1 & (2-\lambda) \end{vmatrix} = 0$$

$$(2-\lambda)\{(3-\lambda)(2-\lambda) - 2\} - \{(2-\lambda) + 2\} + \{1 + (3-\lambda)\} = 0$$

$$(2-\lambda)\{6 - 5\lambda + \lambda^2 - 2\} - \{4 - \lambda\} + \{4 - \lambda\} = 0$$

$$(2-\lambda)\{\lambda^2 - 5\lambda + 4\} = 0$$

$$(2-\lambda)(\lambda - 4)(\lambda - 1) = 0$$

$$\therefore \quad \lambda_1 = 2 \qquad \lambda_2 = 4 \qquad \lambda_3 = 1$$

For $\lambda_1 = 2$:

$$\begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ -1 & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 2 \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$2x_1 + x_2 + x_3 = 2x_1$$

$$x_1 + 3x_2 + 2x_3 = 2x_2$$

$$-1x_1 + x_2 + 2x_3 = 2x_3$$

$$\Rightarrow \quad x_1 = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

For $\lambda_2 = 4$: $\quad x_2 = \begin{bmatrix} 3 \\ 5 \\ 1 \end{bmatrix}$

For $\lambda_3 = 1$: $\quad x_3 = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$

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2.

$$\begin{vmatrix} (1-\lambda) & 2 & 2 \\ 1 & (3-\lambda) & 1 \\ 2 & 2 & (1-\lambda) \end{vmatrix} = 0$$

$$(1-\lambda)\{(3-\lambda)(1-\lambda) - 2\} - 2\{(1-\lambda) - 2\} + 2\{2 - 2(3-\lambda)\} = 0$$

$$(1-\lambda)\{3 - 4\lambda + \lambda^2 - 2\} - 2\{-1 - \lambda\} + 2\{-4 + 2\lambda\} = 0$$

$$(1-\lambda)\{\lambda^2 - 4\lambda + 1\} + 2 + 2\lambda - 8 + 4\lambda = 0$$

$$(1-\lambda)\lambda(\lambda - 4) + 1 - \lambda - 6 + 6\lambda = 0$$

$$(1-\lambda)\lambda(\lambda - 4) + 5\lambda - 5 = 0$$

$$\lambda(1-\lambda)(\lambda - 4) - 5(1-\lambda) = 0$$

$$(1-\lambda)(\lambda^2 - 4\lambda - 5) = 0$$

$$(1-\lambda)(\lambda - 5)(\lambda + 1) = 0$$

$$\therefore \quad \lambda_1 = 1 \qquad \lambda_2 = -1 \qquad \lambda_3 = 5$$

For $\lambda_1 = 1$:
$$\begin{bmatrix} 1 & 2 & 2 \\ 1 & 3 & 1 \\ 2 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 1 \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$x_1 + 2x_2 + 2x_3 = x_1$$

$$x_1 + 3x_2 + x_3 = x_2$$

$$2x_1 + 2x_2 + x_3 = x_3$$

$$\Rightarrow \quad x_1 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

For $\lambda_2 = -1$:
$$x_2 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

For $\lambda_3 = 5$:
$$x_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

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3.

$$\begin{vmatrix} (2-\lambda) & 0 & 1 \\ -1 & (4-\lambda) & -1 \\ -1 & 2 & (0-\lambda) \end{vmatrix} = 0$$

$$(2-\lambda)\{(4-\lambda)(-\lambda) + 2\} + \{-2 + (4-\lambda)\} = 0$$

$$(2-\lambda)\{-4\lambda + \lambda^2 + 2\} + (2-\lambda) = 0$$

$$(2-\lambda)\{\lambda^2 - 4\lambda + 2 + 1\} = 0$$

$$(2-\lambda)(\lambda^2 - 4\lambda + 3) = 0$$

$$(2-\lambda)(\lambda - 3)(\lambda - 1) = 0$$

$$\therefore \lambda_1 = 2 \qquad \lambda_2 = 3 \qquad \lambda_3 = 1$$

For $\lambda_1 = 2$:

$$\begin{bmatrix} 2 & 0 & 1 \\ -1 & 4 & -1 \\ -1 & 2 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 2 \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\Rightarrow \quad x_1 = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$$

For $\lambda_2 = 3$: $\quad x_2 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$

For $\lambda_3 = 1$: $\quad x_3 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

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4.

$$\begin{vmatrix} (1-\lambda) & -4 & -2 \\ 0 & (3-\lambda) & 1 \\ 1 & 2 & (4-\lambda) \end{vmatrix} = 0$$

$$(1-\lambda)\{(3-\lambda)(4-\lambda) - 2\} + 4\{-1\} - 2\{-(3-\lambda)\} = 0$$

$$(1-\lambda)\{12 - 7\lambda + \lambda^2 - 2\} - 4 + 2\{3-\lambda\} = 0$$

$$(1-\lambda)\{\lambda^2 - 7\lambda + 10\} - 4 + 6 - 2\lambda = 0$$

$$(1-\lambda)(\lambda^2 - 7\lambda + 10) + 2(1-\lambda) = 0$$

$$(1-\lambda)(\lambda^2 - 7\lambda + 12) = 0$$

$$(1-\lambda)(\lambda-4)(\lambda-3) = 0$$

$$\therefore \quad \lambda_1 = 1 \qquad \lambda_2 = 4 \qquad \lambda_3 = 3$$

For $\lambda_1 = 1$:

$$\begin{bmatrix} 1 & -4 & -2 \\ 0 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 1 \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$x_1 - 4x_2 - 2x_3 = x_1$$

$$3x_2 + x_3 = x_2$$

$$x_1 + 2x_2 + 4x_3 = x_3$$

$$\Rightarrow \quad x_1 = \begin{bmatrix} 4 \\ 1 \\ -2 \end{bmatrix}$$

For $\lambda_2 = 4$:
$$x_2 = \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$$

For $\lambda_3 = 3$:
$$x_3 = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$$

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5.

$$\begin{vmatrix} (3-\lambda) & 0 & 3 \\ 0 & (3-\lambda) & 3 \\ 2 & 3 & (1-\lambda) \end{vmatrix} = 0$$

$$(3-\lambda)\{(3-\lambda)(1-\lambda) - 9\} + 3\{-2(3-\lambda)\} = 0$$

$$(3-\lambda)\{3 - 4\lambda + \lambda^2 - 9\} - 6\{3-\lambda\} = 0$$

$$(3-\lambda)\{\lambda^2 - 4\lambda - 12\} = 0$$

$$(3-\lambda)(\lambda-6)(\lambda+2) = 0$$

$$\therefore \ \lambda_1 = 3 \qquad \lambda_2 = 6 \qquad \lambda_3 = -2$$

For $\lambda_1 = 3$:

$$\begin{bmatrix} 3 & 0 & 3 \\ 0 & 3 & 3 \\ 2 & 3 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 3 \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$3x_1 + \qquad + 3x_3 = 3x_1$$

$$3x_2 + 3x_3 = 3x_2$$

$$2x_1 + 3x_2 + x_3 = 3x_3$$

$$\Rightarrow \quad x_1 = \begin{bmatrix} 3 \\ -2 \\ 0 \end{bmatrix}$$

For $\lambda_2 = 6$:

$$x_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

For $\lambda_3 = -2$:

$$x_3 = \begin{bmatrix} 3 \\ 3 \\ -5 \end{bmatrix}$$

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Module 4

SELF-TEST 1

1.

$$a_0 = \frac{1}{\pi}\int_0^{2\pi}\left(1 - \frac{x}{\pi}\right) dx$$

$$= \frac{1}{\pi}\left[x - \frac{x^2}{2\pi}\right]_0^{2\pi}$$

$$= \frac{1}{\pi}\left[2\pi - \frac{4\pi^2}{2\pi}\right]$$

$$= 0$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi}\left(1 - \frac{x}{\pi}\right)\cos nx\, dx$$

$$= \frac{1}{\pi}\left[\frac{\sin nx}{n} - \frac{x \sin nx}{n\pi}\right]_0^{2\pi} + \frac{1}{\pi^2 n}\int_0^{2\pi}\sin nx\, dx$$

$$= 0$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi}\left(1 - \frac{x}{\pi}\right)\sin nx\, dx$$

$$= \frac{1}{\pi}\left\{\left[\left(1 - \frac{x}{\pi}\right)\frac{-\cos nx}{n}\right]_0^{2\pi} + \frac{1}{n}\int_0^{2\pi}\cos nx\, dx\right\}$$

$$= \frac{2}{n\pi}$$

$$f(x) = \sum_{n=1}^{\infty}\left\{\frac{2}{n\pi}\sin nx\right\}$$

$$= \frac{2}{\pi}\left\{\sin x + \frac{1}{2}\sin 2x + \frac{1}{3}\sin 3x + \ldots\ldots\right\}$$

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2.

$$a_0 = \frac{1}{\pi}\left\{\int_{-\pi}^{0} (\pi + x)\,dx + \int_{0}^{\pi} (\pi - x)\,dx\right\}$$

$$= \frac{1}{\pi}\left\{\left[\pi x + \frac{x^2}{2}\right]_{-\pi}^{0} + \left[\pi x - \frac{x^2}{2}\right]_{0}^{\pi}\right\}$$

$$= \frac{1}{\pi}\left\{\frac{\pi^2}{2} + \frac{\pi^2}{2}\right\}$$

$$= \pi$$

$$a_n = \frac{1}{\pi}\left\{\int_{-\pi}^{0} (\pi + x)\cos nx\,dx + \int_{0}^{\pi} (\pi - x)\cos nx\,dx\right\}$$

$$= \frac{1}{\pi}\left\{\int_{-\pi}^{0} x\cos nx\,dx - \int_{0}^{\pi} x\cos nx\,dx\right\}$$

$$= \frac{1}{\pi}\left\{-2\int_{0}^{\pi} x\cos nx\,dx\right\}$$

$$= \frac{-2}{\pi}\left\{\left[\frac{x\sin nx}{n}\right]_{0}^{\pi} - \int_{0}^{\pi} \frac{\sin nx}{n}\,dx\right\}$$

$$= \frac{-2}{\pi}\left[\frac{\cos nx}{n^2}\right]_{0}^{\pi}$$

$$= \frac{-2}{n^2\pi}\{(-1)^n - 1\}$$

$$b_n = 0$$

$$f(x) = \frac{\pi}{2} - \frac{2}{\pi}\sum_{n=1}^{\infty}\left\{\left[\frac{(-1)^n - 1}{n^2}\right]\cos nx\right\}$$

$$= \frac{\pi}{2} - \frac{2}{\pi}\left[\frac{-2}{1}\cos x + \frac{-2}{9}\cos 3x + \frac{-2}{25}\cos 5x + \ldots\ldots\right]$$

$$= \frac{\pi}{2} + \frac{4}{\pi}\left[\cos x + \frac{1}{9}\cos 3x + \frac{1}{25}\cos 5x + \ldots\ldots\right]$$

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3.

$$a_0 = \frac{2}{\pi}\left\{\int_0^{\frac{\pi}{3}} a\,dx - \int_{\frac{2\pi}{3}}^{\pi} a\,dx\right\}$$

$$= \frac{2a}{\pi}\left\{[x]_0^{\frac{\pi}{3}} - [x]_{\frac{2\pi}{3}}^{\pi}\right\}$$

$$= \frac{2a}{\pi}\left\{\frac{\pi}{3} - \pi + \frac{2\pi}{3}\right\}$$

$$= 0$$

$$a_n = \frac{2a}{\pi}\left\{\int_0^{\frac{\pi}{3}} \cos 2nx\,dx - \int_{\frac{2\pi}{3}}^{\pi} \cos 2nx\,dx\right\}$$

$$= \frac{a}{n\pi}\left\{\sin 2nx \,\Big|_0^{\frac{\pi}{3}} - \sin 2nx \,\Big|_{\frac{2\pi}{3}}^{\pi}\right\}$$

$$= 0$$

$$b_n = \frac{2a}{\pi}\left\{\int_0^{\frac{\pi}{3}} \sin 2nx\,dx - \int_{\frac{2\pi}{3}}^{\pi} \sin 2nx\,dx\right\}$$

$$= \frac{a}{n\pi}\left\{-\cos 2nx \,\Big|_0^{\frac{\pi}{3}} + \cos 2nx \,\Big|_{\frac{2\pi}{3}}^{\pi}\right\}$$

$$= \frac{a}{n\pi}\left\{\cos 0 - \cos\frac{2n\pi}{3} + \cos 2n\pi - \cos\frac{4n\pi}{3}\right\}$$

$$f(x) = \frac{a}{\pi}\left\{3\sin 2t + \frac{3}{2}\sin 4t + \frac{3}{4}\sin 8t + \frac{3}{5}\sin 10t + \ldots..\right\}$$

$$= \frac{3a}{\pi}\left\{\sin 2t + \frac{1}{2}\sin 4t + \frac{1}{4}\sin 8t + \frac{1}{5}\sin 10t + \ldots..\right\}$$

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4.

$$a_0 = \frac{4}{\pi}\int_0^{\frac{\pi}{2}} (x\pi - x^2)\,dx$$

$$= \frac{4}{\pi}\left[\frac{\pi x^2}{2} - \frac{x^3}{3}\right]_0^{\frac{\pi}{2}}$$

$$= \frac{4}{\pi}\left[\frac{\pi^3}{8} - \frac{\pi^3}{24}\right]$$

$$= \frac{\pi^2}{3}$$

$$a_n = \frac{4}{\pi}\left\{\int_0^{\frac{\pi}{2}} \pi x\cos 2nx\,dx - \int_0^{\frac{\pi}{2}} x^2\cos 2nx\,dx\right\}$$

$$= \frac{4}{\pi}\left\{\frac{\pi x\sin 2nx}{2n}\bigg|_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \frac{\pi\sin 2nx}{2n}\,dx - \frac{x^2\sin 2nx}{2n}\bigg|_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} \frac{x\sin 2nx}{n}\,dx\right\}$$

$$= \frac{4}{\pi}\left\{\frac{\pi\cos 2nx}{4n^2}\bigg|_0^{\frac{\pi}{2}} - \frac{x\cos 2nx}{2n^2}\bigg|_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} \frac{\cos 2nx}{2n^2}\,dx\right\}$$

$$= \frac{4}{\pi}\left\{\frac{\pi(-1)^n}{4n^2} - \frac{\pi}{4n^2} - \frac{\pi(-1)^n}{4n^2} + \frac{\sin 2nx}{4n^3}\bigg|_0^{\frac{\pi}{2}}\right\}$$

$$= \frac{4}{\pi}\left\{-\frac{\pi}{4n^2}\right\}$$

$$= -\frac{1}{n^2}$$

$$f(x) = \frac{\pi^2}{6}\sum_{n=1}^{\infty}\left(-\frac{1}{n^2}\cos 2nx\right)$$

$$= \frac{\pi^2}{6} - \left\{\cos 2x + \frac{1}{4}\cos 4x + \frac{1}{9}\cos 6x + \ldots\ldots\right\}$$

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