MAT1510 ASSIGNMENT 5

SOLUTIONS

Question 1: 5 Marks

Consider the circle in Figure 1. What is the area in square centimetres of the sector of the circle with central angle θ if $r = 9$ cm and $\theta = 60°$?

1. 4860 2. $\frac{27\pi}{2}$ 3. $\frac{27\pi}{4}$ 4. None of the preceding.

$$\text{Area of Sector} = \tfrac{1}{2}r^2\theta = \tfrac{1}{2}(9cm)^2\left(\tfrac{60°}{1}\times\tfrac{\pi}{180°}\right) = \tfrac{27\pi}{2}cm^2$$

Question 2: 5 Marks

Consider the circle in Figure 1. What is the length of the circular arc (in metres) subtended by θ if $r = 9$ metres and $\theta = 60°$?

1. 540 2. 6π 3. 3π 4. None of the preceding.

$$\text{Arc Length} = r\theta = (9m)\left(60°\times\tfrac{\pi}{180°}\right) = 3\pi\ m$$

Question 3: 5 Marks

Consider the circle in Figure 1. What is area of the shaded segment (in square units) if θ is measured in radians?

1. $\frac{1}{2}(\theta - \sin(\theta))r^2$ 2. $\frac{1}{2}(\sin(\theta) - \theta)r^2$
3. $\frac{1}{2}(\theta + \sin(\theta))r^2$ 4. None of the preceding.

Area of shaded area = Area of Sector – Area of triangle

$$= \tfrac{1}{2}r^2\theta - \tfrac{1}{2}r^2\sin\theta = \tfrac{1}{2}r^2(\theta - \sin\theta)$$

Question 4: 5 Marks

Consider the circle in Figure 1. What is the area of the sector of the circle if $r = 12$ and $\theta = 55°$?

1. 7920 2. $\frac{11\pi}{3}$ 3. 22π 4. None of the preceding.

$$\text{Area of Sector} = \tfrac{1}{2}r^2\theta = \tfrac{1}{2}(12)^2\left(\tfrac{55°}{1}\times\tfrac{\pi}{180°}\right) = 22\pi$$

Question 5: 5 Marks

Consider the circle in Figure 1. What is the area of the **unshaded** part of the circle if $r = 12$ and $\theta = \frac{\pi}{4}$ radians?

1. $\frac{1}{2}(\frac{\pi}{4} + \frac{1}{\sqrt{2}})12^2$ 2. $\frac{\pi}{8}12^2$
3. $\frac{1}{2}(\frac{\pi}{4} - \frac{1}{\sqrt{2}})12^2$ 4. None of the preceding.

Area of shaded area = Area of Sector – Area of triangle

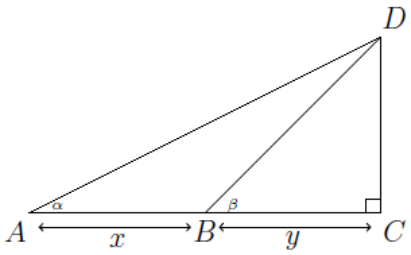
$$= \tfrac{1}{2}r^2(\theta - \sin\theta) = \tfrac{1}{2}(12)^2\left(\tfrac{\pi}{4} - \sin\tfrac{\pi}{4}\right) = \tfrac{1}{2}(12)^2\left(\tfrac{\pi}{4} - \tfrac{1}{\sqrt{2}}\right)$$

Area of unshaded area = πr^2 – Area of shaded area

$$= \pi(12)^2 - \tfrac{1}{2}(12)^2\left(\tfrac{\pi}{4} - \tfrac{1}{\sqrt{2}}\right) = \tfrac{1}{2}(12)^2\left[2\pi - \left(\tfrac{\pi}{4} - \tfrac{1}{\sqrt{2}}\right)\right] = \tfrac{1}{2}(12)^2\left[\tfrac{7\pi}{4} + \tfrac{1}{\sqrt{2}}\right]$$

Question 6: 5 Marks

Consider the figure



Consider the statements and select the correct option below.

(a) $y = \frac{x\sin(\alpha)\cos(\beta)}{\sin(\beta-\alpha)}$;

(b) $y = \frac{x}{\cot(\alpha)-\cot(\beta)}$;

(c) $y = \frac{x\cot(\beta)}{\cot(\alpha)-\cot(\beta)}$;

1. Only (a) is true
2. Only (c) is true
3. Both (a) and (b) are true
4. Both (a) and (c) are true

$$\frac{DB}{\sin(\alpha)} = \frac{x}{\sin(\beta-\alpha)}$$

$$DB = \frac{x\sin(\alpha)}{\sin(\beta-\alpha)}$$

$$\frac{y}{DB} = \cos(\beta)$$

$$y = DB\cos(\beta) = \frac{x\sin(\alpha)}{\sin(\beta-\alpha)} \times \cos(\beta)$$

$$= \frac{x\sin(\alpha)\cos(\beta)}{\sin(\beta-\alpha)} \text{.....(a)}$$

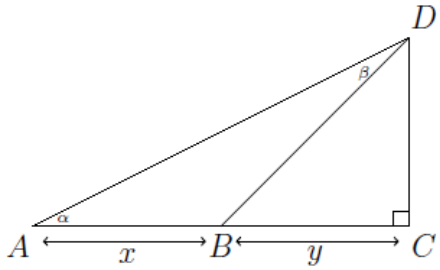
$$= \frac{x\sin(\alpha)\cos(\beta)}{\sin(\beta)\cos(\alpha)-\sin(\alpha)\cos(\beta)}$$

$$= \frac{\frac{x\sin(\alpha)\cos(\beta)}{\sin(\alpha)\sin(\beta)}}{\frac{\sin(\beta)\cos(\alpha)}{\sin(\alpha)\sin(\beta)} - \frac{\sin(\alpha)\cos(\beta)}{\sin(\alpha)\sin(\beta)}}$$

$$= \frac{x\cot(\beta)}{\cot(\alpha)-\cot(\beta)} \quad \text{..... (c)}$$

Question 7: 5 Marks

Consider the figure



Consider the statements and select the correct option below.

(a) $y = \frac{x\sin(\alpha)\cos(\alpha+\beta)}{\sin(\beta)}$;

(b) $y = x\sin(\alpha)(cos(\alpha)\cot(\beta) - \sin(\alpha))$;

(c) $y = \frac{x}{\cot(\alpha)-\cot(\alpha+\beta)}$;

1. Only (a) is true
2. Only (b) is true
3. Both (a) and (b) are true
4. Both (b) and (c) are true

$$\frac{DB}{\sin(\alpha)} = \frac{x}{\sin(\beta)}$$

$$DB = \frac{x\sin(\alpha)}{\sin(\beta)}$$

$$\frac{y}{DB} = \cos(\alpha+\beta)$$

$$y = DB\cos(\alpha+\beta) = \frac{x\sin(\alpha)}{\sin(\beta)} \times \cos(\alpha+\beta)$$

$$= \frac{x\sin(\alpha)\cos(\alpha+\beta)}{\sin(\beta)} \quad \text{.....(a)}$$

$$= \frac{x\sin(\alpha)\left[\cos(\alpha)\cos(\beta)-\sin(\alpha)\sin(\beta)\right]}{\sin(\beta)}$$

$$= \frac{x\sin(\alpha)\left[\cos(\alpha)\cos(\beta)\right]}{\sin(\beta)} - \frac{x\sin(\alpha)\left[\sin(\alpha)\sin(\beta)\right]}{\sin(\beta)}$$

$$= x\sin(\alpha)\cos(\alpha)\cot(\beta) - x\sin(\alpha)\sin(\alpha)$$

$$= x\sin(\alpha)\left[\cos(\alpha)\cot(\beta)-\sin(\alpha)\right] \quad \text{.....(b)}$$

Question 8: 5 Marks

Consider the statements and select the correct option below.

(a) For any real number α, $(\sin(\alpha)+\cos(\alpha))^2+(\sin(\alpha)-\cos(\alpha))^2=2$;

(b) For any real number α, $(\sin(\alpha)+\cos(\alpha))^2-(\sin(\alpha)-\cos(\alpha))^2=2\sin(2\alpha)$;

1. Only (a) is true
2. Only (b) is true
3. Both (a) and (b) are true
4. Neither (a) nor (b) are true

$$\left[\sin(\alpha)+\cos(\alpha)\right]^2+\left[\sin(\alpha)-\cos(\alpha)\right]^2$$
$$=\sin^2(\alpha)+2\sin(\alpha)\cos(\alpha)+\cos^2(\alpha)+\left[\sin^2(\alpha)-2\sin(\alpha)\cos(\alpha)+\cos^2(\alpha)\right]$$
$$=2\left[\sin^2(\alpha)+\cos^2(\alpha)\right]=2[1]=2 \qquad \text{TRUE} \quad \text{.....(a)}$$

$$\left[\sin(\alpha)+\cos(\alpha)\right]^2-\left[\sin(\alpha)-\cos(\alpha)\right]^2$$
$$=\sin^2(\alpha)+2\sin(\alpha)\cos(\alpha)+\cos^2(\alpha)-\left[\sin^2(\alpha)-2\sin(\alpha)\cos(\alpha)+\cos^2(\alpha)\right]$$
$$=4\sin(\alpha)\cos(\alpha)=2\left[\sin(2\alpha)\right] \qquad \text{TRUE} \quad \text{.....(b)}$$

Question 9: 5 Marks

Consider the statements and select the correct option below.

(a) There exist real numbers α and β such that $\frac{\sin(\alpha+\beta)+\sin(\alpha-\beta)}{2\cos(\beta)} \neq \sin(\alpha)$;

(b) For any real numbers α and β, $\tan(\alpha+\beta) = \frac{\tan(\alpha)+\tan(\beta)}{\tan(\alpha)\tan(\beta)-1}$;

1. Only (a) is true
2. Only (b) is true
3. Both (a) and (b) are true
4. Neither (a) nor (b) are true

$$\frac{\sin(\alpha+\beta)+\sin(\alpha-\beta)}{\cos(\beta)}$$

$$= \frac{\sin(\alpha)\cos(\beta)+\cancel{\sin(\beta)\cos(\alpha)}+\sin(\alpha)\cos(\beta)-\cancel{\sin(\beta)\cos(\alpha)}}{\cos(\beta)}$$

$$= \frac{2\sin(\alpha)\cos(\beta)}{\cos(\beta)} = 2\sin(\alpha) \neq \sin(\alpha)$$ .....(a) NOT TRUE for all real α and β

$$\tan(\alpha+\beta) = \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)}$$

$$= \frac{\sin(\alpha)\cos(\beta)+\sin(\beta)\cos(\alpha)}{\cos(\alpha)\cos(\beta)-\sin(\beta)\sin(\alpha)}$$

$$= \frac{\frac{\sin(\alpha)\cancel{\cos(\beta)}}{\cos(\alpha)\cancel{\cos(\beta)}}+\frac{\sin(\beta)\cancel{\cos(\alpha)}}{\cancel{\cos(\alpha)}\cos(\beta)}}{\frac{\cancel{\cos(\alpha)\cos(\beta)}}{\cancel{\cos(\alpha)\cos(\beta)}}-\frac{\sin(\beta)\sin(\alpha)}{\cos(\alpha)\cos(\beta)}}$$

$$= \frac{\tan(\alpha)+\tan(\beta)}{1-\tan(\alpha)\tan(\beta)}$$ .....(b) FALSE

Question 10: 5 Marks

If we evaluate the expression $\sin(\frac{\pi}{6}) + \cos^2(\frac{\pi}{6}) - \sin^2(\frac{\pi}{6}) - \tan(\frac{\pi}{4})$ **without using a calculator**, we find

1. 0
2. -1
3. $\sqrt{3}-1$
4. None of the preceding

$$\sin\left(\tfrac{\pi}{6}\right)+\cos^2\left(\tfrac{\pi}{6}\right)-\sin^2\left(\tfrac{\pi}{6}\right)-\tan\left(\tfrac{\pi}{4}\right)$$

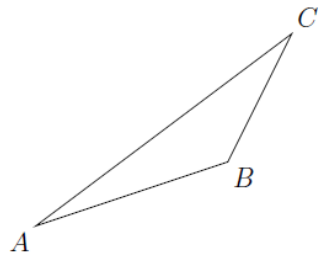
$$= \left(\tfrac{1}{2}\right)+\left(\tfrac{\sqrt{3}}{2}\right)^2-\left(\tfrac{1}{2}\right)^2-(1)$$

$$= \left(\tfrac{1}{2}\right)+\left(\tfrac{3}{4}\right)-\left(\tfrac{1}{4}\right)-(1)$$

$$= 0$$

Question 11: 5 Marks

Consider the triangle



with sides a, b and c opposite the respective angles. If $a = 5$, $b = 7$ and $c = 4$, measured in centimetres, then the area in square centimetres of the triangle is

1. 12 2. $4\sqrt{3}$ 3. $4\sqrt{6}$ 4. None of the preceding.

$$a^2 = b^2 + c^2 - 2bc\cos(A)$$

$$5^2 = 7^2 + 4^2 - 2\times 7\times 4\times\cos(A)$$

$$\cos(A) = \frac{5^2 - 7^2 - 4^2}{-2\times 7\times 4} = \frac{5}{7}$$

$$\sin(A) = \sqrt{1-\left(\tfrac{5}{7}\right)^2} = \frac{2\sqrt{6}}{7}$$

$$Area = \tfrac{1}{2}bc\sin(A) = \tfrac{1}{2}(7)(4)\left(\frac{2\sqrt{6}}{7}\right) = 4\sqrt{6}$$

Question 12: 5 Marks

From a point A at sea two boats set out. The first boat travels in a straight line in the direction $\alpha°$ measured clockwise from north to a point B. The second boat travels in a straight line in the direction $\beta°$ measured anticlockwise from south to a point C. Given AB is c km, AC is b km, and BC is a km. Consider the statements and select the correct option below.

(a) $a^2 - b^2 - c^2 = 2bc\cos(\alpha + \beta)$;

(b) $a^2 - 2bc\cos(\alpha)cos(\beta) = c^2 + b^2 - 2bc\sin(\alpha)\sin(\beta)$

1. Only (a) is true
2. Only (b) is true
3. Both (a) and (b) are true
4. Neither (a) nor (b) are true

$$a^2 = b^2 + c^2 - 2bc\cos(A)$$

$$a^2 = b^2 + c^2 - 2\times b\times c\times\cos(180° - (\alpha+\beta))$$

$$a^2 = b^2 + c^2 - 2\times b\times c\times -\cos(\alpha+\beta)$$

$$a^2 = b^2 + c^2 + 2bc\times\cos(\alpha+\beta)$$

$$a^2 - b^2 - c^2 = 2bc\times\cos(\alpha+\beta) \quad \text{.....(a) isTRUE}$$

$$a^2 - b^2 - c^2 = 2bc\times\left[\cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)\right]$$

$$a^2 - 2bc\times\left[\cos(\alpha)\cos(\beta)\right] = b^2 + c^2 - 2bc\times\left[\sin(\alpha)\sin(\beta)\right] \quad \text{.. (b) is TRUE}$$

Question 13: 5 Marks

Suppose that in the situation of Question 12 we know that distance from A to C is 5 km, the distance from A to B is 3 km, $\alpha = 15°$ and $\beta = 30°$. What is the distance from B to C?

1. $\sqrt{34+15\sqrt{2}}$ km
2. 4 km
3. $\sqrt{34+15\sqrt{3}}$ km
4. None of the preceding.

$$a^2 = b^2 + c^2 - 2bc\cos(A)$$

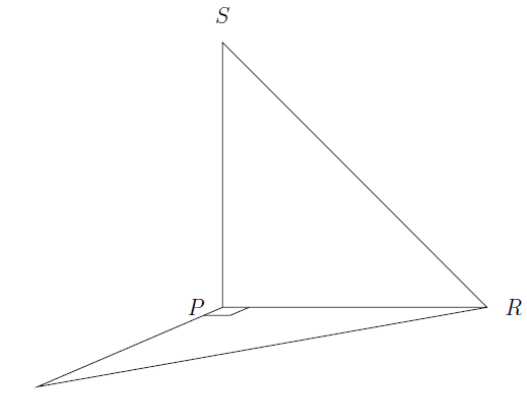
$$BC^2 = 5^2 + 3^2 + 2\times 5\times 3\times \cos(15°+30°)$$

$$BC^2 = 5^2 + 3^2 + 2\times 5\times 3\times \cos(45°)$$

$$BC^2 = 5^2 + 3^2 + 2\times 5\times 3\times \left(\tfrac{1}{\sqrt{2}}\right) = 34+15\sqrt{2}$$

$$BC = \sqrt{34+15\sqrt{2}} \quad km$$

Question 14: 5 Marks



P, Q and R are three points in the same horizontal plane. SP is perpendicular to the horizontal plane at P. $Q\hat{P}R = 90°$, $P\hat{Q}R = 30°$, $S\hat{R}P = 45°$ and $QR = 2$ units. If x is the length of SP in the same units, then

1. 1
2. $\sqrt{3}$
3. $\sqrt{2}$
4. None of the preceding.

In ΔPQR:

$$\frac{PR}{2} = \sin(30°)$$

$$PR = 2\sin(30°) = 2\times\tfrac{1}{2} = 1$$

$$\therefore x = SP = PR = 1$$

Question 15: 5 Marks

Consider the statements and select the correct option below.

(a) $\cos(x) = 1 - \frac{\sin(x)}{\csc(x)+\cot(x)}$

(b) $\sin(x) = 1 - \frac{\cos(x)}{\sec(x)+\tan(x)}$

1. Only (a) is true
2. Only (b) is true
3. Both (a) and (b) are true
4. Neither (a) nor (b) are true

$$1-\frac{\sin(x)}{\csc(x)+\cot(x)}$$

$$=1-\frac{\sin(x)}{\frac{1}{\sin(x)}+\frac{\cos(x)}{\sin(x)}}=1-\frac{\sin(x)}{\frac{1+\cos(x)}{\sin(x)}}$$

$$=1-\frac{\sin^2(x)}{1+\cos(x)}=1-\frac{1-\cos^2(x)}{1+\cos(x)}$$

$$=1-\frac{\left(1+\cos(x)\right)\left(1-\cos(x)\right)}{1+\cos(x)}=1-\left(1-\cos(x)\right)=\cos(x) \quad \text{...(a) TRUE}$$

$$1-\frac{\cos(x)}{\sec(x)+\tan(x)}$$

$$=1-\frac{\cos(x)}{\frac{1}{\cos(x)}+\frac{\sin(x)}{\cos(x)}}=1-\frac{\cos(x)}{\frac{1+\sin(x)}{\cos(x)}}$$

$$=1-\frac{\cos^2(x)}{1+\sin(x)}=1-\frac{1-\sin^2(x)}{1+\sin(x)}$$

$$=1-\frac{\left(1+\sin(x)\right)\left(1-\sin(x)\right)}{1+\sin(x)}=1-\left(1-\sin(x)\right)=\sin(x) \quad \text{...(b) TRUE}$$

Question 16: 5 Marks

Which **one** of the following equations is **not** and identity?

1. $\frac{1}{\cot^2(\theta)} + \sin^2(\theta) + \cos^2(\theta) = \sec^2(\theta)$
2. $\frac{1}{\tan^2(\theta)} + \sin^2(\theta) = \csc^2(\theta) - \cos^2(\theta)$
3. $1 - 2\sin^2(\theta) = \cos(2\theta)$
4. $2\cos^2(\theta) = 1 + \sin(2\theta)$
5. $\tan^2(\theta) + \cot^2(\theta) = \sec^2(\theta) + \csc^2(\theta) - 2$

(1)1

$$\frac{1}{\cot^2(\theta)} + \sin^2(\theta) + \cos^2(\theta)$$

$$= \frac{\sin^2(\theta)}{\cos^2(\theta)} + 1 = \frac{\sin^2(\theta) + \cos^2(\theta)}{\cos^2(\theta)} = \frac{1}{\cos^2(\theta)} = \sec^2(\theta) \quad \therefore \text{IDENTITY}$$

(2)

$$\frac{1}{\tan^2(\theta)} + \sin^2(\theta) + \cos^2(\theta)$$

$$= \frac{\cos^2(\theta)}{\sin^2(\theta)} + 1 = \frac{\sin^2(\theta) + \cos^2(\theta)}{\sin^2(\theta)} = \frac{1}{\sin^2(\theta)} = \csc^2(\theta) \quad \therefore \text{IDENTITY}$$

(3)

$$\cos(2\theta)$$

$$= \cos^2(\theta) - \sin^2(\theta) = 1 - \sin^2(\theta) - \sin^2(\theta) = 1 - 2\sin^2(\theta) \quad \therefore \text{IDENTITY}$$

(4)

$$1 + 2\sin(2\theta) = \sin^2(\theta) + \cos^2(\theta) + 2\left(2\sin(\theta)\cos\left(\theta\right)\right) \neq 2\cos^2(\theta) \quad \therefore \text{NOT an IDENTITY}$$

(5)

$$\text{LHS} = \tan^2(\theta) + \cot^2(\theta) + 2$$

$$= (\tan^2(\theta) + 1) + (\cot^2(\theta) + 1)$$

$$= \sec^2(\theta) + \csc^2(\theta)$$

$$= \text{RHS} \qquad \therefore \text{IDENTITY}$$

Question 17: 5 Marks

Which one of the following is the set of solutions of $2\cos(3x) + \cos(x)\cos(3x) = \cos(x) + 2$?

1. $\{\frac{\pi}{4} + \frac{2k\pi}{3} | k \text{ is an integer}\}$
2. $\{\frac{k\pi}{3} | k \text{ is an integer}\}$
3. $\{\frac{2k\pi}{3} | k \text{ is an integer}\}$
4. None of the preceding.

$$2\cos(3x)+\cos(x)\cos(3x)=\cos(x)+2$$
$$\cos(3x)\left[2+\cos x\right]-\left[2+\cos x\right]=0$$
$$\left[\cos(3x)-1\right]\left[2+\cos(x)\right]=0$$
$$\cos(3x)=1 \quad \text{OR} \quad \cos(x)\neq -2$$
$$3x=0+2\pi k$$
$$x=\tfrac{2}{3}\pi k, \quad k\in\mathbb{Z}$$
$$x\in\left\{\tfrac{2}{3}\pi k, \quad k\in\mathbb{Z}\right\}$$

Question 18: 5 Marks

Which one of the following is the set of solutions of $\sin(3x) + \cos(3x) = 0$ on the interval $(-\pi, \pi)$?

1. $\{-\frac{5\pi}{12}, -\frac{\pi}{12}, \frac{\pi}{4}, \frac{7\pi}{12}\}$
2. $\{-\frac{3\pi}{4}, -\frac{\pi}{12}, \frac{\pi}{4}, \frac{7\pi}{12}, \frac{11\pi}{12}\}$
3. $\{-\frac{3\pi}{4}, -\frac{5\pi}{12}, -\frac{\pi}{12}, \frac{\pi}{4}, \frac{7\pi}{12}, \frac{11\pi}{12}\}$
4. None of the preceding.

$$\sin(3x)+\cos(3x)=0$$
$$\sin(3x)=-\cos(3x)$$
$$\frac{\sin(3x)}{\cos(3x)}=-\frac{\cos(3x)}{\cos(3x)}$$
$$\tan(3x)=-1$$
$$3x=\left(\pi-\tfrac{\pi}{4}\right)+k\pi \quad ,k\in\mathbb{Z}$$
$$x=\tfrac{\pi}{4}+\tfrac{k}{3}\pi, \quad ,k\in\mathbb{Z}$$
$$x\in\left\{-\tfrac{3\pi}{4},-\tfrac{5\pi}{12},-\tfrac{\pi}{12},\tfrac{\pi}{4},\tfrac{7\pi}{12},\tfrac{11\pi}{12}\right\}$$

Question 19: 5 Marks

Which one of the following is the set of solutions of $4\sin^5(x) - 3\sin^3(x) = 0$ on the interval $(0, 2\pi)$?

1. $\{\frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{5\pi}{3}\}$
2. $\{0, \frac{\pi}{3}, \pi, \frac{4\pi}{3}, 2\pi\}$
3. $\{0, \frac{\pi}{3}, \pi, \frac{4\pi}{3}\}$
4. None of the preceding.

$$4\sin^5(x)-3\sin^3(x)=0$$
$$\sin^3(x)\left[4\sin^2(x)-3\right]=0$$
$$\sin^3(x)=0 \quad \text{OR} \quad \sin^2(x)=\tfrac{3}{4}$$
$$\sin(x)=0 \quad \text{OR} \quad \sin(x)=\pm\sqrt{\tfrac{3}{4}}=\pm\tfrac{\sqrt{3}}{2}$$
$$x\neq 0 \text{ or } x=\pi \text{ or } x\neq 2\pi \qquad x=\tfrac{\pi}{3} \text{ or } x=\tfrac{2\pi}{3} \text{ or } x=\tfrac{4\pi}{3} \text{ or } x=\tfrac{5\pi}{3}$$
$$x\in\left\{\tfrac{\pi}{3},\tfrac{2\pi}{3},\pi,\tfrac{4\pi}{3},\tfrac{5\pi}{3}\right\}$$

Question 20: 5 Marks

Which one of the following is the set of solutions of $\sin(3x) + \tan(3x) = 0$?

1. $\{\frac{\pi}{6} + \frac{k\pi}{3} | k \text{ is an integer}\} \cup \{\frac{2k\pi}{3} | k \text{ is an integer}\}$
2. $\{\frac{\pi}{6} + \frac{k\pi}{3} | k \text{ is an integer}\} \cup \{\frac{\pi}{3} + \frac{2k\pi}{3} | k \text{ is an integer}\}$
3. $\{\frac{k\pi}{3} | k \text{ is an integer}\}$
4. None of the preceding.

$$\sin(3x) + \tan(3x) = 0$$

$$\sin(3x) + \frac{\sin(3x)}{\cos(3x)} = 0$$

$$\sin(3x)\cos(3x) + \sin(3x) = 0$$

$$\sin(3x)\left[\cos(3x) + 1\right] = 0$$

$$\sin(3x) = 0 \quad \text{OR} \quad \cos(3x) = -1$$

$$3x = 0 + 2\pi k \quad \text{or} \quad 3x = \pi + 2\pi k \qquad 3x = \pi + 2\pi k$$

$$x = \frac{2\pi k}{3} \quad \text{or} \quad x = \frac{\pi}{3} + \frac{2\pi k}{3} \qquad x = \frac{\pi}{3} + \frac{2\pi k}{3}$$

$$k \in \mathbb{Z}$$

$$\therefore x \in \left\{\frac{k\pi}{3}, \quad k \in \mathbb{Z}\right\}$$