QUESTION 1

If $A = \begin{bmatrix} 3 & 1 \\ -1 & 5 \end{bmatrix}$, find an eigenvalue and an eigenvector of $\boldsymbol{A}$.

Answer:

For eigenvalues put $|A - \lambda I| = 0$

$$\begin{vmatrix} 3-\lambda & 1 \\ -1 & 5-\lambda \end{vmatrix} = 0$$

$$(3-\lambda)(5-\lambda)+1=0$$

$$15-8\lambda+\lambda^2+1=0$$

$$\lambda^2-8\lambda+16=0$$

$$(\lambda-4)^2=0$$

$$\lambda = 4\,(\text{twice})$$

For an eigenvalue corresponmding to $\lambda = 4$

$$\begin{bmatrix} (3-4) & 1 \\ -1 & (5-4) \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$-x_1 + x_2 = 0$$

$$x_1 = x_2$$

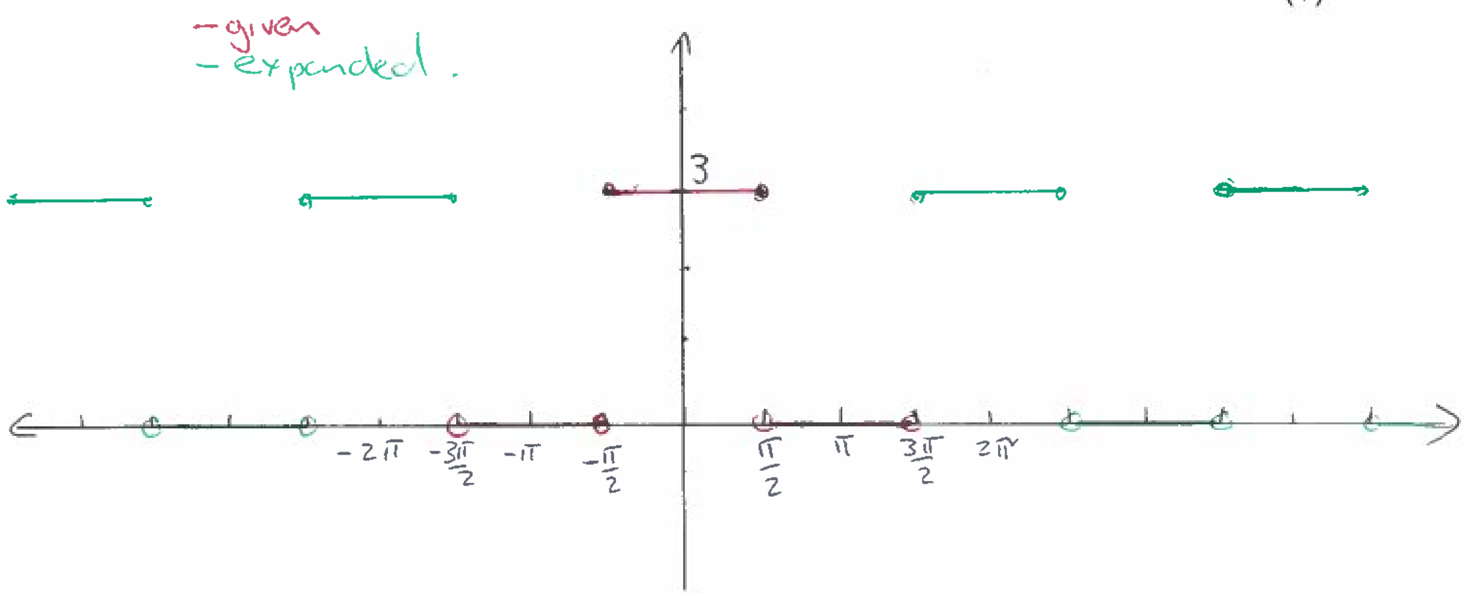
Choose $x_1 = 1$

Thus an eigenvalue is $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ **[8]**

QUESTION 2

Given the function defined by $f(t) = \begin{cases} 0 & -\pi < t < -\frac{\pi}{2} \\ 3 & -\frac{\pi}{2} \le t \le \frac{\pi}{2} \\ 0 & \frac{\pi}{2} < t < \pi \end{cases}$, with period 2π:

2.1 Sketch the function. (3)



2.2 even (1)

2.3 Find the Fourier series for *f(t)*. (8)

For even functions $b_n = 0$.

Period = 2π, thus L = π

$$a_0 = \tfrac{1}{L}\int_{-L}^{L} f(t)\,dt$$

$$= \tfrac{1}{\pi}\left[\int_{-\pi}^{-\frac{\pi}{2}} 0\,dt + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 3\,dt + \int_{\frac{\pi}{2}}^{\pi} 0\,dt\right]$$

$$= \tfrac{1}{\pi}\left[3t\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$= \tfrac{3}{\pi}\left[\tfrac{\pi}{2} - \left(-\tfrac{\pi}{2}\right)\right]$$

$$= 3$$

or because it is an even function $a_0 = \tfrac{2}{\pi}\int_0^{\frac{\pi}{2}} 3\,dt$

$$a_n = \tfrac{1}{L}\int_{-L}^{L} f(t)\cos\left(\tfrac{n\pi t}{L}\right)dt$$

$$= \tfrac{1}{\pi}\left[\int_{-\pi}^{-\frac{\pi}{2}} 0\,dt + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 3\cos(nt)\,dt + \int_{\frac{\pi}{2}}^{\pi} 0\,dt\right]$$

$$= \tfrac{3}{n\pi}\left[\sin(nt)\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$= \tfrac{3}{n\pi}\left[\sin\left(\tfrac{n\pi}{2}\right) - \sin\left(-\tfrac{n\pi}{2}\right)\right]$$

$$= \tfrac{3}{n\pi}\left[\sin\left(\tfrac{n\pi}{2}\right) + \sin\left(\tfrac{n\pi}{2}\right)\right] \qquad \text{Remember } \sin(-x) = -\sin(x)$$

$$= \tfrac{6}{n\pi}\left[\sin\left(\tfrac{n\pi}{2}\right)\right]$$

$$= \begin{cases} 0 & \text{n even} \\ \tfrac{6}{n\pi}\left[\sin\left(\tfrac{n\pi}{2}\right)\right] & \text{odd} \end{cases}$$

$$f(t) = \frac{3}{2} + \frac{6}{\pi}\sum_{n=1}^{\infty}\left(\frac{1}{n}\sin\left(\tfrac{n\pi}{2}\right)\right)\cos(nt)$$

$$= \frac{3}{2} + \frac{6}{\pi}(\cos t) - \frac{2}{\pi}(\cos 3t) + \ldots\ldots\ldots$$

[12]