Assignment 03 Semester 2

ONLY FOR SEMESTER 2 STUDENTS
Assignment 03 (Compulsory)
Due Date: 29 September
Unique number: 733008

This assignment contributes 20% to your year mark.

Source: Paper May 2016

QUESTION 1

If $A = \begin{bmatrix} -1 & -1 & 1 \\ -4 & 2 & 4 \\ -1 & 1 & 5 \end{bmatrix}$, find an eigenvector corresponding to the eigenvalue $\lambda = 2$.

Verify that $\lambda = 2$ is an eigenvalue of *A*. (8)

[8]

QUESTION 2

Determine whether each of the following functions is odd, even or neither:

2.1 $f(x) = 3x + 2$ (1)

2.2 $f(x) = \cos 3x$ (1)

2.3 $f(x) = \begin{cases} -\pi & -2 < x < 0 \\ \pi & 0 < x < 2 \end{cases}$ (1)

QUESTION 3

A function *f(x)* is defined over one period by $f(x) = \begin{cases} -x & -\pi < x < 0 \\ 0 & 0 < x < \pi \end{cases}$

3.1 Sketch the function. (1)

3.2 Find the Fourier series expansion for *f(x)*, given that $a_n = \dfrac{(-1)^n - 1}{n^2}$. (9)

[10]

Maximum: [20]

Open Rubric

QUESTION 7

Eigenvector for $\lambda = 2$:

$$\begin{bmatrix} -3 & -1 & 1 \\ -4 & 0 & 4 \\ -1 & 1 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\therefore -3x_1 - x_2 + x_3 = 0$ (1)

$-4x_1 + 4x_3 = 0$ (2)

$-x_1 + x_2 + 3x_3 = 0$ (3)

From (2) $x_1 = x_3$

into (1): $-3x_1 - x_2 + x_1 = 0$

$-2x_1 = x_2$

$x_1 = -\frac{1}{2}x_2$

Choose $x_2 = 2$

then $x_1 = -1 = x_3$

$\therefore$ For $\lambda = 2$ an eigenvector is $\begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix}$ 5

Verify $\lambda = 2$:

or: Find all λ.

$$\begin{vmatrix} 1-\lambda & -1 & 1 \\ -4 & 2-\lambda & 4 \\ -1 & 1 & 5-\lambda \end{vmatrix} = 0$$

and solve. $\lambda = 2, -2, 6$

$$\begin{vmatrix} -3 & -1 & 1 \\ -4 & 0 & 4 \\ -1 & 1 & 3 \end{vmatrix}$$ Develop along 2nd column.

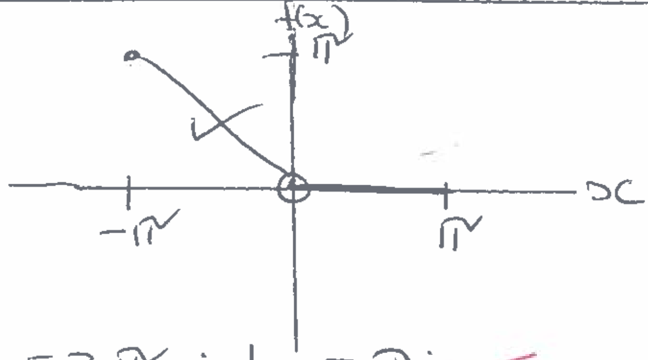
$$= -(-1)\begin{vmatrix} -4 & 4 \\ -1 & 3 \end{vmatrix} + (-1)\begin{vmatrix} -3 & 1 \\ -4 & 4 \end{vmatrix}$$ 3

$$= (-8) - (-8)$$

$$= 0$$

(8)

8.1



8.2 Period $= 2\pi \therefore L = \pi$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$$
$$= \frac{1}{\pi}\int_{-\pi}^{0} (-x)\,dx$$
$$= \frac{1}{\pi}\left[-\frac{x^2}{2}\right]_{-\pi}^{0}$$
$$= \frac{\pi}{2}$$

4

$$a_n = \frac{(-1)^n - 1}{n^2} \quad \text{given}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{0} (-x)\sin nx\,dx$$
$$= \frac{1}{\pi}\left[\frac{x}{n}\cos nx\Big|_{-\pi}^{0} - \frac{1}{n}\int_{-\pi}^{0}\cos nx\right]$$
$$= \frac{1}{\pi}\left[0 + \frac{\pi}{n}\cos n\pi - \frac{1}{n^2}\sin nx\Big|_{-\pi}^{0}\right]$$
$$= \frac{1}{n}(-1)^n - 0$$

4

$$\therefore f(x) = \frac{\pi}{4} + \sum_{n=1}^{\infty}\left[\left(\frac{(-1)^n - 1}{n^2}\right)\cos nx + \frac{(-1)^n}{n}\sin nx\right]$$

1

$$= \frac{\pi}{4} - \frac{2}{\pi}\left(\cos x + \frac{1}{9}\cos 3x + \ldots\right) - \left(\sin x - \frac{1}{2}\sin 2x + \frac{1}{3}\sin 3x \ldots\right)$$

[10]

Check: Calculation of a_n (not asked.

$$a_n = \frac{1}{\pi}\int_{-\pi}^{0}(-x)\cos nx\,dx = -\frac{1}{\pi}\int_{-\pi}^{0} x\cos nx\,dx$$

Use integration by parts

$$= -\frac{1}{\pi}\left[\cancel{-\frac{\pi}{n}\sin n\pi}_{\,0} - \frac{1}{n}\int_{-\pi}^{0}\sin nx\,dx\right]$$

$$= +\frac{1}{\pi n}\left[-\frac{1}{n}\cos nx\right]_{-\pi}^{0}$$

$$= -\frac{1}{\pi n^2}\left[\cos 0 - \cos n(\pi)\right]$$

$$= -\frac{1}{\pi n^2}\left[1 - (-1)^n\right]$$

$$= \frac{1}{\pi n^2}\left[(-1)^n - 1\right]$$