Assignment 02 Semester 2

ONLY FOR SEMESTER 2 STUDENTS
Assignment 02(Compulsory)
Due Date: 18 September
Unique number:680334

This assignment is a written assignment based on Study Guide 1 and 2

This assignment contributes 70% to your year mark.

Source:Paper May 2016

QUESTION 1

Find the general solutions of the following differential equations using **D-operator** methods:

1.1 $D^2y - 3Dy = 0$ (3)

Roots:
$m^2 - 3 = 0$
$\therefore m = 3$ twice.

$y = Ae^{3x} + Bxe^{3x}$

1.2 $(D-2)^3 y = 60e^{2x}x^2$ (6)

1.3 $(D^2 + D + 1)y = x + \sin x$ (7)

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QUESTION 2

Solve for ***y*** in the following set of simultaneous differential equations by using **D-operator** methods:

$$(2D-1)x + (D+1)y = 5\sin t$$
$$(3D-1)x + (2D+1)y = e^t$$
(8)

[8]

QUESTION 3

3.1 Determine

3.1.1 $L\{t\cos t\}$ (1)

3.1.2 $L\{e^{3t}\sin 5t + 3t^8\}$ (3)

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QUESTION 2

2.1 $(D-2)^3 y = 60e^{2x}x^2$

CF: $(m-2)^3 = 0$

$\therefore m = 2$ (3 times)

$\therefore y_{CF} = Ae^{+2x} + Bxe^{+2x} + Cx^2e^{+2x}$

$$y_{PI} = \frac{1}{(D-2)^3}\{60e^{2x}x^2\}$$

$$= 60e^{2x}\frac{1}{((D+2)-2)^3}\{x^2\}$$

$$= 60e^{2x}\frac{1}{D^3}\{x^2\}$$

$$= 60e^{2x}\frac{1}{D^2}\left\{\frac{x^3}{3}\right\}$$

$$= 20e^{2x}\frac{1}{D}\left\{\frac{x^4}{4}\right\}$$

$$= 5e^{2x}\left(\frac{x^5}{5}\right)$$

$$= e^{2x}x^5$$

$y_{gen} = Ae^{+2x} + Bxe^{+2x} + Cx^2e^{+2x} + x^5e^{2x}$

(6)

2.2 $(D^2+D+1)y = x+\sin x$

CF $m^2+m+1 = 0$

$$m = \frac{-1 \pm \sqrt{1-4}}{2}$$

$$= \frac{-1}{2} \pm \frac{\sqrt{3}}{2}j$$

$$\therefore y_{CF} = e^{-\frac{1}{2}x}\left(A\cos\frac{\sqrt{3}}{2}x + B\sin\frac{\sqrt{3}}{2}x\right)$$

$$y_{PI} = \frac{1}{D^2+D+1}\{x\} + \frac{1}{D^2+D+1}\{\sin x\}$$

$$= (1-D+\ldots)\{x\} + \frac{1}{-1+D+1}\{\sin x\}$$

$$= (x-1) + \frac{1}{D}\{\sin x\}$$

$$= (x-1) - \cos x$$

$$\therefore y_{gen} = e^{-\frac{1}{2}x}\left(A\cos\frac{\sqrt{3}}{2}x + B\sin\frac{\sqrt{3}}{2}x\right) + (x-1) - \cos x$$

(7)

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QUESTION 3

Alternative methods: elimination
or Find x then substitute to find y.

$(2D-1) \quad + (D+1) \quad = 5\sin t$

$(3D-1)x + (2D+1)y = e^t$

$$\begin{vmatrix} (2D-1) & (D+1) \\ (3D-1) & (2D+1) \end{vmatrix} y = \begin{vmatrix} (2D-1) & 5\sin t \\ (3D-1) & e^t \end{vmatrix}$$

$\therefore [(2D-1)(2D+1) - (D+1)(3D-1)]y = (2D-1)e^t - (3D-1)(5\sin t)$

$[(4D^2-1) - (3D^2+2D-1)]y = 2e^t - e^t - 15\cos t + 5\sin t$

$[D^2 - 2D]y = e^t - 15\cos t + 5\sin t$

y_{CF}: $m^2 - 2m = 0$

$\therefore m = 0$ or $m = 2$

$\therefore y_{CF} = A + Be^{2t}$

$$y_{PI} = \frac{1}{D^2-2D}\{e^t\} - 15\frac{1}{D^2-2D}\{\cos t\} + 5\frac{1}{D^2-2D}\{\sin t\}$$

$$= -e^t + 15\frac{1}{2D+1}\{\cos t\} - 5\frac{1}{2D+1}\{\sin t\}$$

$$= -e^t + 15\frac{2D-1}{4D^2-1}\{\cos t\} - 5\frac{2D-1}{4D^2-1}\{\sin t\}$$

$$= -e^t - \frac{15}{5}(-2\sin t - \cos t) + \frac{5}{5}(2\cos t - \sin t)$$

$$= -e^t + 6\sin t + 3\cos t + 2\cos t - \sin t$$

$$= -e^t + 5\sin t + 5\cos t$$

$\therefore y_{gen} = A + Be^{2t} - e^t + 5\sin t + 5\cos t.$

(9)

QUESTION 4

4.1 $L\{t\cos t\} = \dfrac{s^2-1}{(s^2+1)^2}$ (1)

4.2 $L^{-1}\left\{e^{s}\left(\dfrac{s^2-1}{(s^2+1)^2}\right)\right\} = H(t+1)\cdot(t+1)\cos(t+1)$ (2)

4.3 $\dfrac{11-3s}{s^2+2s-3} = \dfrac{11-3s}{(s+3)(s-1)} = \dfrac{A}{(s-1)} + \dfrac{B}{s+3}$

$\therefore 11-3s = A(s+3) + B(s-1)$

Put $s=1$: $8 = 4A$

$\therefore A = 2$

Put $s=-3$ $20 = -4B$

$\therefore B = -5$

$\therefore \dfrac{11-3s}{(s+3)(s-1)} = \dfrac{2}{s-1} + \dfrac{(-5)}{s+3}$

$\therefore L^{-1}\left\{\dfrac{11-3s}{(s+3)(s-1)}\right\} = L^{-1}\left(\dfrac{2}{s-1}\right) + L^{-1}\left\{\dfrac{-5}{s+3}\right)$

$= 2e^{t} - 5e^{-3t}$ (4)

[7]

QUESTION 5

$$y''(x) + 6y'(x) + 13y(x) = 0 \quad y(0) = 3 \quad y'(0) = 7$$

$$\left(s^2 Y(s) - s y(0) - y'(0)\right) + 6\left(sY(s) - y(0)\right) + 13Y(s) = 0$$

$$s^2 Y(s) - 3s - 7 + 6sY(s) - 18 + 13Y(s) = 0$$

$$(s^2 + 6s + 13)Y(s) = 3s + 25 \qquad 4$$

$$Y(s) = \frac{3s + 25}{(s+3)^2 + 2^2}$$

$$= \frac{3(s+3) + 16}{(s+3)^2 + 2^2}$$

$$\therefore y(x) = 3L^{-1}\left\{\frac{(s+3)}{(s+3)^2 + 2^2}\right\} + 8L^{-1}\left\{\frac{2}{(s+3)^2 + 2^2}\right\}$$

$$= 3e^{-3x}\cos 2x + 8e^{-3x}\sin 2x$$

(8)

[8]

QUESTION 6 (D-operator)

$$x''(t)+4x'(t)+5x(t)=80\sin 5t \qquad x(0)=x'(0)=0$$

$$(D^2+4D+5)x = 80\sin 5t$$

x_{CF} $m^2+4m+5=0$

$$m=\frac{-4\pm\sqrt{16-20}}{2}$$

$$=-2\pm j$$

$$\therefore x_{CF}=e^{-2t}(A\cos t+B\sin t)$$

$$x_{PI}=\frac{1}{D^2+4D+5}\{80\sin 5t\}$$

$$=80\frac{1}{-(5)^2+4D+5}\{\sin 5t\}$$

$$=80\frac{1}{4D-20}\{\sin 5t)$$

$$=20\frac{(D+5)}{(D-5)(D+5)}\{\sin 5t)$$

$$=20\frac{(D+5)}{D^2-25}\{\sin 5t)$$

$$=\frac{20}{-50}(5\cos 5t+5\sin 5t)$$

$$=-2\cos 5t-2\sin 5t$$

(8)

$$x_{gen}=e^{-2t}(A\cos t+B\sin t)-2\cos 5t-2\sin 5t$$

$$\therefore 0=A-2 \implies A=2$$

$$x'(t)=-2e^{-2t}(A\cos t+B\sin t)+e^{-2t}(-A\sin t+B\cos t)$$

$$0=-4+B+10 \implies B=-6$$

$\cancel{+}$ 10 cos 5t + 10 sin 5t

$$x(t)=e^{-2t}(2\cos t-\cancel{6}\,14\sin t)-2\cos 5t-2\sin 5t$$

QUESTION 6 Laplace transforms. alternative method.

$$x''(t) + 4x'(t) + 5x(t) = 80 \sin 5t.$$

$$\left(s^2 X(s) - s\,x(0) - x'(0)\right) + 4\left(sX(s) - x(0)\right) + 5X(s) = \frac{80(5)}{s^2+25}$$

$$(s^2 + 4s + 5)\,X(s) = \frac{80(5)}{s^2+25}$$

$$X(s) = \frac{400}{(s^2+25)(s^2+4s+5)}$$

$$= \frac{As+B}{s^2+25} + \frac{Bs+C}{s^2+4s+5}$$

$$= \frac{2s+18}{s^2+4s+5} - \frac{2s+10}{s^2+25}$$

$(s+2)^2 + 1$

Inverse Laplace

$$x(t) = 2e^{-2t}(\cos t + 7\sin t) - 2\sin 5t - 2\cos 5t$$