

Semester 2 Assignment 01
Due Date: 21 August

This assignment contributes 10% to your year mark.

Source: Paper May 2016

$$1.1 \quad x \frac{dy}{dx} \sin\left(\frac{y}{x}\right) = y \sin\left(\frac{y}{x}\right) - x$$

$$\frac{dy}{dx} \sin\left(\frac{y}{x}\right) = \frac{y}{x} \sin\left(\frac{y}{x}\right) - 1$$

Put $y = vx$

$$\text{Then } \frac{dy}{dx} = v + x \frac{dv}{dx}$$

$$\left(v + x \frac{dv}{dx}\right) \sin v = v \sin v - 1$$

$$v \sin v + x \sin v \frac{dv}{dx} = v \sin v - 1$$

$$x \sin v \frac{dv}{dx} = -1$$

$$(\sin v) dv = -\frac{dx}{x}$$

$$-\cos v = -\ln x + C$$

$$\cos\left(\frac{y}{x}\right) = \ln x + C$$

(6)

$$1.2 \quad \frac{dy}{dx} + y = 2xy^2$$

$$y^{-2} \frac{dy}{dx} + y^{-1} = 2x$$

$$\text{Let } z = y^{-1}$$

$$\text{then } \frac{dz}{dx} = -y^{-2} \frac{dy}{dx}$$

$$\frac{dz}{dx} + (-1)z = -2x$$

$$\int P dx = \int (-1) dx = -x$$

$$z = e^x \int -2xe^{-x} dx$$

use integration by parts

$$= e^x \left[(-2x)(-e^{-x}) - \int (-e^{-x})(-2) dx \right]$$

$$= 2e^x \left[xe^{-x} + e^{-x} + C \right]$$

(7)

$$y^{-1} = 2x + 2 + Ce^x$$

$$1.3 \quad (1+y)^2 = (1+x^2) \frac{dy}{dx} \quad (3)$$

$$\frac{dx}{1+x^2} = \frac{dy}{(1+y)^2}$$

$$\arctan x = -\frac{1}{1+y} + C$$

$$1.4 \quad \sec 3x dy + y^2 e^{\sin 3x} dx = 0 \quad \text{Separable} \quad (4)$$

$$y^2 e^{\sin 3x} dx = -\sec 3x dy$$

$$\frac{e^{\sin 3x}}{\sec 3x} dx = -\frac{dy}{y^2}$$

$$\cos 3x e^{\sin 3x} dx = -(y^{-2}) dy$$

$$\frac{1}{3} e^{\sin 3x} = \frac{1}{y} + C$$

[20]

Maximum: [20]