Tutorial Letter 202/1/2015

Time Series III

STA3704

Semester 1

Department of Statistics

Solutions to Assignment 2

BAR CODE

Question 1

(a) The AR(2) process characteristic equation is $1 - \phi_1 B - \phi_2 B^2 = 0$.

AR(2) is stationary if the absolute roots of this quadratic equation (in B) $1 - \phi_1 B - \phi_2 B^2 = 0$ are both greater than 1.

That is, $\phi_2 B^2 + \phi_1 B - 1 = 0$

The real roots are:

$$B = \left| \frac{-\phi_1 \ \pm \ \sqrt{\phi_1^2 \ + \ 4\phi_2}}{2\phi_2} \right|.$$

Therefore for an AR(2) process to be stationary, then

$$\left| \frac{-\phi_1 \ \pm \ \sqrt{\phi_1^2 \ + \ 4\phi_2}}{2\phi_2} \right| > 1$$

(5 marks)

(b) For an AR(2) model,

$\gamma_k = \phi_1 \gamma_{k-1} + \phi_2 \gamma_{k-2} + cov(Y_{t-k}, \ \varepsilon_t)$

for $k = 0$,

$\gamma_0 = \phi_1 \gamma_1 + \phi_2 \gamma_2 + \sigma_\varepsilon^2 \qquad (1)$

for $k > 0$,

$\gamma_k = \phi_1 \gamma_{k-1} + \phi_2 \gamma_{k-2}$

Noting that $\rho_k = \dfrac{\gamma_k}{\gamma_0}$,

$\therefore \hat{\rho}_0 = 1$, for $k = 0$ and

$\rho_k = \phi_1 \rho_{k-1} + \phi_2 \rho_{k-2}$ for $k \geq 1$ (10 marks)

(c) Substituting $k = 1$ and $k = 2$ in ρ_k above and solving simultaneously, we have

$$\hat{\phi}_1 = \frac{\hat{\rho}_1(1 - \hat{\rho}_2)}{1 - \hat{\rho}_1^2} = \frac{0.736(1 - 0.304)}{1 - (0.736)^2} = 1.1177;$$

$$\hat{\phi}_2 = \frac{\hat{\rho}_2 - \hat{\rho}_1^2}{1 - \hat{\rho}_1^2} = \frac{0.304 - (0.736)^2}{1 - (0.736)^2} = -0.5186$$

(7 marks)

Question 2

(a) (i) Cuts off after $lag\ 1$, that is, has nonzero correlation only at $lag\ 1$. Could be positive or negative but must be between -0.5 and $+0.5$.

(ii) Cuts of after $lag\ 1$ of the twice differenced series.

(iii) Autocorrelations decay exponentially starting from lag 0. If $\phi > 0$, then all autocorrelations are positive, If $\phi < 0$, then all autocorrelations alternate negative, positive, negative (or damped since wave)

(iv) Decay exponentially after the first difference starting from $lag\ 0$.

(v) Autocorrelations tail off as exponential decay after $lag\ 0$, that is, starting from $lag\ 1$.

(10 marks)

(b) (i) MA(1): $Z_t = e_t - \theta e_{t-1}$

(ii) IMA(2,1): $(1-B)^2 Z_t = e_t - \theta e_{t-1}$

(iii) AR(1): $Z_t = \phi Z_{t-1} + e_t$ or $(1-\phi B)Z_t = e_t$

(iv) ARI(1,2): $(1-B)^2 Z_t = \phi Z_{t-1} + e_t$

(v) ARMA(1,1): $Z_t = \phi Z_{t-1} + e_t - \theta e_{t-1}$ or $(1-\phi B)Z_t = (1-\theta B)e_t$

(2 marks each = 10 marks)

(c)\ (i) Plot and examine the series for stationarity and choose proper transformation.

Plot and examine the sample ACF and PACF of the original series to determine if differencing is necessary.

Compute and examine the sample ACF and PACF of the properly transformed and differenced series to identify the orders of p and q for the relevant time series model.

Test the deterministic trend θ_0 when $d > 0$. (4 marks)

(ii) Logarithm

Reciprocal

Square–root

Inverse square root (3 marks)

(iii) To test if there is still a pattern in the residuals. If there is, it means that some factors have still not be accounted for by the model. That is, the model is incomplete.

(4 marks)

Question 3

(a) $(1-0.6B)(X_t - 9) = e_t \Rightarrow X_t - 9 = 0.6X_{t-1} - 9 + e_t$

the general forecasting equation is

$\hat{X}(l) = 9 + (0.6)^l (X_t - 9), l \geqslant 1$

Thus, the respective forecasts of X_{101} and X_{102} are:

$\hat{X}(1) = \hat{X}_{101} = 9 + (0.6)(8.9 - 9) = 8.94$ (5 marks)

$\hat{X}(2) = \hat{X}_{102} = 9 + (0.6)^2(8.9 - 9) = 8.964$ (5 marks)

(b) 95% Confidence limits for the forecasts in part (a) are

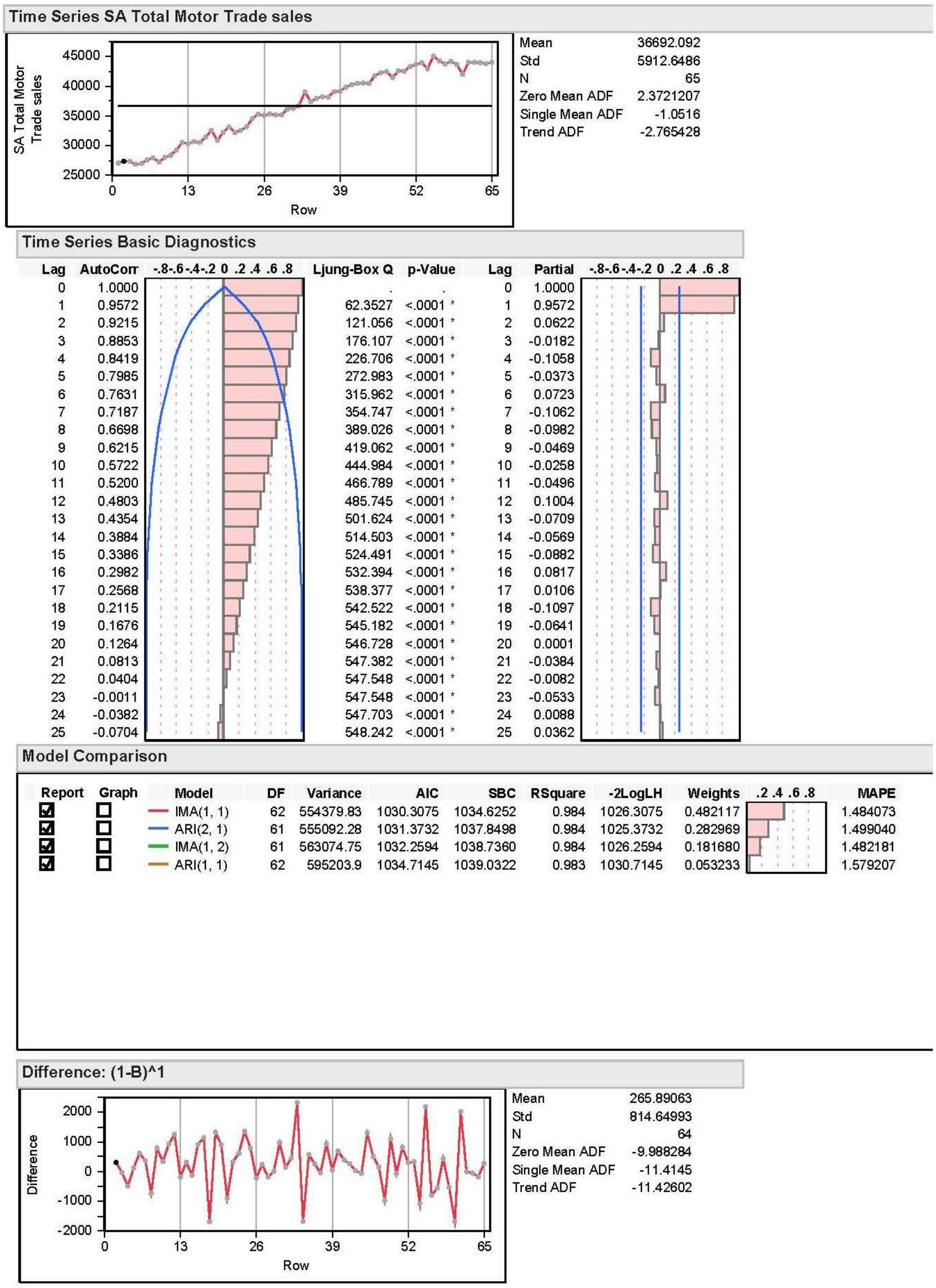
X_{101} is $8.94 \pm 1.96\sqrt{0.1}$ or $(8.32, 9.56)$ (3 marks)

For X_{102}, we have $8.964 \pm 1.96\sqrt{1 + (0.6)^2}\sqrt{0.1}$ or $(8.241, 9.687)$ (3 marks)

Question 4

(a) The time plot has an upward trend, the ACF shows a slow decay and the PACF cuts off at lag 1, hence, the series is not stationary. (3 marks)

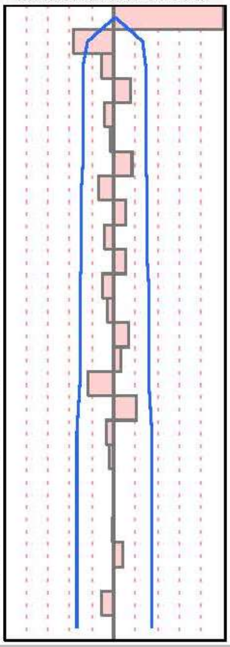
(Time Plot, ACF and PACF charts) (5 marks)

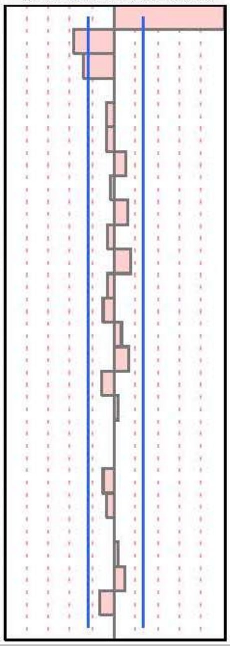


Time Series SA Total Motor Trade sales

Difference: (1-B)^1

Lag	AutoCorr	Ljung-Box Q	p-Value	Lag	Partial
0	1.0000	.	.	0	1.0000
1	-0.3622	8.7962	0.0030 *	1	-0.3622
2	-0.1176	9.7377	0.0077 *	2	-0.2863
3	0.1534	11.3671	0.0099 *	3	-0.0064
4	-0.0881	11.9130	0.0180 *	4	-0.0660
5	-0.0325	11.9886	0.0349 *	5	-0.0689
6	0.1616	13.8893	0.0309 *	6	0.1155
7	-0.1336	15.2114	0.0334 *	7	-0.0296
8	0.1144	16.1991	0.0396 *	8	0.1267
9	-0.0851	16.7555	0.0527	9	-0.0545
10	0.1114	17.7258	0.0598	10	0.1490
11	-0.1030	18.5714	0.0692	11	-0.0601
12	-0.0542	18.8102	0.0932	12	-0.0999
13	0.1445	20.5407	0.0825	13	0.0659
14	0.0644	20.8909	0.1045	14	0.1464
15	-0.2415	25.9174	0.0389 *	15	-0.1135
16	0.2082	29.7309	0.0194 *	16	0.0449
17	-0.0749	30.2347	0.0247 *	17	0.0008
18	-0.0369	30.3595	0.0341 *	18	-0.0008
19	-0.0010	30.3596	0.0474 *	19	-0.1010
20	0.0008	30.3596	0.0642	20	-0.0685
21	-0.0129	30.3759	0.0847	21	0.0060
22	0.0866	31.1308	0.0935	22	0.0388
23	-0.0013	31.1310	0.1196	23	0.1009
24	-0.1128	32.4737	0.1157	24	-0.1189
25	-0.0022	32.4743	0.1447	25	0.0047





Model: IMA(1, 1)

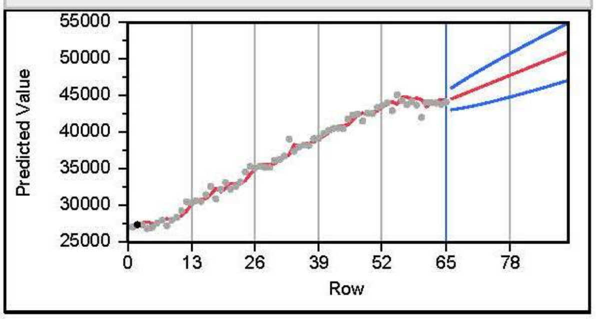
Model Summary

DF	62	Stable	Yes
Sum of Squared Errors	34371549.8	Invertible	Yes
Variance Estimate	554379.835		
Standard Deviation	744.566877		
Akaike's 'A' Information Criterion	1030.30748		
Schwarz's Bayesian Criterion	1034.62525		
RSquare	0.98421386		
RSquare Adj	0.98395924		
MAPE	1.4840729		
MAE	540.690236		
-2LogLikelihood	1026.30748		

Parameter Estimates

Term	Lag	Estimate	Std Error	t Ratio	Prob>|t|	Constant Estimate
MA1	1	0.49152	0.10726	4.58	<.0001 *	270.365524
Intercept	0	270.36552	47.37186	5.71	<.0001 *	

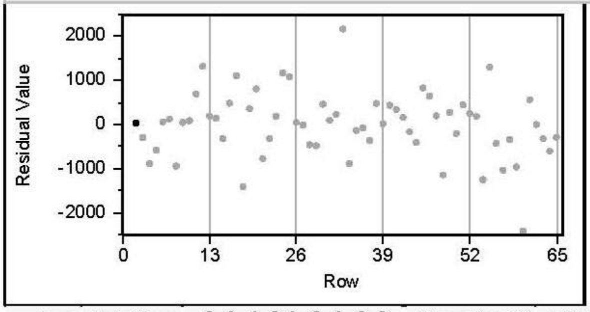
Forecast



Time Series SA Total Motor Trade sales

Model: IMA(1, 1)

Residuals



Lag	AutoCorr	Ljung-Box Q	p-Value
0	1.0000	.	.
1	0.0061	0.0025	0.9604
2	-0.0743	0.3789	0.8274
3	0.1054	1.1478	0.7656
4	-0.0419	1.2712	0.8662
5	0.0129	1.2831	0.9367
6	0.1411	2.7333	0.8415
7	-0.0462	2.8915	0.8949
8	0.0843	3.4277	0.9047
9	-0.0296	3.4948	0.9414
10	0.0625	3.8001	0.9559
11	-0.0834	4.3545	0.9584
12	-0.0299	4.4272	0.9744
13	0.1550	6.4169	0.9297
14	0.0584	6.7047	0.9455
15	-0.1687	9.1594	0.8690
16	0.1170	10.3631	0.8470
17	-0.0555	10.6399	0.8747
18	-0.0751	11.1583	0.8875
19	-0.0316	11.2523	0.9151
20	-0.0013	11.2524	0.9394
21	0.0243	11.3105	0.9564
22	0.0869	12.0695	0.9559
23	-0.0195	12.1087	0.9688
24	-0.1308	13.9142	0.9486
25	-0.0270	13.9934	0.9618

Lag	Partial
0	1.0000
1	0.0061
2	-0.0744
3	0.1069
4	-0.0505
5	0.0309
6	0.1243
7	-0.0394
8	0.1038
9	-0.0688
10	0.1047
11	-0.1324
12	-0.0053
13	0.1356
14	0.0442
15	-0.1407
16	0.0815
17	-0.0452
18	-0.0531
19	-0.0948
20	0.0142
21	0.0613
22	0.0446
23	0.0118
24	-0.1423
25	0.0509

Model: IMA(1, 2)

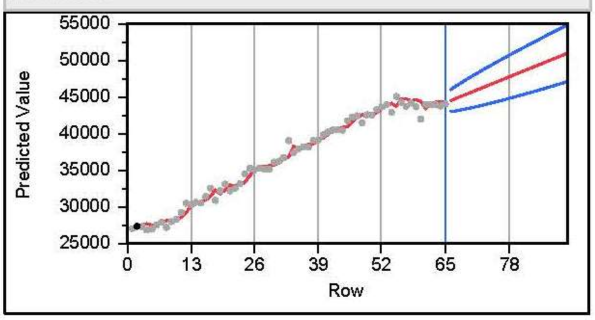
Model Summary

DF	61	Stable	Yes
Sum of Squared Errors	34347559.8	Invertible	Yes
Variance Estimate	563074.751		
Standard Deviation	750.383069		
Akaike's 'A' Information Criterion	1032.25936		
Schwarz's Bayesian Criterion	1038.73601		
RSquare	0.98422343		
RSquare Adj	0.98370616		
MAPE	1.4821813		
MAE	540.677331		
-2LogLikelihood	1026.25936		

Parameter Estimates

Term	Lag	Estimate	Std Error	t Ratio	Prob>|t|	Constant Estimate
MA1	1	0.47287	0.13477	3.51	0.0009 *	271.118161
MA2	2	0.02992	0.13604	0.22	0.8267	
Intercept	0	271.11816	46.52195	5.83	<.0001 *	

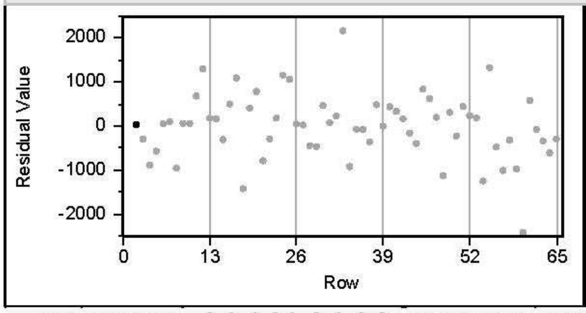
Forecast



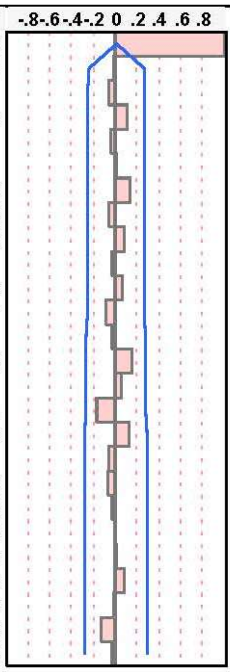
Time Series SA Total Motor Trade sales

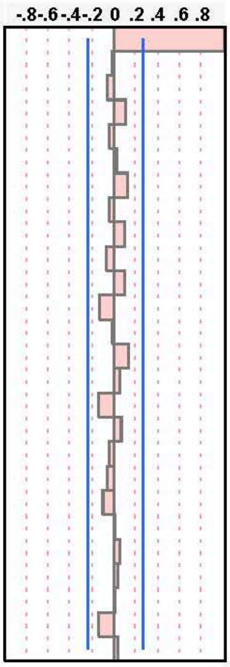
Model: IMA(1, 2)

Residuals



Lag	AutoCorr	Ljung-Box Q	p-Value	Lag	Partial
0	1.0000	.	.	0	1.0000
1	-0.0094	0.0059	0.9388	1	-0.0094
2	-0.0561	0.2203	0.8957	2	-0.0562
3	0.1182	1.1876	0.7560	3	0.1175
4	-0.0381	1.2897	0.8631	4	-0.0404
5	0.0151	1.3060	0.9343	5	0.0286
6	0.1440	2.8165	0.8315	6	0.1281
7	-0.0512	3.0108	0.8840	7	-0.0411
8	0.0890	3.6086	0.8906	8	0.1019
9	-0.0341	3.6981	0.9301	9	-0.0723
10	0.0666	4.0452	0.9453	10	0.1036
11	-0.0807	4.5647	0.9504	11	-0.1274
12	-0.0301	4.6382	0.9690	12	-0.0125
13	0.1502	6.5067	0.9258	13	0.1335
14	0.0591	6.8015	0.9421	14	0.0512
15	-0.1714	9.3329	0.8595	15	-0.1447
16	0.1220	10.6424	0.8310	16	0.0765
17	-0.0591	10.9568	0.8588	17	-0.0421
18	-0.0720	11.4332	0.8751	18	-0.0524
19	-0.0303	11.5194	0.9051	19	-0.0943
20	-0.0019	11.5197	0.9316	20	0.0133
21	0.0199	11.5586	0.9508	21	0.0626
22	0.0832	12.2550	0.9519	22	0.0438
23	-0.0196	12.2945	0.9657	23	0.0146
24	-0.1286	14.0404	0.9457	24	-0.1461
25	-0.0292	14.1326	0.9594	25	0.0437





Model: ARI(2, 1)

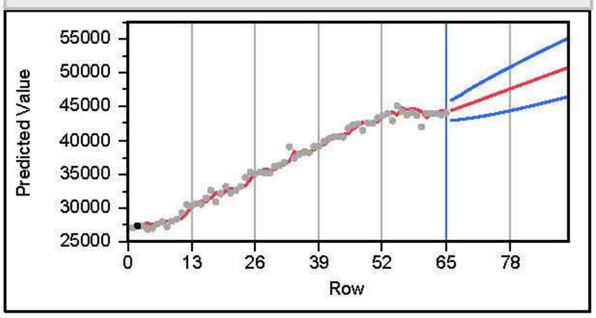
Model Summary

DF	61	Stable	Yes
Sum of Squared Errors	33860629.1	Invertible	Yes
Variance Estimate	555092.28		
Standard Deviation	745.045153		
Akaike's 'A' Information Criterion	1031.37318		
Schwarz's Bayesian Criterion	1037.84983		
RSquare	0.9844515		
RSquare Adj	0.98394171		
MAPE	1.49903992		
MAE	546.99924		
-2LogLikelihood	1025.37318		

Parameter Estimates

Term	Lag	Estimate	Std Error	t Ratio	Prob>|t|	Constant Estimate
AR1	1	-0.4593	0.11906	-3.86	0.0003 *	464.982253
AR2	2	-0.2795	0.11806	-2.37	0.0211 *	
Intercept	0	267.4033	52.77585	5.07	<.0001 *	

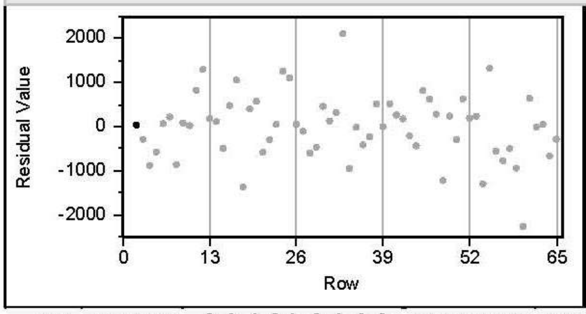
Forecast



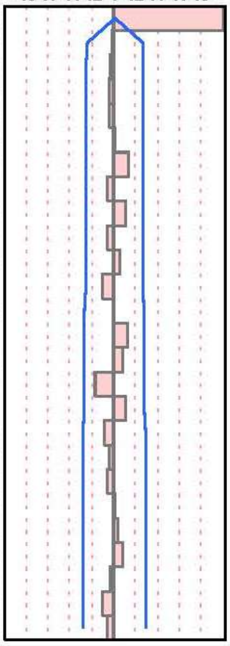
Time Series SA Total Motor Trade sales

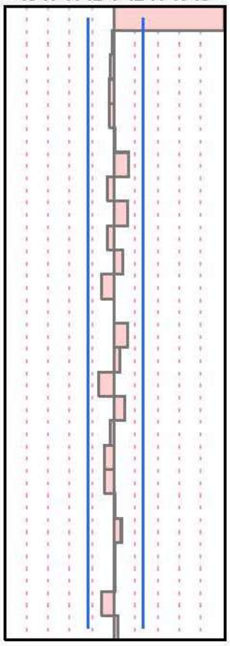
Model: ARI(2, 1)

Residuals



Lag	AutoCorr	Ljung-Box Q	p-Value	Lag	Partial
0	1.0000	.	.	0	1.0000
1	-0.0092	0.0057	0.9398	1	-0.0092
2	-0.0266	0.0537	0.9735	2	-0.0266
3	-0.0418	0.1747	0.9816	3	-0.0423
4	-0.0380	0.2764	0.9913	4	-0.0397
5	0.0108	0.2848	0.9979	5	0.0077
6	0.1399	1.7102	0.9443	6	0.1368
7	-0.0571	1.9514	0.9625	7	-0.0577
8	0.1175	2.9923	0.9348	8	0.1259
9	-0.0572	3.2434	0.9539	9	-0.0498
10	0.0604	3.5284	0.9661	10	0.0773
11	-0.0991	4.3114	0.9599	11	-0.1076
12	-0.0003	4.3114	0.9771	12	-0.0041
13	0.1211	5.5257	0.9617	13	0.1321
14	0.0869	6.1641	0.9622	14	0.0500
15	-0.1629	8.4520	0.9044	15	-0.1460
16	0.1117	9.5500	0.8890	16	0.1008
17	-0.0792	10.1134	0.8988	17	-0.0342
18	-0.0467	10.3135	0.9212	18	-0.0823
19	-0.0580	10.6295	0.9357	19	-0.0828
20	0.0105	10.6401	0.9550	20	0.0092
21	0.0454	10.8421	0.9658	21	0.0705
22	0.0885	11.6305	0.9645	22	0.0194
23	-0.0274	11.7079	0.9747	23	0.0192
24	-0.1014	12.7931	0.9694	24	-0.1098
25	-0.0495	13.0581	0.9758	25	0.0350





Model: ARI(1, 1)

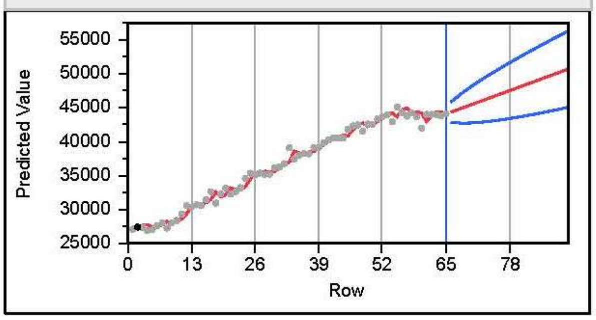
Model Summary

DF	62	Stable	Yes
Sum of Squared Errors	36902641.7	Invertible	Yes
Variance Estimate	595203.899		
Standard Deviation	771.494588		
Akaike's 'A' Information Criterion	1034.71448		
Schwarz's Bayesian Criterion	1039.03225		
RSquare	0.98305788		
RSquare Adj	0.98278462		
MAPE	1.57920665		
MAE	573.052691		
-2LogLikelihood	1030.71448		

Parameter Estimates

Term	Lag	Estimate	Std Error	t Ratio	Prob>|t|	Constant Estimate
AR1	1	-0.3567	0.11536	-3.09	0.0030 *	360.411396
Intercept	0	265.6578	70.25187	3.78	0.0004 *	

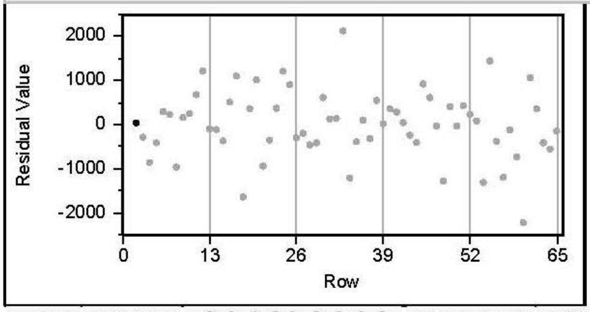
Forecast



Time Series SA Total Motor Trade sales

Model: ARI(1, 1)

Residuals



Lag	AutoCorr	-.8-.6-.4-.2 0 .2 .4 .6 .8	Ljung-Box Q	p-Value	Lag	Partial	-.8-.6-.4-.2 0 .2 .4 .6 .8
0	1.0000		.	.	0	1.0000	
1	-0.1076		0.7768	0.3781	1	-0.1076	
2	-0.2382		4.6412	0.0982	2	-0.2527	
3	0.1147		5.5517	0.1356	3	0.0598	
4	-0.0647		5.8459	0.2110	4	-0.1114	
5	-0.0121		5.8564	0.3204	5	0.0119	
6	0.1417		7.3190	0.2924	6	0.1012	
7	-0.0599		7.5849	0.3706	7	-0.0203	
8	0.0587		7.8449	0.4488	8	0.1150	
9	-0.0176		7.8686	0.5474	9	-0.0422	
10	0.0673		8.2232	0.6070	10	0.1431	
11	-0.1104		9.1937	0.6040	11	-0.1456	
12	-0.0531		9.4228	0.6665	12	-0.0246	
13	0.1917		12.4667	0.4898	13	0.1265	
14	0.0437		12.6281	0.5560	14	0.0618	
15	-0.2014		16.1262	0.3737	15	-0.1195	
16	0.1402		17.8566	0.3324	16	0.0873	
17	-0.0269		17.9214	0.3938	17	-0.0307	
18	-0.0788		18.4916	0.4237	18	-0.0257	
19	-0.0161		18.5159	0.4883	19	-0.1161	
20	-0.0047		18.5180	0.5533	20	-0.0250	
21	0.0191		18.5540	0.6137	21	0.0287	
22	0.1067		19.6979	0.6019	22	0.0644	
23	-0.0124		19.7137	0.6591	23	0.0360	
24	-0.1477		22.0185	0.5782	24	-0.1298	
25	0.0124		22.0351	0.6337	25	0.0778	

(b) Yes, since there is a linear upward trend, difference of order 1 may be recommended.

(3 marks)

(c) (See the fitted models above). (10 marks)

Using the diagnostic tests, the model comparison statistics and the residual statistics, it could be seen that the four models could actually modelled the process since their residuals are white noised. But the AIC, SBC, Variance, and the estimates of each model pointed to IMA(1,1) as the best model. (5 marks)

(d) Using IMA(1,1) model, the forecasted values are as follows:

Actual SA Total Motor Trade sales	Row	Predicted SA Total Motor Trade sales	Std Err Pred SA Total Motor Trade sales	Residual SA Total Motor Trade sales	Upper CL (0.95) SA Total Motor Trade sales	Lower CL (0.95) SA Total Motor Trade sales
	66	44490.3157	744.56688		45949.63995	43030.99142
	67	44760.6812	835.29328		46397.82595	43123.53647
	68	45031.0467	917.08772		46828.50563	43233.58783
	69	45301.4123	992.16173		47246.01352	43356.81099
	70	45571.7778	1061.9416		47653.14504	43490.41052
	71	45842.1433	1127.4107		48051.82773	43632.45888
	72	46112.5088	1189.2813		48443.45732	43781.56034
	73	46382.8744	1248.0885		48829.08294	43936.66577
	74	46653.2399	1304.2469		49209.51686	44096.96289
	75	46923.6054	1358.0851		49585.40322	44261.80758
	76	47193.9709	1409.8688		49957.26302	44430.67883
	77	47464.3364	1459.8168		50325.5248	44603.14809

(5 marks)

TOTAL MARKS **[100 marks]**