Tutorial Letter 202/1/2015

Time Series III STA3704

Semester 1

Department of Statistics

Solutions to Assignment 2

BAR CODE





Learn without limits.

Question 1

(a) The AR(2) process characteristic equation is $1 - \phi_1 B - \phi_2 B^2 = 0$.

AR(2) is stationary if the absolute roots of this quadratic equation (in B) $1 - \phi_1 B - \phi_2 B^2 = 0$ are both greater than 1.

That is, $\phi_2 B^2 + \phi_1 B - 1 = 0$

The real roots are:

$$B = \left| \frac{-\phi_1 \pm \sqrt{\phi_1^2 + 4\phi_2}}{2\phi_2} \right|.$$

Therefore for an AR(2) process to be stationary, then

$$\left| \frac{-\phi_1 \pm \sqrt{\phi_1^2 + 4\phi_2}}{2\phi_2} \right| > 1$$

(5 marks)

(b) For an AR(2) model,

$$\gamma_{k} = \phi_{1}\gamma_{k-1} + \phi_{2}\gamma_{k-2} + cov(Y_{t-k}, \varepsilon_{t})$$

for $k = 0$,
 $\gamma_{0} = \phi_{1}\gamma_{1} + \phi_{2}\gamma_{2} + \sigma_{\varepsilon}^{2}$ (1)
for $k > 0$,
 $\gamma_{k} = \phi_{1}\gamma_{k-1} + \phi_{2}\gamma_{k-2}$
Noting that $\rho_{k} = \frac{\gamma_{k}}{\gamma_{0}}$,
 $\therefore \hat{\rho}_{0} = 1$, for $k = 0$ and
 $\rho_{k} = \phi_{1}\rho_{k-1} + \phi_{2}\rho_{k-2}$ for $k \ge 1$ (10 marks)
Substituting $k = 1$ and $k = 2$ in ρ_{k} above and solving simultaneously, we have

(c) Substituting
$$k = 1$$
 and $k = 2$ in ρ_k above and solving simultaneously, we have

$$\hat{\phi}_1 = \frac{\hat{\rho}_1(1-\hat{\rho}_2)}{1-\hat{\rho}_1^2} = \frac{0.736(1-0.304)}{1-(0.736)^2} = 1.1177;$$
$$\hat{\phi}_2 = \frac{\hat{\rho}_2 - \hat{\rho}_1^2}{1-\hat{\rho}_1^2} = \frac{0.304 - (0.736)^2}{1-(0.736)^2} = -0.5186$$

(7 marks)

Question 2

- (a) (i) Cuts off after lag 1, that is, has nonzero correlation only at lag 1. Could be positive or negative but must be between -0.5 and +0.5.
 - (ii) Cuts of after lag 1 of the twice differenced series.
 - (iii) Autocorrelations decay exponentially starting from lag 0. If $\phi > 0$, then all autocorrelations are positive, lf $\phi < 0$, then all autocorrelations alternate negative, positive, negative (or damped since wave)
 - (iv) Decay exponentially after the first difference starting from lag 0.
 - (v) Autocorrelations tail off as exponential decay after lag 0, that is, starting from lag 1.

(10 marks)

- (b) (i) MA(1): $Z_t = e_t \theta e_{t-1}$
 - (ii) IMA(2,1): $(1-B)^2 Z_t = e_t \theta e_{t-1}$
 - (iii) AR(1): $Z_t = \phi Z_{t-1} + e_t$ or $(1 \phi B)Z_t = e_t$
 - (iv) ARI(1,2): $(1-B)^2 Z_t = \phi Z_{t-1} + e_t$
 - (v) ARMA(1,1): $Z_t = \phi Z_{t-1} + e_t \theta e_{t-1}$ or $(1 \phi B)Z_t = (1 \theta B)e_t$

(2 marks each = 10 marks)

(c) (i) Plot and examine the series for stationarity and choose proper transformation.
 Plot and examine the sample ACF and PACF of the original series to determine if differencing is necessary.

Compute and examine the sample ACF and PACF of the properly transformed and differenced series to identify the orders of p and q for the relevant time series model. Test the deterministic trend θ_0 when d > 0. (4 marks)

(ii) Logarithm

Reciprocal Square-root

Inverse square root

(3 marks)

(iii) To test if there is still a pattern in the residuals. If there is, it means that some factors have still not be accounted for by the model. That is, the model is incomplete.

(4 marks)

Question 3

(a) $(1 - 0.6B)(X_t - 9) = e_t \Rightarrow X_t - 9 = 0.6X_{t-1} - 9 + e_t$

the general forecasting equation is

 $\begin{aligned} \hat{X}(l) &= 9 + (0.6)^{l} (X_{t} - 9), l \ge 1 \\ \text{Thus, the respective forecasts of } X_{101} \text{ and } X_{102} \text{ are:} \\ \hat{X}(1) &= \hat{X}_{101} = 9 + (0.6)(8.9 - 9) = 8.94 \\ \hat{X}(2) &= \hat{X}_{102} = 9 + (0.6)^{2}(8.9 - 9) = 8.964 \end{aligned}$ (5 marks) (5 marks) (b) 95% Confidence limits for the forecasts in part (a) are

X_{101} is $8.94 \pm 1.96\sqrt{0.1}$ or $(8.32, 9.56)$	(3 marks)
For X_{102} , we have $8.964 \pm 1.96\sqrt{1 + (0.6)^2}\sqrt{0.1}$ or $(8.241, 9.687)$	(3 marks)

Question 4

(a) The time plot has an upward trend, the ACF shows a slow decay and the PACF cuts off at lag 1, hence, the series is not stationary. (3 marks)

(Time Plot, ACF and PACF charts)

(5 marks)



4

Time Series	SA Tota	I Motor	Trade sale	es
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5	-0.0001		11 9886	0.0100	5	-0.0000	
6	0.1616		13 8893	0.0040	6	0.1155	
7	-0.1336		15 2114	0.0334 *	7	-0.0296	
8	0.1144		16 1991	0.0396 *	Ŕ	0.1267	
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18	-0.0369		30.3595	0.0341 *	18	-0.0008	1111
19	-0.0010		30.3596	0.0474	19	-0.1010	
20	8000.0		30.3596	0.0642	20	-0.0685	
21	-0.0129	2444 4 244	30.3759	0.0847	21	0.0060	
22	0.0866		31.1308	0.0935	22	0.0388	
23	-0.0013		31.1310	0.1196	23	0.1009	
24	-0.1128		32.4737	0.1157	24	-0.1189	
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8	0.084	3		3.4	277 0.9	047	8	0.1038		
9	-0.029	6		3.4	948 0.9	414 550	9	-0.0688		
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19	-0.031	6		11.2	523 0.9	151 394	19	-0.0948	111	
20	0.024	3		11.2	105 0.9	564	21	0.0142	111	
22	0.086	9		12.0	695 0.9	559	22	0.0446	111	
23	-0.019	15		12.1	087 0.9	688	23	0.0118	111	
24 25	-0.130	0		13.9	934 0.9	480 618	24 25	0.0509		
odel:	IMA(1, 2)								
Mode	l Summa	ıry								
DF		analas	6	1 Stable	Yes					
Sum of Variance	Squared E	rrors	34347559.	3 Invertible 1	Yes					
Standar	rd Deviatio	n	750.38306	9						
Akaike's	s 'A' Inform	ation Criterion	1032.2593	6						
Schwarz	rz's Bayesia ro	an Criterion	1038.7360	1 3						
RSquar	re Adi		0.9842234	6						
MAPE			1.482181	3						
MAE	(le = 10)		540.67733	1						
-∠LogLi Param	neter Est	timates	1026.2593	5			ľ			
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Time Series SA Total Motor Trade sales

Model: IMA(1, 2)
Residuals
2000 - an 1000 - 0 - -2000 - -2000 -
0 13 26 39 52 65 Row
Lag AutoCorr -8-6-4-20.2.4.6.8 Ljung-Box Q p-Value Lag Partial -8-6-4-20.2.4.6.8 0 1.0000 1 -0.0094 0 0.0059 0.9388 1 -0.0094 2 -0.0661 0 0.0059 0.9388 1 -0.0094 4 -0.0381 0.1182 1.1876 0.7660 3 0.1175 4 -0.0381 0.9343 5 0.0286 0.2203 0.8957 2 -0.0662 1.1876 0.7560 3 0.1175 1.2897 0.8631 4 -0.0411 3.0081 0.9343 5 0.0286 8 0.1019 -0.0723 3 0.1666 -0.0301 3.6981 0.9301 9 -0.0723 10 0.0666 6.8015 0.9421 1 -0.1724 -0.1326 11 -0.0807 -0.1714 9.3329 0.8595 15 -0.1447 16 0.1220 0.9321 14
25 -0.0292 14.1326 0.9594 25 0.0437 14.1326 0.9594 25 0.0437
Model Summary
model Summary
Sum of Squared Errors 33860629.1 Invertible Yes Variance Estimate 555092.28 Standard Deviation 745.045153 Akaike's 'A' Information Criterion 1031.37318 Schwarz's Bayesian Criterion 1037.84983 RSquare 0.98344515 RSquare Adj 0.98394171 MAPE 1.49903992 MAE 546.99924 -2LogLikelihood 1025.37318
Parameter Estimates
Term Lag Estimate Std Error t Ratio Prob> t Estimate AR1 1 -0.4593 0.11906 -3.86 0.0003 [±] 464.982253 AR2 2 -0.2795 0.11806 -2.37 0.0211 [±] Intercept 0 267.4033 52.77585 5.07 <.0001 [±]
55000 9 50000 45000 45000 40000 30000 25000 0 13 26 39 52 65 78 Row

me Serie	s SA Tot	al Motor	Trade sale	S						
Model: A	RI(2, 1)									
Residua	als									
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 ۵	- 00									
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ual /	0-•*	ag 4a	6 0 0 A	*** * * ***	9 ⁰ 0					
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	0	13	26 3 Bow	9 52	65					
	AutoCorr	- 8- 6- 4-	20 24 6 5	Liung-Box O	n-Value	lan	Partial	- 8- 6- 4-	20 24	6.8
0	1.0000					0	1.0000			10 10
1	-0.0092			0.0057	0.9398	1	-0.0092		: : : :	
2	-0.0200	1 1 1		0.1747	0.9735	2	-0.0200	111		1 1
4	-0.0380	111		0.2764	0.9913	4	-0.0397	111		
5	0.0108			0.2848	0.9979	5	0.0077	111		
7	-0.0571	111		1.9514	0.9625	7	-0.0577	111	Ē	1.1
8	0.1175			2.9923	0.9348	8	0.1259			
9 10	-0.0572		H	3.2434	0.9539	9 10	-0.0498		4	
11	-0.0991			4.3114	0.9599	11	-0.1076	111		
12	-0.0003			4.3114	0.9771	12	-0.0041			1.1
13	0.1211		9 - 9 8 8 8	6.1641	0.9617	13	0.1321	111		
15	-0.1629	111		8.4520	0.9044	15	-0.1460	111	Ľ.	11
16	0.1117	1.1.1.		9.5500	0.8890	16 17	0.1008	1 1 1	: 44: :	1 1
17	-0.0792			10.1134	0.8988	18	-0.0342			
19	-0.0580			10.6295	0.9357	19	-0.0828			
20	0.0105			10.6401	0.9550	20	0.0092			
21	0.0454			10.8421	0.9658	21	0.0705			
23	-0.0274		f i i i i	11.7079	0.9747	23	0.0192	111		
24 25	-0.1014		3 4 3 1 3 3 3	12.7931	0.9694	24 25	-0.1098			
Aodel: A	RI(1, 1)		I		0.0700	20	0.0000 [
Model S	Summan	1								
DF			6	2 Stable Yes						
Sum of Se	quared Erro	ors	36902641.	7 Invertible Yes						
Variance	Estimate		595203.89 771 49458	9 8						
Akaike's '	A' Informat	ion Criterio	n 1034.7144	8						
Schwarz's	s Bayesian	Criterion	1039.0322	5						
RSquare RSquare	Adi		0.9830578	8 2						
MAPE			1.5792066	5						
MAE	libood		573.05269	1						
Parame	annoou ater Estir	nates	1030.7144	0						
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Term	Lag	Estimate	Std Error	t Ratio Prob> t	Estimat	e				
AR1 Intercent	1 0	-0.3567	0.11536 70.25187	-3.09 0.0030 * 3.78 0.0004 *	360.41139	b				
Forecas	st			1.1.5 0.0004						
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L 300		Jan 1								
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250	0	13 26	39 5	2 65 78	_					
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odel: A	RI(1, 1)								
Residu	als								
20 10 Sesidral <ali>20 30 30 40 40 40 40 40 40 40 40 40 40 40 40 40</ali>			0 00 0 0 0 0		• •				
	0	13 26	39 Row	52	65				
Lag	AutoCorr	8642 0 .2	.4 .6 .8	Ljung-Box Q	p-Value	Lag	Partial	8642 0 .2	.4 .6 .8
0	1.0000					0	1.0000	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	
1	-0.1076		10 1 1	0.7768	0.3781	1	-0.1076		1 1 1
2	-0.2382			4.6412	0.0982	2	-0.2527		1 1 1
3	0.1147		1.1.1	5.5517	0.1356	3	0.0598		
4	-0.0647	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	1.1.1	5.8459	0.2110	4	-0.1114		1.1.1
5	-0.0121	1 1 1 1 E L 1		5.8564	0.3204	5	0.0119		1.1.1
6	0.1417			7.3190	0.2924	6	0.1012		
7	-0.0599			7.5849	0.3706	7	-0.0203		1.1.1
8	0.0587			7.8449	0.4488	8	0.1150		
9	-0.0176	1 1 1 1 C 1		7.8686	0.5474	9	-0.0422		1.1.1
10	0.0673		1 1 1	8.2232	0.6070	10	0.1431		1.1.1.
11	-0.1104		1 1 2	9.1937	0.6040	11	-0.1456		1 1 1
12	-0.0531			9.4228	0.6665	12	-0.0246		1 1 1
13	0.1917		1 1 1	12.4667	0.4898	13	0.1265		1 1 1
14	0.0437		111	12.6281	0.5560	14	0.0618		1 1 1
15	-0.2014		1 1 1	16.1262	0.3737	15	-0.1195		1 1 1
16	0.1402		1.1.1	17.8566	0.3324	16	0.0873		1 1 1
17	-0.0269	1 1 3 3 4 1	1 1 2	17.9214	0.3938	17	-0.0307		1.1.1
18	-0.0788	111111	114	18.4916	0.4237	18	-0.0257		1 1 1
19	-0.0161	1 1 1 1 1 1	111	18.5159	0.4883	19	-0.1161	· · · · · · · · · · ·	1.1.1
20	-0.0047		1 1 1	18.5180	0.5533	20	-0.0250	333143	1 1 1
21	0.0191	1111	1 1 1	18.5540	0.6137	21	0.0287	1111	1 1 1
22	0.1067	1 1 1 1 1 1	1.1.4	19.6979	0.6019	22	0.0644		1 1 1
23	-0.0124	1 1 1 1 1	112	19./137	0.6591	23	0.0360		1 1 1
24	-0.14//		1 1 1	22.0185	0.5782	24	-0.1298		1 1 1
20	001/4			// 11.151	11 0.3.37	10	1111/78		A 10 10

(b) Yes, since there is a linear upward trend, difference of order 1 may be recommended.

(3 marks)

(c) (See the fitted models above).

(10 marks)

Using the diagnostic tests, the model comparison statistics and the residual statistics, it could be seen that the four models could actually modelled the process since their residuals are white noised. But the AIC, SBC, Variance, and the estimates of each model pointed to IMA(1,1) as the best model. (5 marks)

1	(P)	Lleina	$IN/\Delta/1$	1)	model	tho	forecasted	عميادير	aro	as follows:	
	u)	USING	IIVIA(T	,ı)	mouer,	uie	Inecasteu	values	ale	as 10110ws.	

Actual SA Total Motor Trade sales	Row	Predicted SA Total Motor Trade sales	Std Err Pred SA Total Motor Trade sales	Residual SA Total Motor Trade sales	Upper CL (0.95) SA Total Motor Trade sales	Lower CL (0.95) SA Total Motor Trade sales
	66	44490.3157	744.56688		45949.63995	43030.99142
	67	44760.6812	835.29328	7.	46397.82595	43123.53647
	68	45031.0467	917.08772		46828.50563	43233.58783
	69	45301.4123	992.16173		47246.01352	43356.81099
	70	45571.7778	1061.9416		47653.14504	43490.41052
	71	45842.1433	1127.4107		48051.82773	43632.45888
	72	46112.5088	1189.2813		48443.45732	43781.56034
	73	46382.8744	1248.0885		48829.08294	43936.66577
	74	46653.2399	1304.2469		49209.51686	44096.96289
	75	46923.6054	1358.0851		49585.40322	44261.80758
	76	47193.9709	1409.8688		49957.26302	44430.67883
	77	47464.3364	1459.8168		50325.5248	44603.14809

TOTAL MARKS

(5 marks)

[100 marks]