

STA3702

October/November 2016

STATISTICAL INFERENCE III

Duration 2 Hours

100 Marks

EXAMINERS

FIRST

SECOND

EXTERNAL

PROF P NDLOVU

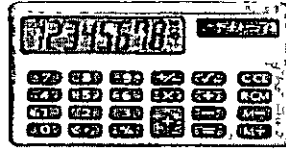
PROF JO OLAOMI

DR EL RAATH

Use of a non-programmable pocket calculator is permissible

Closed book examination

This examination question paper remains the property of the University of South Africa and may not be removed from the examination venue



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This paper consists of 3 pages

INSTRUCTIONS

- 1 Answer all questions
- 2 Show intermediate steps

Abbreviations	
<i>MLE</i>	Maximum Likelihood Estimator
<i>MSE</i>	Mean Square Error
<i>pdf</i>	probability density function
<i>cdf</i>	cumulative distribution function
<i>MVUE</i>	Minimum Variance Unbiased Estimator
<i>MME</i>	Method of Moments Estimator
<i>CRLB</i>	Cramer Rao Lower Bound
<i>pmf</i>	probability mass function
<i>mgf</i>	moment generating function
<i>re</i>	relative efficiency

[TURN OVER]

Question 1

[Total marks=30]

Let X_1, X_2, \dots, X_n be a random sample from a distribution which belongs to the regular 1-parameter exponential family, and with probability density/mass function

$$f(x|\theta) = a(\theta)g(x) \exp\{b(\theta)R(x)\}, \quad x \in \mathcal{X} \subset (-\infty, \infty) \text{ and } \theta \in \Theta \subset (-\infty, \infty)$$

where $a(\theta) > 0$ and $g(x) > 0$

(a) Show that another form of $f(x|\theta)$ is

$$f(x|\theta) = \exp\{b(\theta)R(x) - a^*(\theta) + g^*(x)\} \quad (4)$$

(b) Suppose that X_1, X_2, \dots, X_n is random sample from a distribution with pmf

$$p(x|\theta) = \begin{cases} \theta(1-\theta)^x & \text{if } 0 < \theta < 1 \text{ and } x = 0, 1, 2, \dots \\ 0 & \text{elsewhere} \end{cases}$$

(i) Show that $p(x|\theta)$ belongs to the regular 1-parameter exponential family (4)

(ii) What is the complete sufficient statistic for θ ? **Justify your answer** (3)

(iii) **Refer to part (a) above** $E[R(X)] = \frac{a^{*'}(\theta)}{b'(\theta)}$ Use this information to find the mean of the complete sufficient statistic for θ in part (ii) above (3)

(iv) Find the maximum likelihood estimator (MLE) of θ (8)

(v) Find the method of moments estimator (MME) of θ (4)

(vi) Show that \bar{X} is the minimum variance unbiased estimator (MVUE) of $\frac{1-\theta}{\theta}$ (4)

Question 2

[Total marks=30]

Let X_1, X_2, \dots, X_n be a random sample from a distribution with pdf

$$f(x|\theta) = \begin{cases} \theta \exp\{-\theta x\} & \text{if } x \geq 0 \text{ and } \theta > 0 \\ 0 & \text{otherwise} \end{cases}$$

Given $E[X] = \frac{1}{\theta}$

(a) Prove that $\sum_{i=1}^n X_i$ is a minimal sufficient statistic for θ (8)

(b) Suppose that $n = 200$, $\sum_{i=1}^{200} x_i = 20$, $\sum_{i=1}^{200} x_i^2 = 100$, and $\sum_{i=1}^{200} x_i^3 = 250$ Determine

(i) the method of moments estimate (MME) of θ , (3)

(ii) the maximum likelihood estimate (MLE) of θ . (9)

(iii) the observed information, $I(\mathbf{x})$, of θ , and (4)

(iv) and the approximate standard errors of the estimates of θ in (i) and (ii) (6)

[TURN OVER]

Question 3**[Total marks=18]**

Suppose that 2, 1, 2, 0, 2, 2, 2, 0, 1, 2 are ten independent observations from a population whose discrete distribution has a probability mass function

$$p(x|\theta) = \begin{cases} \frac{e^{-\theta} \theta^x}{x!} & \text{if } \theta > 0 \text{ and } x = 0, 1, 2, \dots \\ 0 & \text{otherwise} \end{cases}$$

- (a) Find the method of moments estimate (*MME*) of θ (4)
- (b) Find the maximum likelihood estimate (*MLE*) of e^θ (9)
- (c) Find the approximate standard error for the estimate in part (a) (5)

Question 4**[Total marks=22]**

A random variable X has probability density function

$$f(x|\theta) = \begin{cases} \theta e^{-\theta x} & \text{if } x > 0 \text{ and } \theta > 0 \\ 0 & \text{otherwise} \end{cases}$$

It is desired to test the null hypothesis $H_0: \theta = 1$ against the alternative $H_1: \theta = 2$. A random sample X_1 of size $n = 1$ is to be used. Given $\int f(x|\theta) dx = -e^{-\theta x}$

- (a) Calculate (i) the level of significance and (ii) the power of the test which rejects H_0 if $X_1 \geq \frac{1}{2}$ (15)
- (b) Consider the test which rejects H_0 if $X_1 \geq c$. Find c for which the level of significance of the test is 0.1 (7)

TOTAL [100]