UNIVERSITY EXAMINATIONS

UNIVERSITEITSEKSAMENS

UNISA university of south africa

STA3702

October/November 2016

STATISTICAL INFERENCE III

Duration 2 Hours

100 Marks

EXAMINERS

FIRST	PROF P NDLOVU
SECOND	PROF JO OLAOMI
EXTERNAL	DR EL RAATH

Use of a non-programmable pocket calculator is permissible

Closed book examination

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This paper consists of 3 pages

INSTRUCTIONS

1 Answer all questions

2 Show intermediate steps

Abbreviations	
MLE	Maximum Likelihood Estimator
MSE	Mean Square Error
pdf	probability density function
cdf	cumulative distribution function
$MVUE$	Minimum Variance Unbiased Estimator
MME	Method of Moments Estimator
$CRLB$	Cramer Rao Lower Bound
pmf	probability mass function
mgf	moment generating function
re	relative efficiency

[TURN OVER]

Question 1 **[Total marks=30]**

Let $X_1, X_2, \ldots, X_n$ be a random sample from a distribution which belongs to the regular 1-parameter exponential family, and with probability density/mass function

$$f(x|\theta) = a(\theta)g(x)\exp\{b(\theta)R(x)\}, \quad x \in \chi \subset (-\infty, \infty) \text{ and } \theta \in \Theta \subset (-\infty, \infty)$$

where $a(\theta) > 0$ and $g(x) > 0$

(a) Show that another form of $f(x|\theta)$ is

$$f(x|\theta) = \exp\{b(\theta)R(x) - a^*(\theta) + g^*(x)\}$$

(4)

(b) Suppose that $X_1, X_2, \ldots, X_n$ is random sample from a distribution with *pmf*

$$p(x|\theta) = \begin{cases} \theta(1-\theta)^x & \text{if } 0 < \theta < 1 \text{ and } x = 0, 1, 2, \ldots, \\ 0 & \text{elsewhere} \end{cases}$$

(i) Show that $p(x|\theta)$ belongs to the regular 1-parameter exponential family **(4)**

(ii) What is the complete sufficient statistic for θ? **Justify your answer** **(3)**

(iii) **Refer to part (a) above** $E[R(X)] = \dfrac{a^{*\prime}(\theta)}{b'(\theta)}$ Use this information to find the mean of the complete sufficient statistic for θ in part (ii) above **(3)**

(iv) Find the maximum likelihood estimator (MLE) of θ **(8)**

(v) Find the method of moments estimator (MME) of θ **(4)**

(vi) Show that $\bar{X}$ is the minimum variance unbiased estimator ($MVUE$) of $\dfrac{1-\theta}{\theta}$ **(4)**

Question 2 **[Total marks=30]**

Let $X_1, X_2, \ldots, X_n$ be a random sample from a distribution with *pdf*

$$f(x|\theta) = \begin{cases} \theta\exp\{-\theta x\} & \text{if } x \geq 0 \text{ and } \theta > 0 \\ 0 & \text{otherwise} \end{cases}$$

Given $E[X] = \frac{1}{\theta}$

(a) Prove that $\sum_{i=1}^{n} X_i$ is a minimal sufficient statistic for θ **(8)**

(b) Suppose that $n = 200$, $\sum_{i=1}^{200} x_i = 20$, $\sum_{i=1}^{200} x_i^2 = 100$, and $\sum_{i=1}^{200} x_i^3 = 250$. Determine

(i) the method of moments estimate (MME) of θ, **(3)**

(ii) the maximum likelihood estimate (MLE) of θ, **(9)**

(iii) the observed information, $I(\mathbf{x})$, of θ, and **(4)**

(iv) and the approximate standard errors of the estimates of θ in (i) and (ii) **(6)**

[TURN OVER]

Question 3 **[Total marks=18]**

Suppose that 2, 1, 2, 0, 2, 2, 2, 0, 1, 2 are ten independent observations from a population whose discrete distribution has a probability mass function

$$p(x|\theta) = \begin{cases} \frac{e^{-\theta}\theta^{x}}{x!} & \text{if } \theta > 0 \text{ and } x = 0, 1, 2, \ldots \\ 0 & \text{otherwise} \end{cases},$$

(a) Find the method of moments estimate (MME) of θ **(4)**

(b) Find the maximum likelihood estimate (MLE) of e^{θ} **(9)**

(c) Find the approximate standard error for the estimate in part (a) **(5)**

Question 4 **[Total marks=22]**

A random variable X has probability density function

$$f(x|\theta) = \begin{cases} \theta e^{-\theta x} & \text{if } x > 0 \text{ and } \theta > 0 \\ 0 & \text{otherwise} \end{cases}$$

It is desired to test the null hypothesis $H_0: \theta = 1$ against the alternative $H_1: \theta = 2$. A random sample X_1 of size $n = 1$ is to be used. **Given** $\int f(x|\theta)dx = -e^{-\theta x}$

(a) Calculate (i) the level of significance and (ii) the power of the test which rejects H_0 if $X_1 \geq \frac{1}{2}$ **(15)**

(b) Consider the test which rejects H_0 if $X_1 \geq c$. Find c for which the level of significance of the test is 0.1. **(7)**

TOTAL [100]