Tutorial letter 105/2/2018

Applied Statistics II

STA2601

Semester 2

Department of Statistics

TRIAL EXAMINATION PAPER

Dear Student
Congratulations if you obtained examination admission by submitting assignment 1. I would like to take the opportunity of wishing you well in the coming examinations. I hope you found the module stimulating.

The examination

Please note the following with regard to the examination:

* The duration of the examination paper is **two-hours**. You will be able to complete the set paper in 2 hours, but there will be no time for dreaming or sitting on questions you are unsure about. Make sure that you take along a functional scientific calculator that you can operate with ease as it can save you some time. My advice to you would be to do those questions you find easy *first;* then go back to the ones that need more thinking. I do not mind to mark questions in whatever order you do them, just *make sure that you number them clearly!*
* A copy of the list of formulae is attached to the trial examination paper. Please ensure that you know how to test the various hypotheses.
* All the necessary statistical tables will be supplied (see the trial paper).
* Pocket calculators are necessary for doing the calculations.
* Working through (and understanding!) ALL the examples and exercises in the study guide, workbook and in the assignments as well as the trial paper will provide beneficial supplementary preparation.
* Make sure that you know all the theory as well as the practical applications.
* All the chapters in the study guide are equally important and don't try to spot!
* Start preparing early and don't hesitate to call or email me if something is unclear.

The enclosed trial examination papers should give you a good indication of what to expect in the examination.

Best wishes with your preparation for the examination and do not hesitate to contact me if you have any questions about STA2601.

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Trial paper 1

Reserve two hours for yourself and do the trial paper under exam conditions on your own!

Duration: 2 hours **100 Marks**

INSTRUCTIONS

1. Answer ALL questions.
2. Marks will not be given for answers only. Show clearly how you solve each problem.
3. For all hypothesis-testing problems always give
   (i) the null and alternative hypothesis to be tested;
   (ii) the test statistic to be used; and
   (iii) the critical region for rejecting the null hypothesis.
4. Justify your answer completely if you make use of JMP output to answer a question.

May/June 2018 Paper One Final Examination

QUESTION 1

(a) Name one distribution which is symmetric about zero. (1)

(b) Complete the following:

(i) The statistic T is called an unbiased estimator for the parameter θ if .... (1)

(ii) Let $X_1, ..., X_n$ be a random sample from a population with unknown variance σ^2.

An unbiased estimator for the population variance σ^2 is given by $\widehat{\sigma^2} =$ .... (1)

(c) Give, in general terms, the three main steps when calculating a maximum likelihood estimator for a parameter θ if the p.d.f. is $f(X;\theta)$. (Give formulae where appropriate.) (4)

[7]

QUESTION 2

(a) Let $X_1,\ X_2$ be independent random variables such that

$$E(X_1) = c_1\theta_1 \quad \text{and} \quad E(X_2) = c_1\theta_1 + c_2\theta_2$$

where $\theta_1$and θ_2 are unknown parameters and c_1 and c_2 known constants. Find the **least squares estimators** for $\theta_1$and θ_2

(7)

(b) Let $X_1, ..., X_n$ be a random sample from a $n(\mu;\ \sigma^2)$ distribution.

Let $A_1 = \frac{1}{n}\sum_{i=1}^{n}(X_i - \overline{X})^2$. Show that $E(A_1) = \left[\frac{(n-1)}{n}\right]\sigma^2$.

(5)

[12]

QUESTION 3

One hundred weaner lambs were weighed before being sent to market and the weights (in kilograms) grouped into the following table of observed frequencies:

Class interval (lamb mass)	Observed frequency	*Expected frequency
< 13.5	1	0
$13.5 - 15.0$	1	1
$15.0 - 16.5$	2	4
$16.5 - 18.0$	14	12
$18.0 - 19.5$	17	24
$19.5 - 21.0$	31	28
$21.0 - 22.5$	24	20
$22.5 - 24.0$	7	8
$24.0 - 25.5$	2	2
> 25.5	1	1

Note: The expected frequencies were computed under the assumption of a $n\,(20;\;4)$ distribution.

The observed frequencies can be represented in a histogram as follows:

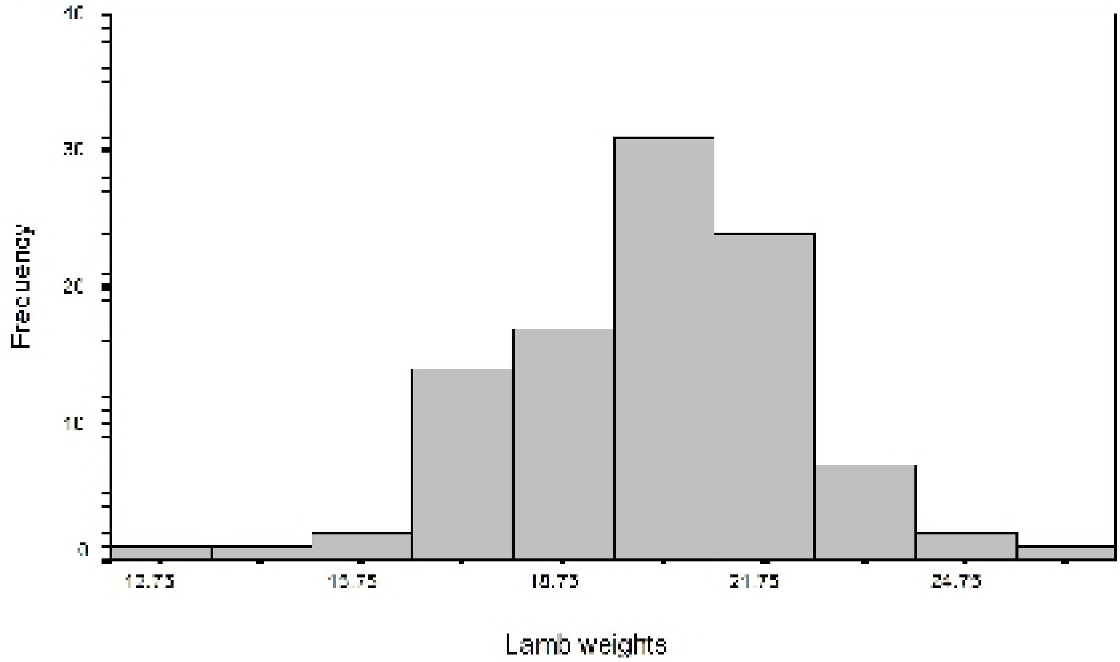


Figure 1: Histogram of lamb weights

(a) Does the histogram suggest that the sample originates from a normal distribution? Why (not)? (2)

(b) Compute the chi-square goodness-of-fit statistic Y^2 to test whether the sample originates from a normal distribution with $\mu = 20$ and $\sigma^2 = 4$. (6)

(c) A statistical package computed the following statistics:

$$\overline{x} = 20 \qquad \Sigma\,(x_i - \overline{x})^2 = 426.4 \qquad \Sigma\,(x_i - \overline{x})^3 = -282.3 \qquad \Sigma\,(x_i - \overline{x})^4 = 6\,958$$

Compute the statistics B_1 and B_2 as given in the formula sheet on page 8, as an alternative test for normality. Perform the two-sided tests for skewness and kurtosis at the 10% level of significance. (14)

(d) Explain the differences (if any) between the conclusions of the two different tests for normality in (b) and (c). (3)

[25]

QUESTION 4

(a) A cell phone company conducts a survey in order to find out if awareness about 2 of its top selling cell phone models is equal among its customers based on a radom sample of $n = 12$ customers. The table below shows results obtained from the survey:

		Degree of awareness	
		Yes	**No**
Cell phone model	**Model 1**	6	1
	Model 2	1	4

Test the hypothesis that the customers are equally aware of models 1 and 2 at the 0.05 level of significance (9)

(b) In a random sample of 39 observations from a bivariate normal distribution, it was computed that $r = 0.2$ (i.e. the sample correlation coefficient).

(i) Find a 95% confidence interval for ρ. (8)

(ii) How can you use this confidence interval to test $H_0 : \rho = 0$ against $H_1 : \rho \neq 0$ at the 5% level of significance? (2)

[19]

QUESTION 5

The durability of tyres is tested by using a machine with a metallic device that wears down the tyres. The time it takes (in hours) for a tyre to blow is then recorded. "Safe Taxi" taxi company is trying to decide which brand of tyres to use for the coming year. Random samples from two different brands of tyres were drawn, the blowout times measured and the following statistics were computed from the data:

Brand A $N_1 = 25$ $\sum_{i=1}^{25} X_i = 83$ hours $\sum_{i=1}^{25} \left(X_i - \overline{X}\right)^2 = 11.0976$

Brand B $N_2 = 49$ $\sum_{j=1}^{49} Y_j = 196$ hours $\sum_{j=1}^{49} \left(Y_j - \overline{Y}\right)^2 = 11.5248$

(a) Do you think it is reasonable to assume that the two groups may be considered as *independent groups*? (2)

(b) Use the 5% level of significance and test whether the **variances of the two populations** from which these samples were drawn, differ significantly. (9)

(c) Test at the 5% level of significance whether the mean blowout time for tyres of Brand B is significantly higher than the mean blowout time for the tyres of Brand A. (Show how you interpolate for the critical value.) (7)

(d) Comment on the assumptions that you have to make in order to perform the test in (c). (3)

[21]

QUESTION 6

In order to determine the effect of a foliar-spray on the production of tomato plants, 12 tomato plants were sprayed with different doses of the foliar-spray. The following data were observed.

Dose x_i	**Yield** y_i	$(x_i - \overline{x})$	$(x_i - \overline{x})^2$	$y_i\,(x_i - \overline{x})$
1	14	-3	9	-42
1	11	-3	9	-33
1	16	-3	9	-48
3	23	-1	1	-23
3	19	-1	1	-19
3	20	-1	1	-20
5	20	1	1	20
5	30	1	1	30
5	27	1	1	27
7	35	3	9	105
7	31	3	9	93
7	30	3	9	90
48	276	0	60	180

Consider the simple linear regression model $y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$ where the ε_i's are independent $n\left(0;\ \sigma^2\right)$ random variables.

The following SAS JMP output is obtained.

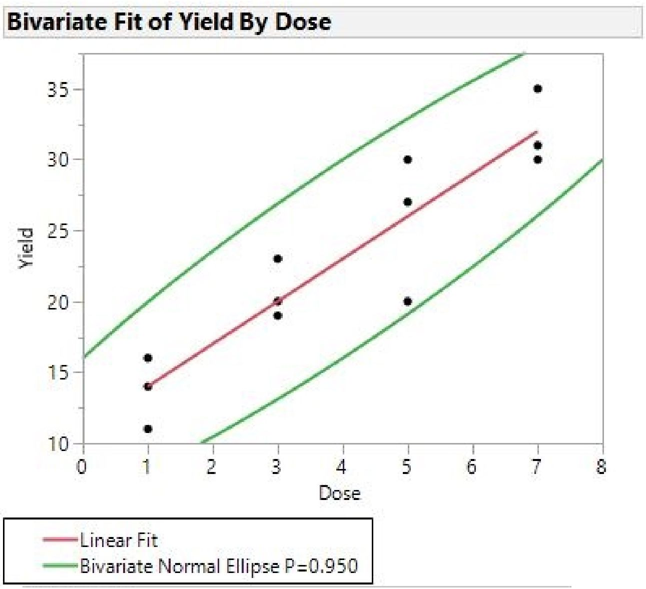


Figure 2a: The scatter plot

Linear Fit

Yield = 11 + 3*Dose

Summary of Fit

RSquare	0.857143
RSquare Adj	0.842857
Root Mean Square Error	3
Mean of Response	23
Observations (or Sum Wgts)	12

Lack Of Fit

Source	DF	Sum of Squares	Mean Square	F Ratio
Lack Of Fit	2	2.000000	1.0000	0.0909
Pure Error	8	88.000000	11.0000	Prob > F
Total Error	10	90.000000		0.9140
				Max RSq
				0.8603

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	1	540.00000	540.000	60.0000
Error	10	90.00000	9.000	Prob > F
C. Total	11	630.00000		<.0001*

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	11	1.774824	6.20	0.0001*
Dose	3	0.387298	7.75	<.0001*

Bivariate Normal Ellipse P=0.950

Variable	Mean	Std Dev	Correlation	Signif. Prob	Number
Dose	4	2.335497	0.92582	<.0001*	12
Yield	23	7.567875			

Figure 2a: The simple linear regression model

(a) Show how the regression line of y on x is obtained and *show all workings*. (7)

(b) Find a 95% confidence interval for the slope of the regression line computed in (a). (4)

(c) What is the expected yield for $x = 4$? (1)

(d) Find a 95% confidence interval for the expected yield of a new observation at $x = 4$. (4)

[16]

[100]

Trial paper 2

Reserve two hours for yourself and do the trial paper under exam conditions on your own!

Duration: 2 hours **100 Marks**

INSTRUCTIONS

1. Answer ALL questions.
2. Marks will not be given for answers only. Show clearly how you solve each problem.
3. For all hypothesis-testing problems always give
   - (i) the null and alternative hypothesis to be tested;
   - (ii) the test statistic to be used; and
   - (iii) the critical region for rejecting the null hypothesis.
4. Justify your answer completely if you make use of JMP output to answer a question.

May/June 2018 Paper Two Final Examination

QUESTION 1

(a) Give the definition of an unbiased estimator. (1)

(b) Explain what is meant by "*the significance level of a test*" (2)

(c) Explain what is meant by "*the power of a test*". (2)

(d) Name two methods of obtaining point estimators. (2)

(e) Name three methods of testing whether a sample comes from a normal distribution. (3)

[10]

QUESTION 2

Let $X_1; X_2; \ldots; X_n$ be independent random variables such that

$$E(X_i) = \theta_1, \qquad i = 1, \ \ldots, \ (n-1)$$

$$\text{and } E(X_n) = \theta_1 + \theta_2..$$

Find the least squares estimates of θ_1 and θ_2.

[8]

QUESTION 3

An aluminium company is experimenting with a new design for batteries. The main objective is to maximize the expected service life of a battery. Thirty batteries of the new design are tested and failed at the following ages (in days):

632	752	813	856	948	977	1 023	1 121	1 159	1 168	1 185	1 253	1 296	1 311	1 342
1 356	1 469	1 478	1 503	1 536	1 586	1 609	1 683	1 699	1 712	1 821	1 944	1 982	1 992	2 194

You may assume that

$$\sum_{i=1}^{n} X_i = 41\,400 \qquad \sum_{i=1}^{n}(X_i - \overline{X})^2 = 4\,623\,074 \qquad \sum_{i=1}^{n}(X_i - \overline{X})^3 = 144\,776\,994$$

We wish to test the null hypothesis that the observations come from a **normal distribution** by using a goodness-of-fit test. The **30 observed values** were classified into the following **six classes with equal probsability for each class interval.**

Equal probability intervals	Expected frequency	Count marks	Observed frequency
$-\infty < X \leq 1\,000.79$	5	𝍸 |	6
$1\,000.79 < X \leq 1\,210.41$	5	𝍸	5
$1\,210.41 < X \leq 1\,380$	5	𝍸	5
$1\,380 < X \leq 1\,549.59$	5	||||	4
$1\,549.59 < X \leq 1\,759.21$	5	𝍸	5
$1\,759.21 < X \leq \infty$	5	𝍸	5
Total	30		

(a) State the null and alternative hypothesis. (2)

(b) Under H_0 the distribution is not completely specified and we have to estimate the two unknown parameters by using the **maximum likelihood estimators** $\widehat{\mu}$ and $\widehat{\sigma}^2$ for the goodness-of-fit test. Calculate the values of the two unknown parameters. (5)

(c) Show that the first interval is $-\infty < X \leq 1\,000.79$. (4)

(d) Use the output in Figure 1 to make a conclusion on whether the data follows a normal distribution. Comment using all available information. Use $\alpha = 0.10$.

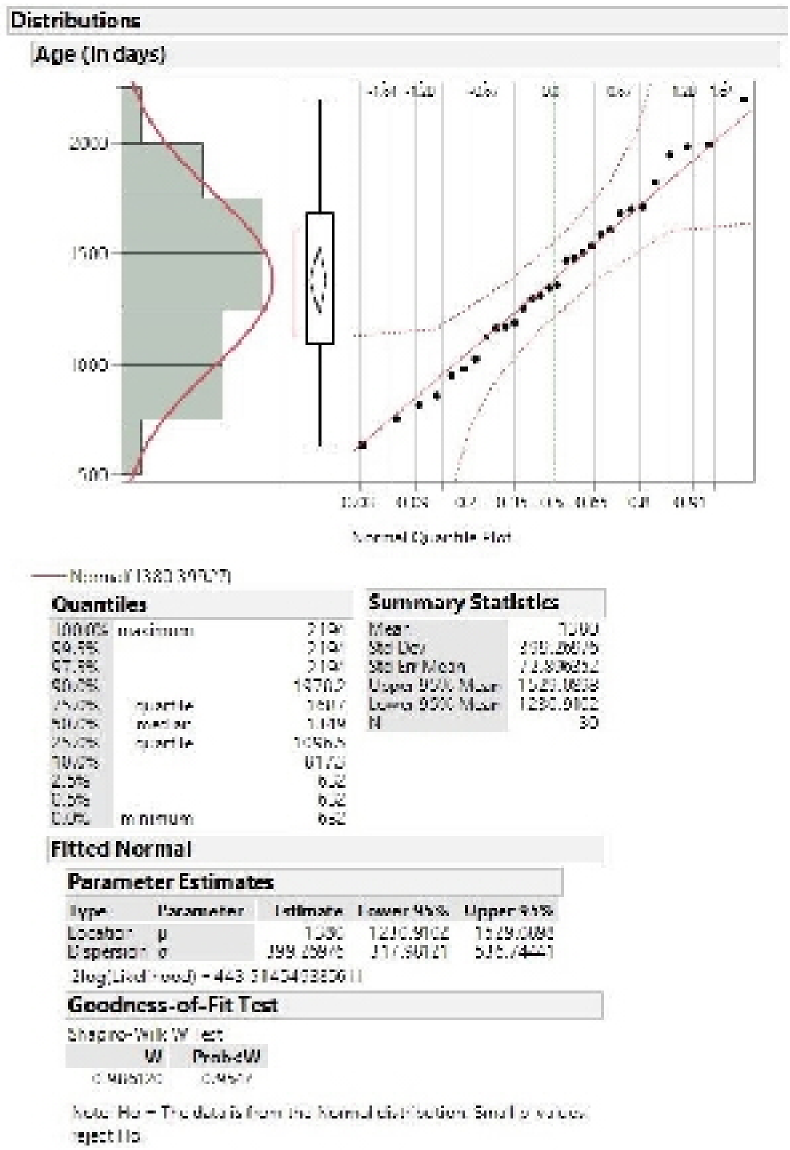


Figure 1

(6)

(e) The following SAS JMP output was obtained:

Test Mean

Hypothesized Value	1300
Actual Estimate	1380
DF	29
Std Dev	399.27

	t Test
Test Statistic	1.0974
Prob > |t|	0.2815
Prob > t	0.1407
Prob < t	0.8593

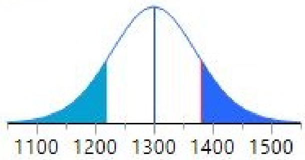


Figure 2

(i) Suppose that it is known that the standard (or the "old") design for the batteries has a mean service life of $1\,300$ days. Can management conclude that the new design is superior to the standard design with respect to mean service life? (Test at the 2.5% level of significance.) (4)

(ii) Show that the 95% (two-sided) confidence interval for the mean service life, μ, of batteries of the new design is $1\,230.91$ to $1\,529.09$. (5)

(iii) Show that the 95% two-sided confidence interval for the standard deviation (σ) of the new design is to 317.98 to 536.74. (5)

(iv) What assumptions do you make to do the confidence interval in (iii)? (1)

[32]

QUESTION 4

(a) In a summer tea-part in Pretoria, Pretoria, a lady claimed to be able to discern, by taste alone, whether a cup of tea with milk had the tea poured first or the milk poured first. An experiment was performed by a researcher to see if her claim is valid. Twelve cups of tea are prepared and presented to her in random order. Six had the milk poured first, and six had the tea poured first. The lady tasted each one and rendered her opinion.

The results are summarized in a 2 $\times$ 2 table below:

		Lady says		**Row**
		Tea first	**Milk first**	**total**
Poured	**Tea**	5	1	6
first	**Milk**	1	5	6
Column total		6	6	12

Does the information above support the theory that the lady has no discerning ability? Test at the 5% level of significance. (9)

(b) Fifteen patients with high blood pressure are chosen randomly and their blood pressure measured before, and two hours after, taking a certain drug.

Patient	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Before (b)	210	169	187	160	167	176	185	206	173	146	174	201	198	148	154
After (a)	201	165	166	157	147	145	168	180	147	136	151	168	179	129	131

The following SAS JMP output was obtained:

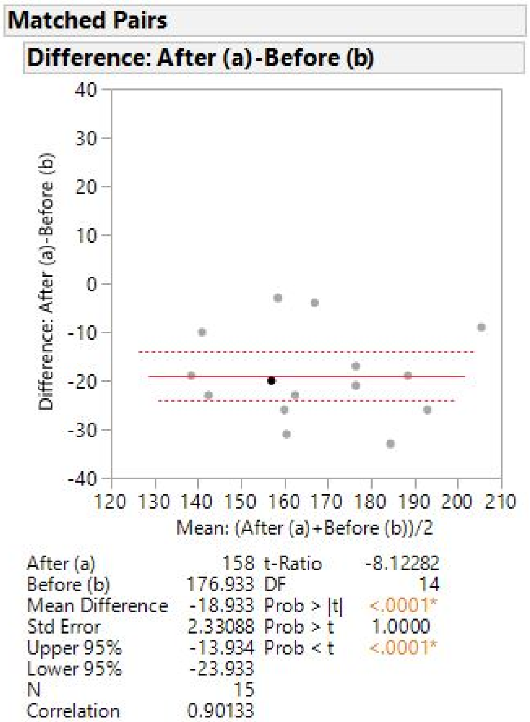


Figure 3

(i) Is this a matched pair or not? Explain. (1)

(ii) Using the 0.05 level of significance, do the results confirm the drug company's claim that the drug lowers blood pressure? **Clearly state the hypothesis implied by the question and how it can be tested. Give the rejection region and the conclusions.** (5)

(c) We wish to test $H_0 : \mu = 30$ against $H_1 : \mu \neq 30$, using a sample of size $n = 10$. from a normal population with mean μ and variance σ^2. What is the power of the test if $\mu = 30 + \sqrt{2}\sigma$? (4)

(d) The scores obtained in maths (X_i) and stats (Y_i) by a random sample of $n = 12$ Year 1 UNISA students gave a sample correlation coefficient $r_1 = 0.73$. Suppose that the same experiment is conducted on a random sample of $n = 20$ Year 2 UNISA students, and a correlation coefficient of $r_2 = 0.89$ is obtained. Test at the 1% level of significance the null hypothesis $H_0 : \rho_1 = \rho_2$ against the alternative hypothesis $H_1 : \rho_1 < \rho_2$. (6)

[25]

QUESTION 5

An agricultural experiment involving a control group and 3 experimental groups was performed to determine the effect of weed-killers on the yield of maize at a certain farm. A random sample of $n = 32$ plots with similar plot sizes and soil type are randomly assigned to 4 groups of 8 plots each. Group 1 was used as the control group, while Groups 2, 3 and 4 were used as experimental groups A, B and C in which weed-killers A, B and C were applied. The quantity of maize planted on each of the 32 plots was the same. The same amounts and types of fertilizer and irrigation methods were used on each plot. The following table shows the amount of yield in tons observed in each plot:

Quantity of yield in tons			
Control (X_1)	**Weed-killer A** (X_2)	**Weed-killer B** (X_3)	**Weed-killer C** (X_4)
4	9	5	8
4	7	7	5
3	8	6	5
4	7	6	7
5	9	6	5
4	7	5	6
3	8	6	7
5	9	7	8

(Regard the data as random samples from normal populations.)

The following SAS JMP output was obtained.

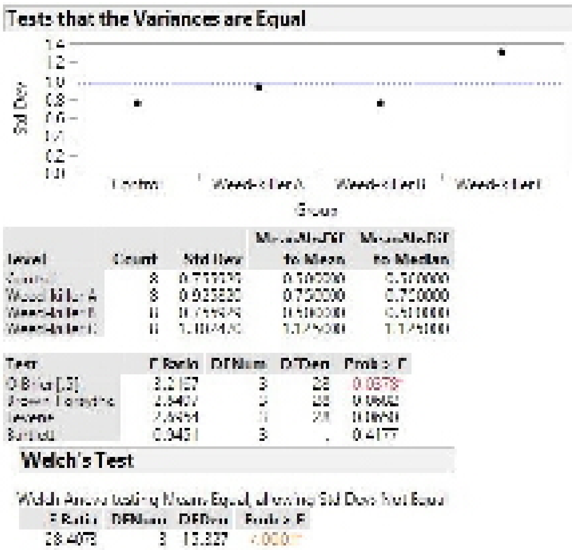


Figure 4

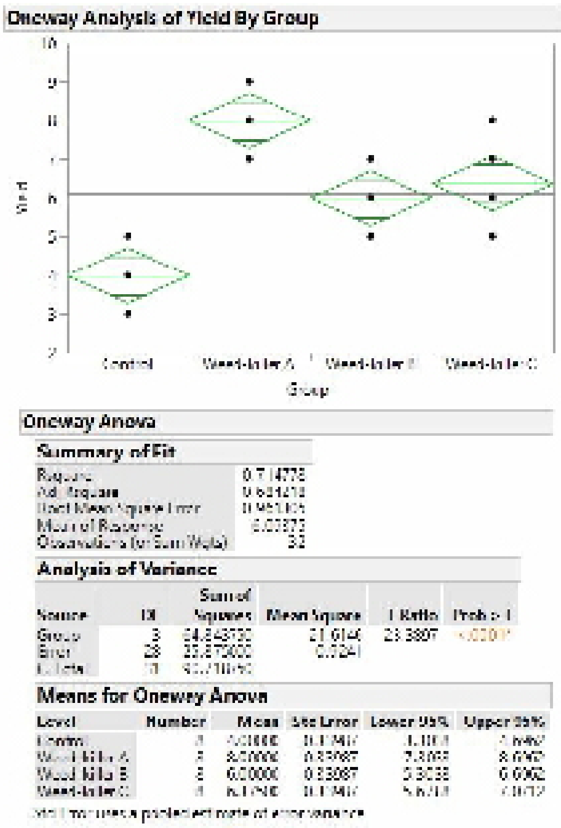
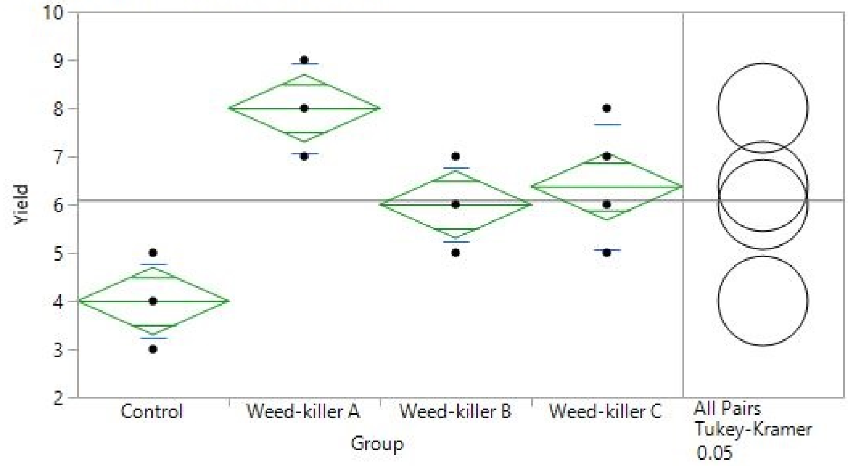


Figure 5

Oneway Analysis of Yield By Group



Means Comparisons

Comparisons for all pairs using Tukey-Kramer HSD

Confidence Quantile

q*	Alpha
2.73031	0.05

HSD Threshold Matrix

Abs(Dif)-HSD

	Weed-killer A	Weed-killer C	Weed-killer B	Control
Weed-killer A	-1.3123	0.3127	0.6877	2.6877
Weed-killer C	0.3127	-1.3123	-0.9373	1.0627
Weed-killer B	0.6877	-0.9373	-1.3123	0.6877
Control	2.6877	1.0627	0.6877	-1.3123

Positive values show pairs of means that are significantly different.

Connecting Letters Report

Level				Mean
Weed-killer A	A			8.0000000
Weed-killer C		B		6.3750000
Weed-killer B		B		6.0000000
Control			C	4.0000000

Levels not connected by same letter are significantly different.

Ordered Differences Report

Level	- Level	Difference	Std Err Dif	Lower CL	Upper CL	p-Value
Weed-killer A	Control	4.000000	0.4806525	2.68767	5.312332	<.0001*
Weed-killer C	Control	2.375000	0.4806525	1.06267	3.687332	0.0002*
Weed-killer A	Weed-killer B	2.000000	0.4806525	0.68767	3.312332	0.0015*
Weed-killer B	Control	2.000000	0.4806525	0.68767	3.312332	0.0015*
Weed-killer A	Weed-killer C	1.625000	0.4806525	0.31267	2.937332	0.0109*
Weed-killer C	Weed-killer B	0.375000	0.4806525	-0.93733	1.687332	0.8628

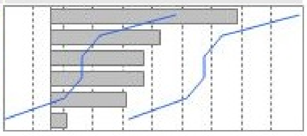


Figure 6

(a) Test at the 5% level of significance whether the population variances differ significantly from one another. (4)

(b) Test at the 5% level of significance whether the population means of the four different groups differ.

(i) State the null and alternative hypotheses.

(ii) State the rejection region and conclusion.

(4)

(c) Looking at output in Figure 6, can you conclude that $\mu_1 \neq \mu_2 = \mu_3$? Justify. (3)

[11]

QUESTION 6

A clinical trial consisting of a random sample of $n = 20$ cardiac patients is conducted in order to investigate the relationship between the dose given (X_i) and the number of cells killed (Y_i). The table below shows readings obtained from the clinical trial:

Patient	**Dose given in cubic cms** (X_i)	**Number of dead cells** (Y_i)	**Patient**	**Dose given in cubic cms** (X_i)	**Number of dead cells** (Y_i)
1	4.5	60	11	6.5	67
2	2	35	12	4	88
3	3.5	55	13	3.5	60
4	4	50	14	4	70
5	6.5	70	15	5.5	90
6	1.5	40	16	4	68
7	2	40	17	4.5	73
8	3	45	18	3.5	66
9	1.5	30	19	5.5	77
10	7	80	20	6	66

The following output was obtained.

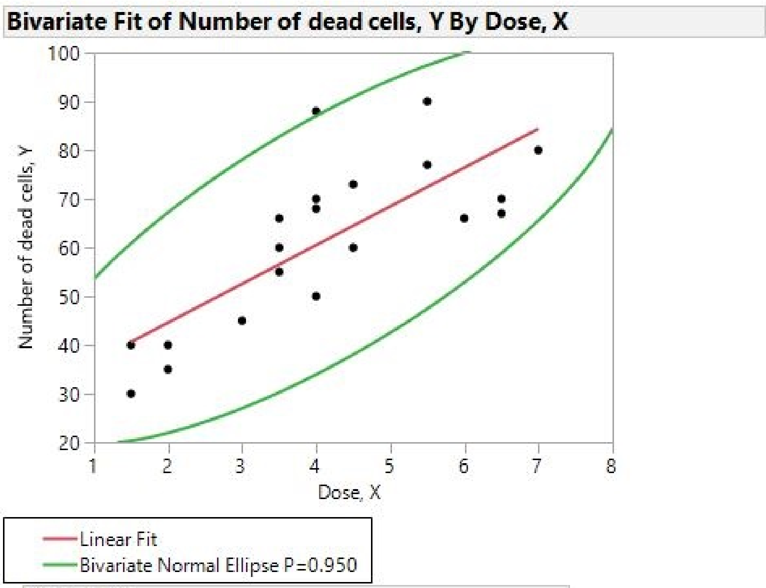


Figure 7a

Linear Fit

Number of dead cells, Y = 28.676996 + 7.9570918*Dose, X

Summary of Fit

RSquare	0.597462
RSquare Adj	0.575099
Root Mean Square Error	11.14775
Mean of Response	61.5
Observations (or Sum Wgts)	20

Lack Of Fit

Source	DF	Sum of Squares	Mean Square	F Ratio
Lack Of Fit	8	1216.2368	152.030	1.4895
Pure Error	10	1020.6667	102.067	Prob > F
Total Error	18	2236.9035		0.2725
				Max RSq
				0.8163

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	1	3320.0965	3320.10	26.7163
Error	18	2236.9035	124.27	Prob > F
C. Total	19	5557.0000		<.0001*

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	28.676996	6.821965	4.20	0.0005*
Dose, X	7.9570918	1.539453	5.17	<.0001*

Bivariate Normal Ellipse P=0.950

Variable	Mean	Std Dev	Correlation	Signif. Prob	Number
Dose, X	4.125	1.661285	0.772957	<.0001*	20
Number of dead cells, Y	61.5	17.10186			

Figure 7b

Assume that a linear relationship $Y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$ where the ε_i's are independent $n(0;\sigma^2)$ random variables, is meaningful. Using Figure 4:

(a) Does the assumption of linearity appear to be reasonable and why? (2)

(b) Give the estimates β_0, β_1 and σ^2 for the model. (3)

(c) What is the equation of the regression line used for the number of dead cells as a function of dose given? (1)

(d) Predict the number of dead cells for 4 cubic cms of dose. (1)

(e) At the 0.01 level, test the null hypothesis $H_0 : \beta_1 = 0$ versus $H_1 : \beta_1 \neq 0$. (4)

(f) Find a 99% confidence interval for the slope of the regression line. (3)

[14]

[100]

Formulae / Formules

$$B_1 = \frac{\frac{1}{n}\sum_{i=1}^{n}(X_i - \overline{X})^3}{[\frac{1}{n}\sum_{i=1}^{n}(X_i - \overline{X})^2]^{\frac{3}{2}}}$$

$$B_2 = \frac{\frac{1}{n}\sum_{i=1}^{n}(X_i - \overline{X})^4}{[\frac{1}{n}\sum_{i=1}^{n}(X_i - \overline{X})^2]^2}$$

$$A = \frac{\frac{1}{n}\sum_{i=1}^{n}\left|X_i - \overline{X}\right|}{\sqrt{\frac{1}{n}\sum_{i=1}^{n}(X_i - \overline{X})^2}}$$

$$\rho = \frac{e^{\eta} - e^{-\eta}}{e^{\eta} + e^{-\eta}}$$

$$T = \sqrt{n-2}\,\frac{U_{11} - U_{22}}{2\sqrt{U_{11}U_{22} - U_{12}^2}}$$

$$T = \frac{(\overline{X}_1 - \overline{X}_2) - (\mu_1 - \mu_2)}{S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$$\upsilon = \frac{\left[\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}\right]^2}{\frac{S_1^4}{n_1^2(n_1-1)} + \frac{S_2^4}{n_2^2(n_2-1)}}$$

$$F = \frac{n\sum_{i=1}^{k}(\overline{X}_i - \overline{X})^2/(k-1)}{\sum_{i=1}^{k}\sum_{j=1}^{n}(X_{ij} - \overline{X}_i)^2/(kn-k)}$$

$\widehat{\beta}_1 = \dfrac{\sum_{i=1}^{n} Y_i(X_i - \overline{X})}{d^2}$ Note: $d^2 = \sum_{i=1}^{n}(X_i - \overline{X})^2$ and $\widehat{\beta}_0 = \dfrac{\sum_{i=1}^{n} Y_i - \widehat{\beta}_1 \sum_{i=1}^{n} X_i}{n} = \overline{Y} - \widehat{\beta}_1\overline{X}$

TABEL I

Oppervlaktes onder die Normaalkromme

$$\Phi(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-\frac{1}{2}x^2}\, dx$$

$$\Phi(-z) = 1 - \Phi(z)$$

Die oppervlakte $\Phi(z)$ is teen z vir $z \geqslant 0$ getabelleer.

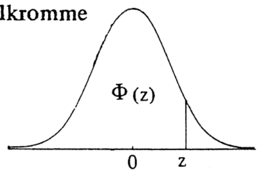


TABLE I

Areas under the Normal Curve

$$\Phi(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-\frac{1}{2}x^2}\, dx$$

$$\Phi(-z) = 1 - \Phi(z)$$

Entries in the table are values of $\Phi(z)$ for $z \geqslant 0$.

z	0,00	0,01	0,02	0,03	0,04	0,05	0,06	0,07	0,08	0,09
0,0	0,5000	0,5040	0,5080	0,5120	0,5160	0,5199	0,5239	0,5279	0,5319	0,5359
0,1	0,5398	0,5438	0,5478	0,5517	0,5557	0,5596	0,5636	0,5675	0,5714	0,5753
0,2	0,5793	0,5832	0,5871	0,5910	0,5948	0,5987	0,6026	0,6064	0,6103	0,6141
0,3	0,6179	0,6217	0,6255	0,6293	0,6331	0,6368	0,6406	0,6443	0,6480	0,6517
0,4	0,6554	0,6591	0,6628	0,6664	0,6700	0,6736	0,6772	0,6808	0,6844	0,6879
0,5	0,6915	0,6950	0,6985	0,7019	0,7054	0,7088	0,7123	0,7157	0,7190	0,7224
0,6	0,7257	0,7291	0,7324	0,7357	0,7389	0,7422	0,7454	0,7486	0,7517	0,7549
0,7	0,7580	0,7611	0,7642	0,7673	0,7704	0,7734	0,7764	0,7794	0,7823	0,7852
0,8	0,7881	0,7910	0,7939	0,7967	0,7995	0,8023	0,8051	0.8078	0,8106	0,8133
0,9	0,8159	0,8186	0,8212	0,8238	0,8264	0,8289	0,8315	0,8340	0,8365	0,8389
1.0	0,8413	0,8438	0,8461	0,8485	0,8508	0,8531	0,8554	0,8577	0,8599	0,8621
1,1	0,8643	0,8665	0,8686	0,8708	0,8729	0,8749	0,8770	0,8790	0,8810	0,8830
1,2	0,8849	0,8869	0,8888	0,8907	0,8925	0,8944	0,8962	0,8980	0,8997	0.9015
1,3	0,9032	0,9049	0,9066	0,9082	0,9099	0,9115	0,9131	0,9147	0,9162	0,9177
1,4	0,9192	0,9207	0,9222	0,9236	0,9251	0,9265	0,9279	0,9292	0.9306	0,9319
1,5	0,9332	0,9345	0,9357	0,9370	0,9382	0,9394	0,9406	0,9418	0,9429	0,9441
1,6	0,9452	0,9463	0,9474	0,9484	0,9495	0,9505	0,9515	0,9525	0,9535	0.9545
1,7	0,9554	0,9564	0,9573	0,9582	0,9591	0,9599	0,9608	0,9616	0.9625	0,9633
1,8	0,9641	0,9649	0,9656	0,9664	0,9671	0,9678	0,9686	0,9693	0,9699	0,9706
1,9	0,9713	0,9719	0,9726	0,9732	0,9738	0,9744	0,9750	0,9756	0,9761	0,9767
2,0	0,9772	0,9778	0,9783	0,9788	0,9793	0,9798	0,9803	0,9808	0,9812	0,9817
2.1	0,9821	0,9826	0,9830	0,9834	0,9838	0,9842	0,9846	0,9850	0,9854	0,9857
2,2	0,9861	0,9864	0,9868	0,9871	0,9875	0,9878	0,9881	0,9884	0,9887	0,9890
2,3	0,98928	0,98956	0,98983	0,99010	0,99036	0,99061	0,99086	0,99111	0,99134	0,99158
2,4	0,99180	0,99202	0,99224	0,99245	0,99266	0,99286	0,99305	0,99324	0,99343	0,99361
2,5	0,99379	0,99396	0,99413	0,99430	0,99446	0,99461	0,99477	0,99492	0,99506	0,99520
2,6	0,99534	0,99547	0,99560	0,99573	0,99585	0,99598	0,99609	0,99621	0,99632	0,99643
2.7	0,99653	0,99664	0,99674	0,99683	0,99693	0,99702	0,99711	0,99720	0,99728	0,99736
2,8	0,99744	0,99752	0,99760	0,99767	0,99774	0,99781	0,99788	0,99795	0,99801	0,99807
2,9	0,99813	0,99819	0,99825	0,99831	0,99836	0,99841	0,99846	0,99851	0,99856	0,99861
3,0	0,99865	0,99869	0,99874	0,99878	0,99822	0,99886	0,99889	0,99893	0,99896	0,99900
3,1	0,99903	0,99906	0,99910	0,99913	0,99916	0,99918	0,99921	0,99924	0,99926	0,99929
3,2	0,99931	0,99934	0,99936	0,99938	0,99940	0,99942	0,99944	0,99946	0,99948	0,99950
3,3	0,99952	0,99953	0,99955	0,99957	0,99958	0,99960	0,99961	0,99962	0,99964	0,99965
3,4	0,99966	0,99968	0,99969	0,99970	0,99971	0,99972	0,99973	0,99974	0,99975	0,99976
3,5	0,99977									
3,6	0,99984									
3,7	0,99989									
3,8	0,99993									
3,9	0,99995									
4,0	0,99997									

TABEL II

Waardes van die Inverse Normaalverdeling

Die inverse funksie $z = \Phi^{-1}(u)$ is teen u vir $u \geqslant 0{,}5$ getabelleer, waar $u = \Phi(z)$ die standaard normaalverdelingsfunksie aandui. Let op dat vir $u = \Phi(z) < 0{,}5$ is $\Phi(-z) = 1 - \Phi(z) > 0{,}5$

TABLE II

Values of the Inverse Normal Distribution

Entries in the table are values of the inverse function $z = \Phi^{-1}(u)$ for $u \geqslant 0{,}5$, where $u = \Phi(z)$ denotes the standard normal distribution function. Note that $\Phi(-z) = 1 - \Phi(z) > 0{,}5$ when $u = \Phi(z) < 0{,}5$.

Φ(z)	0,000	0,001	0,002	0,003	0,004	0,005	0,006	0,007	0,008	0,009
0,50	0,000	0,003	0,005	0,008	0,010	0,013	0,015	0,018	0,020	0,023
0,51	0,025	0,028	0,030	0,033	0,035	0,038	0,040	0,043	0,045	0,048
0,52	0,050	0,053	0,055	0,058	0,060	0,063	0,065	0,068	0,070	0,073
0,53	0,075	0,078	0,080	0,083	0,085	0,088	0,090	0,093	0,095	0,098
0,54	0,100	0,103	0,105	0,108	0,111	0,113	0,116	0,118	0,121	0,123
0,55	0,126	0,128	0,131	0,133	0,136	0,138	0,141	0,143	0,146	0,148
0,56	0,151	0,154	0,156	0,159	0,161	0,164	0,166	0,169	0,171	0,174
0,57	0,176	0,179	0,181	0,184	0,187	0,189	0,192	0,194	0,197	0,199
0,58	0,202	0,204	0,207	0,210	0,212	0,215	0,217	0,220	0,222	0,225
0,59	0,228	0,230	0,233	0,235	0,238	0,240	0,243	0,246	0,248	0,251
0,60	0,253	0,256	0,259	0,261	0,264	0,266	0,269	0,272	0,274	0,277
0,61	0,279	0,282	0,285	0,287	0,290	0,292	0,295	0,298	0,300	0,303
0,62	0,305	0,308	0,311	0,313	0,316	0,319	0,321	0,324	0,327	0,329
0,63	0,332	0,335	0,337	0,340	0,342	0,345	0,348	0,350	0,353	0,356
0,64	0,358	0,361	0,364	0,366	0,369	0,372	0,375	0,377	0,380	0,383
0,65	0,385	0,388	0,391	0,393	0,396	0,399	0,402	0,404	0,407	0,410
0,66	0,412	0,415	0,418	0,421	0,423	0,426	0,429	0,432	0,434	0,437
0,67	0,440	0,443	0,445	0,448	0,451	0,454	0,457	0,459	0,462	0,465
0,68	0,468	0,471	0,473	0,476	0,479	0,482	0,485	0,487	0,490	0,493
0,69	0,496	0,499	0,502	0,504	0,507	0,510	0,513	0,516	0,519	0,522
0,70	0,524	0,527	0,530	0,533	0,536	0,539	0,542	0,545	0,548	0,550
0,71	0,553	0,556	0,559	0,562	0,565	0,568	0,571	0,574	0,577	0,580
0,72	0,583	0,586	0,589	0,592	0,595	0,598	0,601	0,604	0,607	0,610
0,73	0,613	0,616	0,619	0,622	0,625	0,628	0,631	0,634	0,637	0,640
0,74	0,643	0,646	0,650	0,653	0,656	0,659	0,662	0,665	0,668	0,671
0,75	0,674	0,678	0,681	0,684	0,687	0,690	0,693	0,697	0,700	0,703
0,76	0,706	0,710	0,713	0,716	0,719	0,722	0,726	0,729	0,732	0,736
0,77	0,739	0,742	0,745	0,749	0,752	0,755	0,759	0,762	0,765	0,769
0,78	0,772	0,776	0,779	0,782	0,786	0,789	0,793	0,796	0,800	0,803
0,79	0,806	0,810	0,813	0,817	0,820	0,824	0,827	0,831	0,835	0,838
0,80	0,842	0,845	0,849	0,852	0,856	0,860	0,863	0,867	0,871	0,874
0,81	0,878	0,882	0,885	0,889	0,893	0,896	0,900	0,904	0,908	0,912
0,82	0,915	0,919	0,923	0,927	0,931	0,935	0,938	0,942	0,946	0,950
0,83	0,954	0,958	0,962	0,966	0,970	0,974	0,978	0,982	0,986	0,990
0,84	0,994	0,999	1,003	1,007	1,011	1,015	1,019	1,024	1,028	1,032
0,85	1,036	1,041	1,045	1,049	1,054	1,058	1,063	1,067	1,071	1,076
0,86	1,080	1,085	1,089	1,094	1,098	1,103	1,108	1,112	1,117	1,122
0,87	1,126	1,131	1,136	1,141	1,146	1,150	1,155	1,160	1,165	1,170
0,88	1,175	1,180	1,185	1,190	1,195	1,200	1,206	1,211	1,216	1,221
0,89	1,227	1,232	1,237	1,243	1,248	1,254	1,259	1,265	1,270	1,276
0,90	1 282	1,287	1,293	1,299	1,305	1,311	1,317	1,323	1,329	1,335
0,91	1,341	1,347	1,353	1,359	1,366	1,372	1,379	1,385	1,392	1,398
0,92	1,405	1,412	1,419	1,426	1,433	1,440	1,447	1,454	1,461	1,468
0,93	1,476	1,483	1,491	1,499	1,506	1,514	1,522	1,530	1,538	1,546
0,94	1,555	1,563	1,572	1,580	1,589	1,598	1,607	1,616	1,626	1,635
0,95	1,645	1,655	1,665	1,675	1,685	1,695	1,706	1,717	1,728	1,739
0,96	1,751	1,762	1,774	1,787	1,799	1,812	1,825	1,838	1,852	1,866
0,97	1,881	1,896	1,911	1,927	1,943	1,960	1,977	1,995	2,014	2,034
0,98	2,054	2,075	2,097	2,120	2,144	2,170	2,197	2,226	2,257	2,290
0,99	2,326	2,366	2,409	2,457	2,512	2,576	2,652	2,748	2,878	3,090

TABEL III

Die t-verdeling: Boonste Waarskynlikheidspunte

$P = P(t \geqslant t_{\nu,P}) = P(t \leqslant -t_{\nu,P})$

met $t_{\nu,P} = -t_{\nu,1-P}$ sodat

$P(|t| \geqslant t_{\nu,P}) = 2P, \quad t_{\nu,P} > 0.$

Die waardes $t_{\nu,P}$ van die t-verdeling is teen die aantal vryheidsgrade ν en die eenkantige oorskrydingswaarskynlikheid P getabelleer.

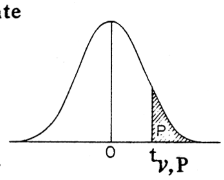


TABLE III

The t-Distribution: Upper Probability Points

$P = P(t \geqslant t_{\nu,P}) = P(t \leqslant -t_{\nu,P})$

with $t_{\nu,P} = -t_{\nu,1-P}$ so that

$P(|t| \geqslant t_{\nu,P}) = 2P, \quad t_{\nu,P} > 0.$

Entries in the table are the values $t_{\nu,P}$ of the t-distribution for various degrees of freedom ν and one-tailed probabilities P.

ν \ P	0,25	0,10	0,05	0,025	0,01	0,005
1	1,000	3,078	6,314	12,706	31,821	63,657
2	0,816	1,886	2,920	4,303	6,965	9,925
3	0,765	1,638	2,353	3,182	4,541	5,841
4	0,741	1,533	2,132	2,776	3,747	4,604
5	0,727	1,476	2,015	2,571	3,365	4,032
6	0,718	1,440	1,943	2,447	3,143	3,707
7	0,711	1,415	1,895	2,365	2,998	3,499
8	0,706	1,397	1,860	2,306	2,896	3,355
9	0,703	1,383	1,833	2,262	2,821	3,250
10	0,700	1,372	1,812	2,228	2,764	3,169
11	0,697	1,363	1,796	2,201	2,718	3,106
12	0,695	1,356	1,782	2,179	2,681	3,055
13	0,694	1,350	1,771	2,160	2,650	3,012
14	0,692	1,345	1,761	2,145	2,624	2,977
15	0,691	1,341	1,753	2,131	2,602	2,947
16	0,690	1,337	1,746	2,120	2,583	2,921
17	0,689	1,333	1,740	2,110	2,567	2,898
18	0,688	1,330	1,734	2,101	2,552	2,878
19	0,688	1,328	1,729	2,093	2,539	2,861
20	0,687	1,325	1,725	2,086	2,528	2,845
21	0,686	1,323	1,721	2,080	2,518	2,831
22	0,686	1,321	1,717	2,074	2,508	2,819
23	0,685	1,319	1,714	2,069	2,500	2,807
24	0,685	1,318	1,711	2,064	2,492	2,797
25	0,684	1,316	1,708	2,060	2,485	2,787
26	0,684	1,315	1,706	2,056	2,479	2,779
27	0,684	1,314	1,703	2,052	2,473	2,771
28	0,683	1,313	1,701	2,048	2,467	2,763
29	0,683	1,311	1,699	2,045	2,462	2,756
30	0,683	1,310	1,697	2,042	2,457	2,750
35	0,682	1,306	1,690	2,030	2,438	2,724
40	0,681	1,303	1,684	2,021	2,423	2,704
60	0,679	1,296	1,671	2,000	2,390	2,660
100	0,677	1,290	1,660	1,984	2,364	2,626
∞	0,675	1,282	1,645	1,960	2,326	2,576

TABEL IV

Die χ^2-verdeling: Boonste Waarskynlikheidspunte

$P = P(\chi^2 \geqslant \chi^2_{\nu,P})$

Die waardes $\chi^2_{\nu,P}$ van die χ^2-verdeling is teen die aantal vryheidsgrade ν en die eenkantige oorskrydingswaarskynlikheid P getabelleer.

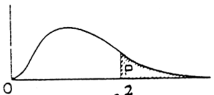


TABLE IV

The χ^2-Distribution: Upper Probability Points

$P = P(\chi^2 \geqslant \chi^2_{\nu,P})$

Entries in the table are the values $\chi^2_{\nu,P}$ of the χ^2-distribution for various degrees of freedom ν and one-tailed probabilities P.

ν \ P	0·990	0·975	0·950	0·900	0·500	0·100	0·050	0·025	0·010	0·005
1	157088.10^{-9}	982069.10^{-9}	393214.10^{-8}	0·0157908	0·454937	2·70554	3·84146	5·02389	6·63490	7·87944
2	0·0201007	0·0506356	0·102587	0·210720	1·38629	4·60517	5·99147	7·37776	9·21034	10·5966
3	0·114832	0·215795	0·351846	0·584375	2·36597	6·25139	7·81473	9·34840	11·3449	12·8381
4	0·297110	0·484419	0·710721	1·063623	3·35670	7·77944	9·48773	11·1433	13·2767	14·8602
5	0·554300	0·831211	1·145476	1·61031	4·35146	9·23635	11·0705	12·8325	15·0863	16·7496
6	0·872085	1·237347	1·63539	2·20413	5·34812	10·6446	12·5916	14·4494	16·8119	18·5476
7	1·239043	1·68987	2·16735	2·83311	6·34581	12·0170	14·0671	16·0128	18·4753	20·2777
8	1·646482	2·17973	2·73264	3·48954	7·34412	13·3616	15·5073	17·5346	20·0902	21·9550
9	2·087912	2·70039	3·32511	4·16816	8·34283	14·6837	16·9190	19·0228	21·6660	23·5893
10	2·55821	3·24697	3·94030	4·86518	9·34182	15·9871	18·3070	20·4831	23·2093	25·1882
11	3·05347	3·81575	4·57481	5·57779	10·3410	17·2750	19·6751	21·9200	24·7250	26·7569
12	3·57056	4·40379	5·22603	6·30380	11·3403	18·5494	21·0261	23·3367	26·2170	28·2995
13	4·10691	5·00874	5·89186	7·04150	12·3398	19·8119	22·3621	24·7356	27·6883	29·8194
14	4·66043	5·62872	6·57063	7·78953	13·3393	21·0642	23·6848	26·1190	29·1413	31·3193
15	5·22935	6·26214	7·26094	8·54675	14·3389	22·3072	24·9958	27·4884	30·5779	32·8013
16	5·81221	6·90766	7·96164	9·31223	15·3385	23·5418	26·2962	28·8454	31·9999	34·2672
17	6·40776	7·56418	8·67176	10·0852	16·3381	24·7690	27·5871	30·1910	33·4087	35·7185
18	7·01491	8·23075	9·39046	10·8649	17·3379	25·9894	28·8693	31·5264	34·8053	37·1564
19	7·63273	8·90655	10·1170	11·6509	18·3376	27·2036	30·1435	32·8523	36·1908	38·5822
20	8·26040	9·59083	10·8508	12·4426	19·3374	28·4120	31·4104	34·1696	37·5662	39·9968
21	8·89720	10·28293	11·5913	13·2396	20·3372	29·6151	32·6705	35·4789	38·9321	41·4010
22	9·54249	10·9823	12·3380	14·0415	21·3370	30·8133	33·9244	36·7807	40·2894	42·7956
23	10·19567	11·6885	13·0905	14·8479	22·3369	32·0069	35·1725	38·0757	41·6384	44·1813
24	10·8564	12·4011	13·8484	15·6587	23·3367	33·1963	36·4151	39·3641	42·9798	45·5585
25	11·5240	13·1197	14·6114	16·4734	24·3366	34·3816	37·6525	40·6465	44·3141	46·9278
26	12·1981	13·8439	15·3791	17·2919	25·3364	35·5631	38·8852	41·9232	45·6417	48·2899
27	12·8786	14·5733	16·1513	18·1138	26·3363	36·7412	40·1133	43·1944	46·9630	49·6449
28	13·5648	15·3079	16·9279	18·9392	27·3363	37·9159	41·3372	44·4607	48·2782	50·9933
29	14·2565	16·0471	17·7083	19·7677	28·3362	39·0875	42·5569	45·7222	49·5879	52·3356
30	14·9535	16·7908	18·4926	20·5992	29·3360	40·2560	43·7729	46·9792	50·8922	53·6720
40	22·1643	24·4331	26·5093	29·0505	39·3354	51·8050	55·7585	59·3417	63·6907	66·7659
50	29·7067	32·3574	34·7642	37·6886	49·3349	63·1671	67·5048	71·4202	76·1539	79·4900
60	37·4848	40·4817	43·1879	46·4589	59·3347	74·3970	79·0819	83·2976	88·3794	91·9517
70	45·4418	48·7576	51·7393	55·3290	69·3344	85·5271	90·5312	95·0231	100·425	104·215
80	53·5400	57·1532	60·3915	64·2778	79·3343	96·5782	101·879	106·629	112·329	116·321
90	61·7541	65·6466	69·1260	73·2912	89·3342	107·565	113·145	118·136	124·116	128·299
100	70·0648	74·2219	77·9295	82·3581	99·3341	118·498	124·342	129·561	135·807	140·169

TABEL V

Die F-verdeling: Boonste 5% Punte

(ν_1 vryheidsgrade in die teller en ν_2 in die noemer)

TABLE V

The F-Distribution: Upper 5% Points

(ν_1 degrees of freedom in numerator and ν_2 in denominator)

ν_2	$\nu_1=1$	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	161	200	216	225	230	234	237	239	241	242	244	246	248	249	250	251	252	253	254
2	18,5	19,0	19,2	19,2	19,3	19,3	19,4	19,4	19,4	19,4	19,4	19,4	19,4	19,5	19,5	19,5	19,5	19,5	19,5
3	10,1	9,55	9,28	9,12	9,01	8,94	8,89	8,85	8,81	8,79	8,74	8,70	8,66	8,64	8,62	8,59	8,57	8,55	8,53
4	7,71	6,94	6,59	6,39	6,26	6,16	6,09	6,04	6,00	5,96	5,91	5,86	5,80	5,77	5,75	5,72	5,69	5,66	5,63
5	6,61	5,79	5,41	5,19	5,05	4,95	4,88	4,82	4,77	4,74	4,68	4,62	4,56	4,53	4,50	4,46	4,43	4,40	4,36
6	5,99	5,14	4,76	4,53	4,39	4,28	4,21	4,15	4,10	4,06	4,00	3,94	3,87	3,84	3,81	3,77	3,74	3,70	3,67
7	5,59	4,74	4,35	4,12	3,97	3,87	3,79	3,73	3,68	3,64	3,57	3,51	3,44	3,41	3,38	3,34	3,30	3,27	3,23
8	5,32	4,46	4,07	3,84	3,69	3,58	3,50	3,44	3,39	3,35	3,28	3,22	3,15	3,12	3,08	3,04	3,01	2,97	2,93
9	5,12	4,26	3,86	3,63	3,48	3,37	3,29	3,23	3,18	3,14	3,07	3,01	2,94	2,90	2,86	2,83	2,79	2,75	2,71
10	4,96	4,10	3,71	3,48	3,33	3,22	3,14	3,07	3,02	2,98	2,91	2,85	2,77	2,74	2,70	2,66	2,62	2,58	2,54
11	4,84	3,98	3,59	3,36	3,20	3,09	3,01	2,95	2,90	2,85	2,79	2,72	2,65	2,61	2,57	2,53	2,49	2,45	2,40
12	4,75	3,89	3,49	3,26	3,11	3,00	2,91	2,85	2,80	2,75	2,69	2,62	2,54	2,51	2,47	2,43	2,38	2,34	2,30
13	4,67	3,81	3,41	3,18	3,03	2,92	2,83	2,77	2,71	2,67	2,60	2,53	2,46	2,42	2,38	2,34	2,30	2,25	2,21
14	4,60	3,74	3,34	3,11	2,96	2,85	2,76	2,70	2,65	2,60	2,53	2,46	2,39	2,35	2,31	2,27	2,22	2,18	2,13
15	4,54	3,68	3,29	3,06	2,90	2,79	2,71	2,64	2,59	2,54	2,48	2,40	2,33	2,29	2,25	2,20	2,16	2,11	2,07
16	4,49	3,63	3,24	3,01	2,85	2,74	2,66	2,59	2,54	2,49	2,42	2,35	2,28	2,24	2,19	2,15	2,11	2,06	2,01
17	4,45	3,59	3,20	2,96	2,81	2,70	2,61	2,55	2,49	2,45	2,38	2,31	2,23	2,19	2,15	2,10	2,06	2,01	1,96
18	4,41	3,55	3,16	2,93	2,77	2,66	2,58	2,51	2,46	2,41	2,34	2,27	2,19	2,15	2,11	2,06	2,02	1,97	1,92
19	4,38	3,52	3,13	2,90	2,74	2,63	2,54	2,48	2,42	2,38	2,31	2,23	2,16	2,11	2,07	2,03	1,98	1,93	1,88
20	4,35	3,49	3,10	2,87	2,71	2,60	2,51	2,45	2,39	2,35	2,28	2,20	2,12	2,08	2,04	1,99	1,95	1,90	1,84
21	4,32	3,47	3,07	2,84	2,68	2,57	2,49	2,42	2,37	2,32	2,25	2,18	2,10	2,05	2,01	1,96	1,92	1,87	1,81
22	4,30	3,44	3,05	2,82	2,66	2,55	2,46	2,40	2,34	2,30	2,23	2,15	2,07	2,03	1,98	1,94	1,89	1,84	1,78
23	4,28	3,42	3,03	2,80	2,64	2,53	2,44	2,37	2,32	2,27	2,20	2,13	2,05	2,01	1,96	1,91	1,86	1,81	1,76
24	4,26	3,40	3,01	2,78	2,62	2,51	2,42	2,36	2,30	2,25	2,18	2,11	2,03	1,98	1,94	1,89	1,84	1,79	1,73
25	4,24	3,39	2,99	2,76	2,60	2,49	2,40	2,34	2,28	2,24	2,16	2,09	2,01	1,96	1,92	1,87	1,82	1,77	1,71
28	4,20	3,34	2,95	2,71	2,56	2,45	2,36	2,29	2,24	2,19	2,12	2,04	1,96	1,91	1,87	1,82	1,77	1,71	1,65
30	4,17	3,32	2,92	2,69	2,53	2,42	2,33	2,27	2,21	2,16	2,09	2,01	1,93	1,89	1,84	1,79	1,74	1,68	1,62
34	4,13	3,28	2,88	2,65	2,49	2,38	2,29	2,23	2,17	2,12	2,05	1,97	1,89	1,84	1,80	1,75	1,69	1,63	1,57
40	4,08	3,23	2,84	2,61	2,45	2,34	2,25	2,18	2,12	2,08	2,00	1,92	1,84	1,79	1,74	1,69	1,64	1,58	1,51
48	4,04	3,19	2,80	2,57	2,41	2,29	2,21	2,14	2,08	2,03	1,96	1,88	1,79	1,75	1,70	1,64	1,59	1,52	1,45
60	4,00	3,15	2,76	2,53	2,37	2,25	2,17	2,10	2,04	1,99	1,92	1,84	1,75	1,70	1,65	1,59	1,53	1,47	1,39
80	3,96	3,11	2,72	2,49	2,33	2,21	2,13	2,06	2,00	1,95	1,88	1,79	1,70	1,65	1,60	1,54	1,48	1,41	1,32
120	3,92	3,07	2,68	2,45	2,29	2,18	2,09	2,02	1,96	1,91	1,83	1,75	1,66	1,61	1,55	1,50	1,43	1,35	1,25
∞	3,84	3,00	2,60	2,37	2,21	2,10	2,01	1,94	1,88	1,83	1,75	1,67	1,57	1,52	1,46	1,39	1,32	1,22	1,00

TABEL VI

Die F-verdeling: Boonste 2,5% Punte

(ν_1 vryheidsgrade in die teller en ν_2 in die noemer)

TABLE VI

The F-Distribution: Upper 2,5% Points

(ν_1 degrees of freedom in numerator and ν_2 in denominator)

ν_2	$\nu_1=1$	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	648	800	864	900	922	937	948	957	963	969	977	985	993	997	1001	1006	1010	1014	1018
2	38,5	39,0	39,2	39,2	39,3	39,3	39,4	39,4	39,4	39,4	39,4	39,4	39,4	39,5	39,5	39,5	39,5	39,5	39,5
3	17,4	16,0	15,4	15,1	14,9	14,7	14,6	14,5	14,5	14,4	14,3	14,3	14,2	14,1	14,1	14,0	14,0	13,9	13,9
4	12,2	10,6	9,98	9,60	9,36	9,20	9,07	8,98	8,90	8,84	8,75	8,66	8,56	8,51	8,46	8,41	8,36	8,31	8,26
5	10,0	8,43	7,76	7,39	7,15	6,98	6,85	6,76	6,68	6,62	6,52	6,43	6,33	6,28	6,23	6,18	6,12	6,07	6,02
6	8,81	7,26	6,60	6,23	5,99	5,82	5,70	5,60	5,52	5,46	5,37	5,27	5,17	5,12	5,07	5,01	4,96	4,90	4,85
7	8,07	6,54	5,89	5,52	5,29	5,12	4,99	4,90	4,82	4,76	4,67	4,57	4,47	4,42	4,36	4,31	4,25	4,20	4,14
8	7,57	6,06	5,42	5,05	4,82	4,65	4,53	4,43	4,36	4,30	4,20	4,10	4,00	3,95	3,89	3,84	3,78	3,73	3,67
9	7,21	5,71	5,08	4,72	4,48	4,32	4,20	4,10	4,03	3,96	3,87	3,77	3,67	3,61	3,56	3,51	3,45	3,39	3,33
10	6,94	5,46	4,83	4,47	4,24	4,07	3,95	3,85	3,78	3,72	3,62	3,52	3,42	3,37	3,31	3,26	3,20	3,14	3,08
11	6,72	5,26	4,63	4,28	4,04	3,88	3,76	3,66	3,59	3,53	3,43	3,33	3,23	3,17	3,12	3,06	3,00	2,94	2,88
12	6,55	5,10	4,47	4,12	3,89	3,73	3,61	3,51	3,44	3,37	3,28	3,18	3,07	3,02	2,96	2,91	2,85	2,79	2,72
13	6,41	4,97	4,35	4,00	3,77	3,60	3,48	3,39	3,31	3,25	3,15	3,05	2,95	2,89	2,84	2,78	2,72	2,66	2,60
14	6,30	4,86	4,24	3,89	3,66	3,50	3,38	3,29	3,21	3,15	3,05	2,95	2,84	2,79	2,73	2,67	2,61	2,55	2,49
15	6,20	4,77	4,15	3,80	3,58	3,41	3,29	3,20	3,12	3,06	2,96	2,86	2,76	2,70	2,64	2,58	2,52	2,46	2,40
16	6,12	4,69	4,08	3,73	3,50	3,34	3,22	3,12	3,05	2,99	2,89	2,79	2,68	2,63	2,57	2,51	2,45	2,38	2,32
17	6,04	4,62	4,01	3,66	3,44	3,28	3,16	3,06	2,98	2,92	2,82	2,72	2,62	2,56	2,50	2,44	2,38	2,32	2,25
18	5,98	4,56	3,95	3,61	3,38	3,22	3,10	3,01	2,93	2,87	2,77	2,67	2,56	2,50	2,44	2,38	2,32	2,26	2,19
19	5,92	4,51	3,90	3,56	3,33	3,17	3,05	2,96	2,88	2,82	2,72	2,62	2,51	2,45	2,39	2,33	2,27	2,20	2,13
20	5,87	4,46	3,86	3,51	3,29	3,13	3,01	2,91	2,84	2,77	2,68	2,57	2,46	2,41	2,35	2,29	2,22	2,16	2,09
21	5,83	4,42	3,82	3,48	3,25	3,09	2,97	2,87	2,80	2,73	2,64	2,53	2,42	2,37	2,31	2,25	2,18	2,11	2,04
22	5,79	4,38	3,78	3,44	3,22	3,05	2,93	2,84	2,76	2,70	2,60	2,50	2,39	2,33	2,27	2,21	2,14	2,08	2,00
23	5,75	4,35	3,75	3,41	3,18	3,02	2,90	2,81	2,73	2,67	2,57	2,47	2,36	2,30	2,24	2,18	2,11	2,04	1,97
24	5,72	4,32	3,72	3,38	3,15	2,99	2,87	2,78	2,70	2,64	2,54	2,44	2,33	2,27	2,21	2,15	2,08	2,01	1,94
25	5,69	4,29	3,69	3,35	3,13	2,97	2,85	2,75	2,68	2,61	2,51	2,41	2,30	2,24	2,18	2,12	2,05	1,98	1,91
28	5,61	4,22	3,63	3,29	3,06	2,90	2,78	2,69	2,61	2,55	2,45	2,34	2,23	2,17	2,11	2,05	1,98	1,91	1,83
30	5,57	4,18	3,59	3,25	3,03	2,87	2,75	2,65	2,57	2,51	2,41	2,31	2,20	2,14	2,07	2,01	1,94	1,87	1,79
34	5,50	4,12	3,53	3,19	2,97	2,81	2,69	2,59	2,52	2,45	2,35	2,25	2,13	2,07	2,01	1,95	1,88	1,80	1,72
40	5,42	4,05	3,46	3,13	2,90	2,74	2,62	2,53	2,45	2,39	2,29	2,18	2,07	2,01	1,94	1,88	1,80	1,72	1,64
48	5,35	3,99	3,40	3,07	2,84	2,69	2,56	2,47	2,39	2,33	2,23	2,12	2,01	1,94	1,88	1,81	1,73	1,65	1,56
60	5,29	3,93	3,34	3,01	2,79	2,63	2,51	2,41	2,33	2,27	2,17	2,06	1,94	1,88	1,82	1,74	1,67	1,58	1,48
80	5,22	3,86	3,28	2,95	2,73	2,57	2,45	2,35	2,28	2,21	2,11	2,00	1,88	1,82	1,75	1,68	1,60	1,51	1,40
120	5,15	3,80	3,23	2,89	2,67	2,52	2,39	2,30	2,22	2,16	2,05	1,94	1,82	1,76	1,69	1,61	1,53	1,43	1,31
∞	5,02	3,69	3,12	2,79	2,57	2,41	2,29	2,19	2,11	2,05	1,94	1,83	1,71	1,64	1,57	1,48	1,39	1,27	1,00

TABEL IX

Die Produkmoment-korrelasiekoëffisiënt:
Boonste Kritieke Waardes (vir ρ=0)

TABLE IX

The Product Moment Correlation Coefficient:
Upper Critical Values (for ρ=0)

n = aantal pare waarnemings

n = number of pairs of observations

	Betekenispeil vir eenkantige toets			Significance level for one-tailed test		
n	0,25	0,10	0,05	0,025	0,01	0,005
3	0,7071	0,9511	0,9877	0,9969	0,9995	0,9999
4	0,5000	0,8000	0,9000	0,9500	0,9800	0,9900
5	0,4040	0,6870	0,8054	0,8783	0,9343	0,9587
6	0,3473	0,6084	0,7293	0,8114	0,8822	0,9172
7	0,3091	0,5509	0,6694	0,7545	0,8329	0,8745
8	0,2811	0,5067	0,6215	0,7067	0,7887	0,8343
9	0,2596	0,4716	0,5822	0,6664	0,7498	0,7977
10	0,2423	0,4428	0,5494	0,6319	0,7155	0,7646
11	0,2281	0,4187	0,5214	0,6021	0,6851	0,7348
12	0,2161	0,3981	0,4973	0,5760	0,6581	0,7079
13	0,2058	0,3802	0,4762	0,5529	0,6339	0,6835
14	0,1968	0,3646	0,4575	0,5324	0,6120	0,6614
15	0,1890	0,3507	0,4409	0,5140	0,5923	0,6411
16	0,1820	0,3383	0,4259	0,4973	0,5742	0,6226
17	0,1757	0,3271	0,4124	0,4821	0,5577	0,6055
18	0,1700	0,3170	0,4000	0,4683	0,5425	0,5897
19	0,1649	0,3077	0,3887	0,4555	0,5285	0,5751
20	0,1602	0,2992	0,3783	0,4438	0,5155	0,5614
21	0,1558	0,2914	0,3687	0,4329	0,5034	0,5487
22	0,1518	0,2841	0,3598	0,4227	0,4921	0,5368
23	0,1481	0,2774	0,3515	0,4132	0,4815	0,5256
24	0,1447	0,2711	0,3438	0,4044	0,4716	0,5151
25	0,1415	0,2653	0,3365	0,3961	0,4622	0,5052
26	0,1384	0,2598	0,3297	0,3882	0,4534	0,4958
27	0,1356	0,2546	0,3233	0,3809	0,4451	0,4896
28	0,1330	0,2497	0,3172	0,3739	0,4372	0,4785
29	0,1305	0,2451	0,3115	0,3673	0,4297	0,4705
30	0,1281	0,2407	0,3061	0,3610	0,4226	0,4629
31	0,1258	0,2366	0,3009	0,3550	0,4158	0,4556
32	0,1237	0,2327	0,2960	0,3494	0,4093	0,4487
35	0,1179	0,2220	0,2826	0,3338	0,3916	0,4296
40	0,1098	0,2070	0,2638	0,3120	0,3665	0,4026
45	0,1032	0,1947	0,2483	0,2940	0,3457	0,3801
50	0,0976	0,1843	0,2353	0,2787	0,3281	0,3610
60	0,0888	0,1678	0,2144	0,2542	0,2997	0,3301
70	0,0820	0,1550	0,1982	0,2352	0,2776	0,3060
80	0,0765	0,1448	0,1852	0,2199	0,2597	0,2864
90	0,0720	0,1364	0,1745	0,2072	0,2449	0,2702
100	0,0682	0,1292	0,1654	0,1966	0,2324	0,2565

TABEL X

Die z-transformasie vir die Korrelasiekoëffisiënt

Die getransformeerde waardes

$$z = \tanh^{-1} r = ½\log_e \frac{1+r}{1-r}$$

is teen die korrelasiekoëffisiënt r getabelleer.

TABLE X

The z-Transformation for the Correlation Coefficient

Entries in the table are the transformed values

$$z = \tanh^{-1} r = ½\log_e \frac{1+r}{1-r}$$

for various values of the correlation coefficient r.

r	0,00	0,01	0,02	0,03	0,04	0,05	0,06	0,07	0,08	0,09
0,0	0,0000	0,0100	0,0200	0,0300	0,0400	0,0500	0,0601	0,0701	0,0802	0,0902
0,1	0,1003	0,1104	0,1206	0,1307	0,1409	0,1511	0,1614	0,1717	0,1820	0,1923
0,2	0,2027	0,2132	0,2237	0,2342	0,2448	0,2554	0,2661	0,2769	0,2877	0,2986
0,3	0,3095	0,3205	0,3316	0,3428	0,3541	0,3654	0,3769	0,3884	0,4001	0,4118
0,4	0,4236	0,4356	0,4477	0,4599	0,4722	0,4847	0,4973	0,5101	0,5230	0,5361
0,5	0,5493	0,5627	0,5763	0,5901	0,6042	0,6184	0,6328	0,6475	0,6625	0,6777
0,6	0,6931	0,7089	0,7250	0,7414	0,7582	0,7753	0,7928	0,8107	0,8291	0,8480
0,7	0,8673	0,8872	0,9076	0,9287	0,9505	0,9730	0,9962	1,0203	1,0454	1,0714
0,8	1,0986	1,1270	1,1568	1,1881	1,2212	1,2562	1,2933	1,3331	1,3758	1,4219
0,9	1,4722	1,5275	1,5890	1,6584	1,7380	1,8318	1,9459	2,0923	2,2976	2,6466

r	0,000	0,001	0,002	0,003	0,004	0,005	0,006	0,007	0,008	0,009
0,90	1,4722	1,4775	1,4828	1,4882	1,4937	1,4992	1,5047	1,5103	1,5160	1,5217
0,91	1,5275	1,5334	1,5393	1,5453	1,5513	1,5574	1,5636	1,5698	1,5762	1,5826
0,92	1,5890	1,5956	1,6022	1,6089	1,6157	1,6226	1,6296	1,6366	1,6438	1,6510
0,93	1,6584	1,6658	1,6734	1,6811	1,6888	1,6967	1,7047	1,7129	1,7211	1,7295
0,94	1,7380	1,7467	1,7555	1,7645	1,7736	1,7828	1,7923	1,8019	1,8117	1,8216
0,95	1,8318	1,8421	1,8527	1,8635	1,8745	1,8857	1,8972	1,9090	1,9210	1,9333
0,96	1,9459	1,9588	1,9721	1,9857	1,9996	2,0139	2,0287	2,0439	2,0595	2,0756
0,97	2,0923	2,1095	2,1273	2,1457	2,1649	2,1847	2,2054	2,2269	2,2494	2,2729
0,98	2,2976	2,3235	2,3507	2,3796	2,4101	2,4427	2,4774	2,5147	2,5550	2,5987
0,99	2,6466	2,6996	2,7587	2,8257	2,9031	2,9945	3,1063	3,2504	3,4534	3,8002

Table A. Percentage points for the distribution of B_1
Lower percentage point $=$ – (tabulated upper percentage point)

Size of sample	Percentage points	Size of sample	Percentage points
n	5%	n	5%
25	0, 711	200	0, 280
30	0, 662	250	0, 251
35	0, 621	300	0, 230
40	0, 587	350	0, 213
45	0, 558	400	0, 200
50	0, 534	450	0, 188
		500	0, 179
60	0, 492	550	0, 171
70	0, 459	600	0, 163
80	0, 432	650	0, 157
90	0, 409	700	0, 151
100	0, 389	750	0, 146
		800	0, 142
125	0, 350	850	0, 138
150	0, 321	900	0, 134
175	0, 298	950	0, 130
200	0, 280	1000	0, 127

Table B. Percentage points of the distribution of B_2

Size of sample n	Percentage points	
	Upper 5%	Lower 5%
50	3, 99	2, 15
75	3, 87	2, 27
100	3, 77	2, 35
125	3, 71	2, 40
150	3, 65	2, 45
200	3, 57	2, 51
250	3, 52	2, 55
300	3, 47	2, 59
350	3, 44	2, 62
400	3, 41	2, 64
450	3, 39	2, 66
500	3, 37	2, 67
550	3, 35	2, 69
600	3, 34	2, 70
650	3, 33	2, 71
700	3, 31	2, 72
800	3, 29	2, 74
900	3, 28	2, 75
1000	3, 26	2, 76

Table C. Percentage points for the distribution of $A = \dfrac{\text{mean deviation}}{\text{standard deviation}}$

Size of sample n	$n-1$	Percentage points			
		Upper 5%	Upper 10%	Lower 10%	Lower 5%
11	10	0,9073	0,8899	0,7409	0,7153
16	15	0,8884	0,8733	0,7452	0,7236
21	20	0,8768	0,8631	0,7495	0,7304
26	25	0,8686	0,8570	0,7530	0,7360
31	30	0,8625	0,8511	0,7559	0,7404
36	35	0,8578	0,8468	0,7583	0,7440
41	40	0,8540	0,8436	0,7604	0,7470
46	45	0,8508	0,8409	0,7621	0,7496
51	50	0,8481	0,8385	0,7636	0,7518
61	60	0,8434	0,8349	0,7662	0,7554
71	70	0,8403	0,8321	0,7683	0,7583
81	80	0,8376	0,8298	0,7700	0,7607
91	90	0,8353	0,8279	0,7714	0,7626
101	100	0,8344	0,8264	0,7726	0,7644

Table D
Tabel D

The hypergeometric probability distribution: $P(X \leq x)$ for $N = 12$

Die hipergeometriese verdeling: $P(X \leq x)$ vir $N = 12$

n	k	x	P	n	k	x	P	n	k	x	P
1	1	0	0,917	4	4	0	0,141	6	2	0	0,227
1	1	1	1,000	4	4	1	0,594	6	2	1	0,773
				4	4	2	0,933	6	2	2	1,000
2	1	0	0,833	4	4	3	0,998				
2	1	1	1,000	4	4	4	1,000	6	3	0	0,091
								6	3	1	0,500
2	2	0	0,682	5	1	0	0,583	6	3	2	0,909
2	2	1	0,985	5	1	1	1,000	6	3	3	1,000
2	2	2	1,000								
				5	2	0	0,318	6	4	0	0,030
3	1	0	0,750	5	2	1	0,848	6	4	1	0,273
3	1	1	1,000	5	2	2	1,000	6	4	2	0,727
								6	4	3	0,970
3	2	0	0,545	5	3	0	0,159	6	4	4	1,000
3	2	1	0,955	5	3	1	0,636				
3	2	2	1,000	5	3	2	0,955	6	5	0	0,008
				5	3	3	1,000	6	5	1	0,121
3	3	0	0,382					6	5	2	0,500
3	3	1	0,873	5	4	0	0,071	6	5	3	0,879
3	3	2	0,995	5	4	1	0,424	6	5	4	0,992
3	3	3	1,000	5	4	2	0,848	6	5	5	1,000
				5	4	3	0,990				
4	1	0	0,667	5	4	4	1,000	6	6	0	0,001
4	1	1	1,000					6	6	1	0,040
				5	5	0	0,027	6	6	2	0,284
4	2	0	0,424	5	5	1	0,247	6	6	3	0,716
4	2	1	0,909	5	5	2	0,689	6	6	4	0,960
4	2	2	1,000	5	5	3	0,955	6	6	5	0,999
				5	5	4	0,999	6	6	6	1,000
4	3	0	0,255	5	5	5	1,000				
4	3	1	0,764								
4	3	2	0,982	6	1	0	0,500				
4	3	3	1,000	6	1	1	1,000				

Table E

Upper 5% percentage points of the ratio, $S^2_{\max}/S^2_{\min}$

υ	$k=2$	3	4	5	6
2	39, 0	87, 5	142	202	266
3	15, 4	27, 8	39, 2	50, 7	62, 0
4	9, 60	15, 5	20, 6	25, 2	29, 5
5	7, 15	10, 8	13, 7	16, 3	18, 7
6	5, 82	8, 38	10, 4	12, 1	13, 7
7	4, 99	6, 94	8, 44	9, 70	10, 8
8	4, 43	6, 00	7, 18	8, 12	9, 03
9	4, 03	5, 34	6, 31	7, 11	7, 80
10	3, 72	4, 85	5, 67	6, 34	6, 92
12	3, 28	4, 16	4, 79	5, 30	5, 72
15	2, 86	3, 54	4, 01	4, 37	4, 68
20	2, 46	2, 95	3, 29	3, 54	3, 76
30	2, 07	2, 40	2, 61	2, 78	2, 91
60	1, 67	1, 85	1, 96	2, 04	2, 11
∞	1, 00	1, 00	1, 00	1, 00	1, 00

$k =$ number of samples

$\upsilon =$ degrees of freedom for each sample variance

Table F:
$100 \times$ (power) of the two-sided t-test with level α

ϕ	6	7	8	9	10	12	15	20	30	60	∞	$\upsilon =$ degrees of freedom
1.2	30	31	32	33	34	35	36	37	38	39	40	
1.3	35	36	37	38	39	40	41	42	43	44	45	
1.4	39	40	41	42	43	45	46	47	49	50	51	
1.5	43	45	46	47	48	50	51	52	54	55	56	
1.6	48	50	52	53	54	55	57	58	59	61	62	
1.7	52	55	57	58	59	60	62	64	65	66	67	
1.8	57	60	62	63	64	65	67	69	70	71	72	
1.9	62	64	65	67	68	69	71	73	74	76	77	
2.0	66	68	70	71	72	74	75	77	78	80	81	
2.1	70	72	74	75	77	78	79	81	82	83	85	
2.2	74	76	78	79	80	81	83	84	86	87	88	
2.3	77	80	81	83	84	85	86	87	88	89	90	
2.4	81	83	85	86	87	88	89	90	91	92	93	$\alpha = 0.05$
2.5	84	86	87	88	89	90	91	92	93	94	94	
2.6	86	88	90	91	91	92	93	94	95	95	96	
2.7	89	90	92	93	93	94	95	95	96	96	97	
2.8	91	92	93	94	95	95	96	96	97	97	98	
2.9	92	94	95	95	96	96	97	97	98	98	98	
3.0	94	95	96	96	97	97	98	98	98	99	99	
3.1	95	96	97	97	98	98	98	99	99	·	·	
3.2	96	97	98	98	98	99	99	·	·	·	·	
3.3	97	98	98	99	99	·	·	·	·	·	·	
3.4	98	98	99	·	·	·	·	·	·	·	·	
3.5	98	99	·	·	·	·	·	·	·	·	·	

Table F (continued):
$100 \times$ (power) of the two-sided t-test with level α

ϕ	6	7	8	9	10	12	15	20	30	60	∞
2.0	31	33	37	40	42	45	48	50	54	57	60
2.2	39	42	46	49	51	54	58	61	64	67	70
2.4	47	51	55	58	60	63	67	70	74	77	80
2.6	55	60	63	67	69	72	76	79	82	85	87
2.8	62	68	71	74	77	80	83	86	88	90	92
3.0	69	75	78	81	83	86	89	91	92	94	95
3.2	75	81	84	87	88	90	93	94	96	97	97
3.4	81	86	88	91	92	94	95	97	98	98	99
3.6	86	90	92	94	95	96	97	98	99	99	·
3.8	90	93	95	96	97	98	99	99	·	·	·
4.0	93	95	97	98	98	99	·	·	·	·	·
4.2	95	97	98	99	99	·	·	·	·	·	·
4.4	96	98	99	·	·	·	·	·	·	·	·
4.6	97	99	·	·	·	·	·	·	·	·	·
4.8	98	·	·	·	·	·	·	·	·	·	·
5.0	99	·	·	·	·	·	·	·	·	·	·

υ = degrees of freedom

$\alpha = 0.01$