Tutorial Letter 104/1/2014

Applied Statistics II

STA2601

Semester 1

Department of Statistics

TRIAL EXAMINATION PAPER

BAR CODE

Dear Student
Congratulations if you obtained examination admission by submitting assignment 1. I would like to take the opportunity of wishing you well in the coming examinations. I hope you found the module stimulating.

The examination

Please note the following with regard to the examination:

* The duration of the examination paper is **two-hours**. You will be able to complete the set paper in 2 hours, but there will be no time for dreaming or sitting on questions you are unsure about. Make sure that you take along a functional scientific calculator that you can operate with ease as it can save you some time. My advice to you would be to do those questions you find easy *first;* then go back to the ones that need more thinking. I do not mind to mark questions in whatever order you do them, just *make sure that you number them clearly!*
* A copy of the list of formulae is attached to the trial examination paper. Please ensure that you know how to test the various hypotheses.
* All the necessary statistical tables will be supplied (see the trial paper).
* Pocket calculators are necessary for doing the calculations.
* Working through (and understanding!) ALL the examples and exercises in the study guide, workbook and in the assignments as well as the trial paper will provide beneficial supplementary preparation.
* Make sure that you know all the theory as well as the practical applications.
* All the chapters in the study guide are equally important and don't try to spot!
* Start preparing early and don't hesitate to call or email me if something is unclear.

Trial paper

Reserve two hours for yourself and do the trial paper under exam conditions on your own!

Duration: 2 hours **100 Marks**

INSTRUCTIONS

1. Answer ALL questions.
2. Marks will not be given for answers only. Show clearly how you solve each problem.
3. For all hypothesis-testing problems always give
   (i) the null and alternative hypothesis to be tested;
   (ii) the test statistic to be used; and
   (iii) the critical region for rejecting the null hypothesis.
4. Justify your answer completely if you make use of JMP output to answer a question.

QUESTION 1

Complete the following statements in your answer book (i.e. **give the missing words** and **do not** waste time to rewrite everything).

(a) We commit a..............................error if we do not reject H_0 when H_0 is false.

$$\beta = P\,(\text{not rejecting } H_0 |\ H_1 \text{ is true})$$

(1)

(b) $\beta_2 = \dfrac{\mu_4}{\sigma_4}$ is the fourth standardized moment and it measures the .... of a distribution. A distribution with $\beta_2 > 3$ is called.................................... . (2)

(c) Repeated measurements on the same individual, for example "paired observations" (X_i, Y_i) for $i = 1, 2, 3, \ldots, n$ **cannot be** considered as ....observations. (1)

[4]

QUESTION 2

(a) Let X_1 and X_2 be independent random variables such that

$$E\,(X_1) = 4\theta_1; \qquad E\,(X_2) = 6\theta_2;$$
$$Var\,(X_1) = Var\,(X_2) = \sigma^2.$$

Determine the least squares estimators of θ_1 and θ_2. (7)

(b) Let X_1, X_2 and X_3 be a random sample of size 3 drawn from a normal population with mean μ and variance σ^2. Consider the following two estimators for μ.

$$T_1 = \frac{X_1 + X_2 + X_3}{3} \qquad \text{(The sample mean)}$$

$$T_2 = \frac{X_1 + 2X_2 + 2X_3}{5} \qquad \text{(A weighted mean)}$$

(i) Show that both T_1 and T_2 are unbiased estimators of μ. (6)

(ii) Which estimator would you prefer and why? (6)

[19]

QUESTION 3

The velocity of the wind (measured in km per hour) at a specific point on the Cape South Coast (and specifically on Christmas day) was measured for 36 consecutive years. Consider the observations $X_1, X_2 \ldots, X_{36}$ to constitute a random sample from the population of Christmas day wind velocities. The measurements are as follows:

(Note that **the sample values are ordered in order of magnitude** to ease classification into intervals.)

5	8	11	15	18	21	22	27	28	29	30	33
34	35	36	37	39	40	41	42	45	48	50	52
53	54	55	57	60	65	75	78	80	83	88	90

You may make use of any of the following calculations:

$$\sum_{i=1}^{36} X_i = 1\,584; \qquad \sum_{i=1}^{36} X_i^2 = 87\,838; \qquad \sum_{i=1}^{36} \left(X_i - \bar{X}\right)^2 = 18\,142$$

Consider the **following six equiprobable** class intervals.

(a) The following 6 equal-probability intervals that are symmetrical with respect to μ are derived assuming $\mu = 45$ and $\sigma = 25$.

Equal probability intervals	Observed frequency (O_i)	Expected frequency (E_i)
$-\infty < X \leq 20.85$		
$20.85 < X \leq 34.20$		
$34.20 < X \leq 45.00$		
$45.00 < X \leq 55.80$		
$55.80 < X \leq 69.15$		
$69.15 < X < \infty$		
Total	36	36

At the $\alpha = 0.05$ level of significance, use a chi-square goodness-of-fit test to test whether the data shown above comes from a normal distribution with mean 45 and variance 625. Show the following three steps:

(i) Show how the last interval is derived. (5)

(ii) Fill in the columns for O_i and E_i (7)

(iii) At the 0.05 level, use the chi-square goodness-of-fit test to test if the 36 observations in the sample come from the normal distribution with mean 45 and variance 625. (7)

(b) Suppose that another area had velocity of the wind taken for 40 consecutive years and the following statistics were obtained.

$$\sum_{i=1}^{40} Y_i = 1\,700 \qquad \text{and} \qquad \sum_{i=1}^{40} \left(Y_i - \bar{Y}\right)^2 = 25\,966.4.$$

[Y_i represents the statistics mark of the i-th student of the second sample.].

(i) Test $H_0 : \sigma_1^2 = \sigma_2^2$ against $H_1 : \sigma_1^2 \neq \sigma_2^2$ at the 5% level of significance. (7)

(ii) Test the hypothesis $H_0 : \mu_X = \mu_Y$ against $H_1 : \mu_X \neq \mu_Y$ at the 5% level of significance, that is, can you conclude that the mean velocities of the two groups are different? (8)

[34]

QUESTION 4

(a) A large electronics firm that hires many workers with disabilities wants to determine whether their disabilities affect such workers's performance. The following table was obtained.

Disability	Performance			Total
	Above average	Average	Below average	
Blind	21	64	17	102
Deaf	16	49	14	79
No disability	29	93	28	150
Total	66	206	59	331

The following JMP output was obtained.

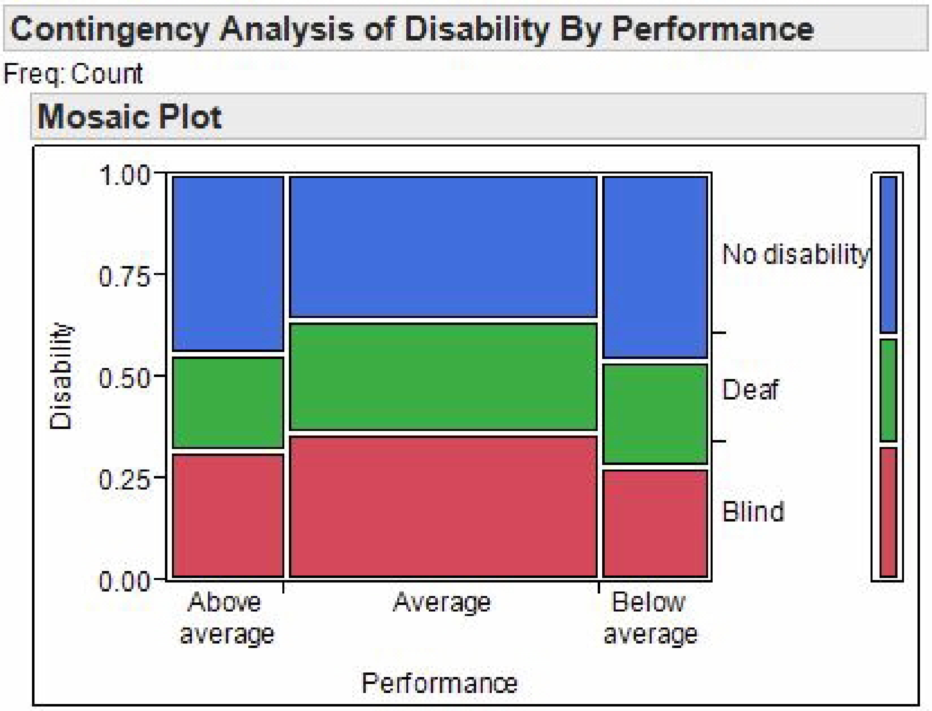


Figure 1: Mosaic Plot

Contingency Analysis of Disability By Performance

Freq: Count

Contingency Table

Performance	Count Total % Col % Row %	Disability: Blind	Deaf	No disability	
	Above average	21 6.93 20.59 31.82	16 5.28 19.75 24.24	29 9.57 24.17 43.94	66 21.78
	Average	64 21.12 62.75 36.36	49 16.17 60.49 27.84	63 20.79 52.50 35.80	176 58.09
	Below average	17 5.61 16.67 27.87	16 5.28 19.75 26.23	28 9.24 23.33 45.90	61 20.13
		102 33.66	81 26.73	120 39.60	303

Tests

N	DF	-LogLike	RSquare (U)
303	4	1.4396628	0.0044

Test	ChiSquare	Prob>ChiSq
Likelihood Ratio	2.879	0.5782
Pearson	2.865	0.5807

Figure 2: Contigency Table

Use the level of significance $\alpha = 0.05$ to decide on the basis of the sample data whether it is reasonable to maintain that the disabilities have no effect on the workers' performance.

Justify your answer by giving attention to the following detail:

(i) State the appropriate null and alternative hypothesis for this test. (2)

(ii) What test statistic is used to test these hypotheses and what is the value of the test statistic? (2)

(iii) Looking at the row percentages in **Figure 2**, can you draw any conclusions? (3)

(iv) What is your final conclusion? (2)

(v) Looking at the Mosaic Plot, does the findings confirm what can be interpreted from the diagram. Substantiate. (3)

(b) If the data on ages and prices of 25 pieces of equipment yielded $r = -0.58$, test the null hypothesis that $\rho = -0.40$ against the alternative hypothesis $\rho < -0.40$ at the 0.05 level of significance. Assume that the sample comes from a bivariate normal distribution with population correlation coefficient ρ. (5)

[17]

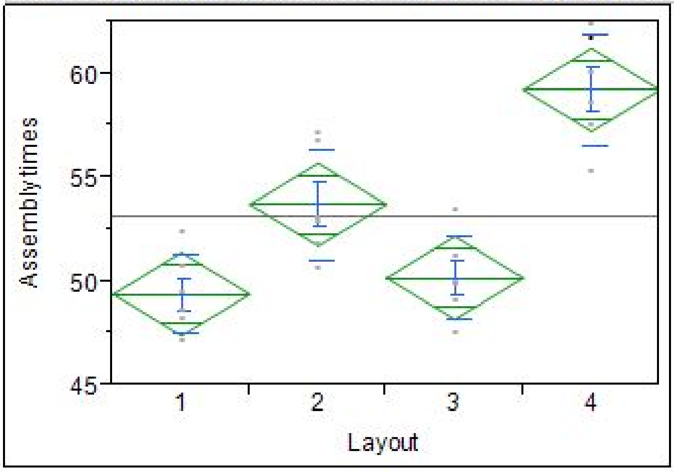
QUESTION 5

An industrial engineer tests 4 different shop-f loor layouts by having each of 6 work crews construct a subassembly and measuring the construction times (minutes) as follows:

Crew	Layout			
A	1	2	3	4
B	48.2	53.1	51.2	58.6
C	49.5	52.9	50.0	60.1
D	50.7	56.8	49.9	62.4
E	48.6	50.6	47.5	57.5
F	47.1	51.8	49.1	55.3
G	52.4	57.2	53.5	61.7

Study the following JMP output and answer the questions given below:

Oneway Analysis of Assembly times By Layout



Oneway Anova

Summary of Fit

Rsquare	0.766488
Adj Rsquare	0.731461
Root Mean Square Error	2.349415
Mean of Response	53.15417
Observations (or Sum Wgts)	24

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio	Prob > F
Layout	3	362.36458	120.788	21.8829	<.0001*
Error	20	110.39500	5.520		
C. Total	23	472.75958			

Means for Oneway Anova

Level	Number	Mean	Std Error	Lower 95%	Upper 95%
1	6	49.4167	0.95914	47.416	51.417
2	6	53.7333	0.95914	51.733	55.734
3	6	50.2000	0.95914	48.199	52.201
4	6	59.2667	0.95914	57.266	61.267

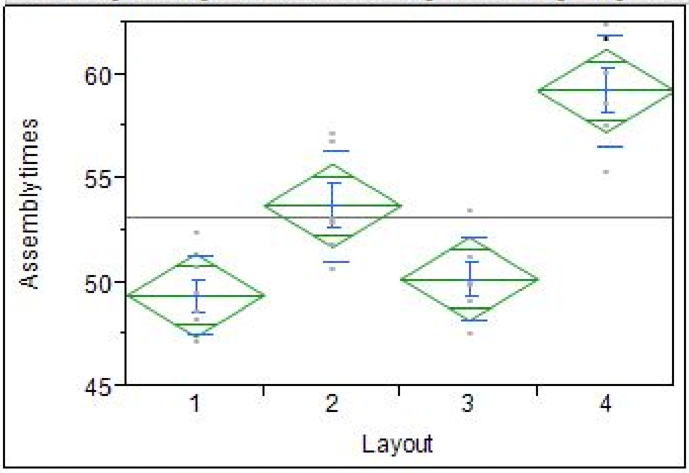
Std Error uses a pooled estimate of error variance

Means and Std Deviations

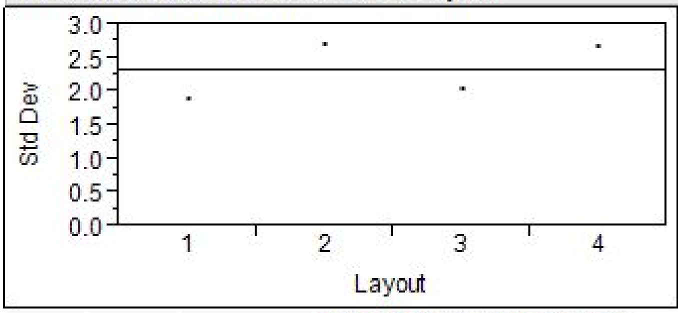
Level	Number	Mean	Std Dev	Std Err Mean	Lower 95%	Upper 95%
1	6	49.4167	1.90096	0.7761	47.422	51.412
2	6	53.7333	2.68601	1.0966	50.915	56.552
3	6	50.2000	2.02583	0.8270	48.074	52.326
4	6	59.2667	2.67333	1.0914	56.461	62.072

Figure 3: JMP Ouput for ANOVA

Oneway Analysis of Assembly times By Layout



Tests that the Variances are Equal



Level	Count	Std Dev	MeanAbsDif to Mean	MeanAbsDif to Median
1	6	1.900965	1.450000	1.450000
2	6	2.686013	2.177778	1.966667
3	6	2.025833	1.433333	1.366667
4	6	2.673325	2.133333	2.133333

Test	F Ratio	DFNum	DFDen	Prob > F
O'Brien[.5]	0.5248	3	20	0.6702
Brown-Forsythe	0.4237	3	20	0.7381
Levene	0.6881	3	20	0.5698
Bartlett	0.2983	3	.	0.8267

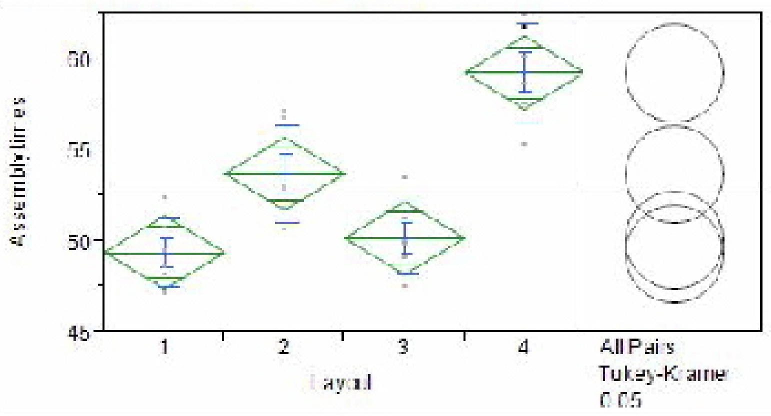
Welch's Test

Welch Anova testing Means Equal, allowing Std Devs Not Equal

F Ratio	DFNum	DFDen	Prob > F
18.6003	3	10.995	0.0001*

Figure 4: JMP Output for Equality of Variances

Oneway Analysis of Assembly times By Layout



Means Comparisons

Comparisons for all pairs using Tukey-Kramer HSD

Confidence Quantile

q*	Alpha
2.70894	0.05

LSD Threshold Matrix

Abs(Dif)-HSD

	4	2	3	1
4	-3.7966	1.7368	5.2701	6.0534
2	1.7368	-3.7966	-0.2632	0.5201
3	5.2701	-0.2632	-3.7966	-3.0132
1	6.0534	0.5201	-3.0132	-3.7966

Positive values show pairs of means that are significantly different.

Connecting Letters Report

Level				Mean
4	A			59.266667
2		B		53.733333
3		B	C	50.200000
1			C	49.416667

Levels not connected by same letter are significantly different.

Ordered Differences Report

Level	- Level	Difference	Std Err Dif	Lower CL	Upper CL	p-Value
4	1	9.850000	1.356135	6.05342	13.64658	<.0001*
4	3	9.066667	1.356135	5.27009	12.86324	<.0001*
4	2	5.533333	1.356135	1.73676	9.32991	0.0030*
2	1	4.316667	1.356135	0.52009	8.11324	0.0223*
2	3	3.533333	1.356135	-0.26324	7.32991	0.0738
3	1	0.783333	1.356135	-3.01324	4.57991	0.9377

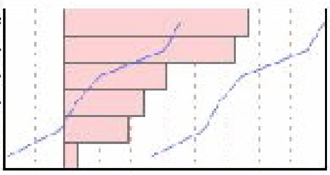


Figure 5: JMP Output for Means Comparison

(a) Use Bartlett's test to determine if the four groups have *equal population variances*? Use $\alpha = 0.05$. (3)

(b) **Do these results indicate that the layouts gave the same result** at the 5% level of significance?

Justify your answer by giving attention to the following detail:

(i) State the appropriate null and alternative hypothesis for this test.

(ii) What test statistic is used to test these hypotheses?

(iii) What is the value of the test statistic? (4)

(c) Discuss the results of the multiple comparisons in **Figure** 5 on all pairs. (5)

[12]

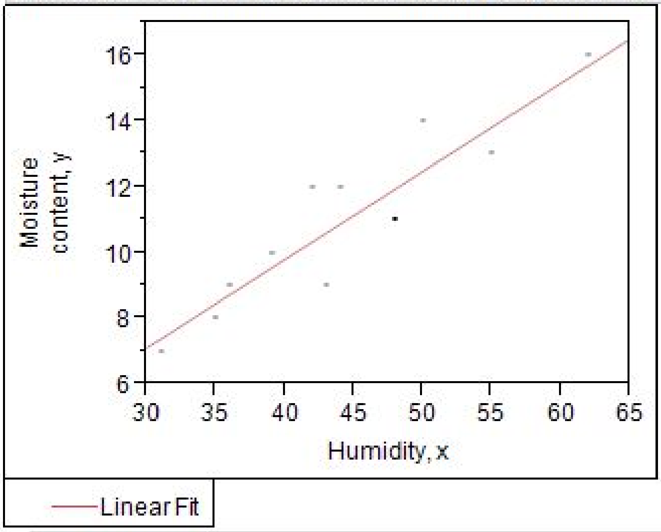
QUESTION 6

Raw material used in the production of a synthetic fibre is stored in a place which has no humidity control. Measurements of the relative humidity in the storage place and the moisture content of a sample of the raw material (both in percentages) on 12 days yielded the following results:

Humidity, x	*Moisture content, y*
42	12
35	8
50	14
43	9
48	11
62	16
31	7
36	9
44	12
39	10
55	13
48	11

The following output was obtained:

Bivariate Fit of Moisture content, y By Humidity, x



Linear Fit

Moisture content, y = -0.949508 + 0.2690321*Humidity, x

Summary of Fit

RSquare	0.836181
RSquare Adj	0.819799
Root Mean Square Error	1.101028
Mean of Response	11
Observations (or Sum Wgts)	12

Lack Of Fit

Source	DF	Sum of Squares	Mean Square	F Ratio
Lack Of Fit	9	12.122624	1.34696	.
Pure Error	1	0.000000	0.00000	Prob > F
Total Error	10	12.122624		.
				Max RSq
				1.0000

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	1	61.877376	61.8774	51.0429
Error	10	12.122624	1.2123	Prob > F
C. Total	11	74.000000		<.0001*

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	-0.949508	1.702495	-0.56	0.5893
Humidity, x	0.2690321	0.037656	7.14	<.0001*

Figure 6: JMP Output for Simple Linear Regression

(a) Brief ly discuss the applicability of simple linear regression for this data set. (1)

(b) What are the estimates of β_0 and β_1? Hence, give the least squares regression line. (3)

(c) Interpret the slope of the regression line. (1)

(d) Test for the significance of the slope, β_1 at the 5% level of significance.

Justify your answer by giving attention to the following detail

(i) State the appropriate null and alternative hypothesis for this test.

(ii) What test statistic is used to test these hypotheses?

(iii) What is the value of the test statistic? (4)

(e) Predict the amount of moisture content of the raw material when the humidity of the storage place is 40%. (1)

(f) Interpret R^2. (1)

(g) Calculate the standard error of estimate for the expected amount of moisture content of the raw material when the humidity of the storage place is 40%. Hint: $d^2 = \sum_{i=1}^{n}(X_i - \overline{X})^2 = 854.9167$ (3)

[14]

[100]

Formulae / Formules

$$B_1=\frac{\frac{1}{n}\sum_{i=1}^{n}(X_i-\overline{X})^3}{[\frac{1}{n}\sum_{i=1}^{n}(X_i-\overline{X})^2]^{\frac{3}{2}}}$$

$$B_2=\frac{\frac{1}{n}\sum_{i=1}^{n}(X_i-\overline{X})^4}{[\frac{1}{n}\sum_{i=1}^{n}(X_i-\overline{X})^2]^2}$$

$$\rho=\frac{e^{\eta}-e^{-\eta}}{e^{\eta}+e^{-\eta}}$$

$$T=\sqrt{n-2}\frac{U_{11}-U_{22}}{2\sqrt{U_{11}U_{22}-U_{12}^2}}$$

$$T=\frac{(\overline{X}_1-\overline{X}_2)-(\mu_1-\mu_2)}{S\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$$

$$\upsilon=\frac{\left[\frac{S_1^2}{n_1}+\frac{S_2^2}{n_2}\right]^2}{\frac{S_1^4}{n_1^2(n_1-1)}+\frac{S_2^4}{n_2^2(n_2-1)}}$$

$$F=\frac{n\sum_{i=1}^{k}(\overline{X}_i-\overline{X})^2/(k-1)}{\sum_{i=1}^{k}\sum_{j=1}^{n}(X_{ij}-\overline{X}_i)^2/(kn-k)}$$

$$\widehat{\beta}_1=\frac{\sum_{i=1}^{n}Y_i(X_i-\overline{X})}{d^2}$$

TABEL I

Oppervlaktes onder die Normaalkromme

$$\Phi(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-\frac{1}{2}x^2}\, dx$$

$$\Phi(-z) = 1 - \Phi(z)$$

Die oppervlakte $\Phi(z)$ is teen z vir $z \geqslant 0$ getabelleer.

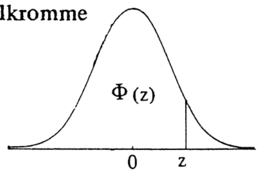


TABLE I

Areas under the Normal Curve

$$\Phi(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-\frac{1}{2}x^2}\, dx$$

$$\Phi(-z) = 1 - \Phi(z)$$

Entries in the table are values of $\Phi(z)$ for $z \geqslant 0$.

z	0,00	0,01	0,02	0,03	0,04	0,05	0,06	0,07	0,08	0,09
0,0	0,5000	0,5040	0,5080	0,5120	0,5160	0,5199	0,5239	0,5279	0,5319	0,5359
0,1	0,5398	0,5438	0,5478	0,5517	0,5557	0,5596	0,5636	0,5675	0,5714	0,5753
0,2	0,5793	0,5832	0,5871	0,5910	0,5948	0,5987	0,6026	0,6064	0,6103	0,6141
0,3	0,6179	0,6217	0,6255	0,6293	0,6331	0,6368	0,6406	0,6443	0,6480	0,6517
0,4	0,6554	0,6591	0,6628	0,6664	0,6700	0,6736	0,6772	0,6808	0,6844	0,6879
0,5	0,6915	0,6950	0,6985	0,7019	0,7054	0,7088	0,7123	0,7157	0,7190	0,7224
0,6	0,7257	0,7291	0,7324	0,7357	0,7389	0,7422	0,7454	0,7486	0,7517	0,7549
0,7	0,7580	0,7611	0,7642	0,7673	0,7704	0,7734	0,7764	0,7794	0,7823	0,7852
0,8	0,7881	0,7910	0,7939	0,7967	0,7995	0,8023	0,8051	0,8078	0,8106	0,8133
0,9	0,8159	0,8186	0,8212	0,8238	0,8264	0,8289	0,8315	0,8340	0,8365	0,8389
1,0	0,8413	0,8438	0,8461	0,8485	0,8508	0,8531	0,8554	0,8577	0,8599	0,8621
1,1	0,8643	0,8665	0,8686	0,8708	0,8729	0,8749	0,8770	0,8790	0,8810	0,8830
1,2	0,8849	0,8869	0,8888	0,8907	0,8925	0,8944	0,8962	0,8980	0,8997	0,9015
1,3	0,9032	0,9049	0,9066	0,9082	0,9099	0,9115	0,9131	0,9147	0,9162	0,9177
1,4	0,9192	0,9207	0,9222	0,9236	0,9251	0,9265	0,9279	0,9292	0,9306	0,9319
1,5	0,9332	0,9345	0,9357	0,9370	0,9382	0,9394	0,9406	0,9418	0,9429	0,9441
1,6	0,9452	0,9463	0,9474	0,9484	0,9495	0,9505	0,9515	0,9525	0,9535	0,9545
1,7	0,9554	0,9564	0,9573	0,9582	0,9591	0,9599	0,9608	0,9616	0,9625	0,9633
1,8	0,9641	0,9649	0,9656	0,9664	0,9671	0,9678	0,9686	0,9693	0,9699	0,9706
1,9	0,9713	0,9719	0,9726	0,9732	0,9738	0,9744	0,9750	0,9756	0,9761	0,9767
2,0	0,9772	0,9778	0,9783	0,9788	0,9793	0,9798	0,9803	0,9808	0,9812	0,9817
2,1	0,9821	0,9826	0,9830	0,9834	0,9838	0,9842	0,9846	0,9850	0,9854	0,9857
2,2	0,9861	0,9864	0,9868	0,9871	0,9875	0,9878	0,9881	0,9884	0,9887	0,9890
2,3	0,98928	0,98956	0,98983	0,99010	0,99036	0,99061	0,99086	0,99111	0,99134	0,99158
2,4	0,99180	0,99202	0,99224	0,99245	0,99266	0,99286	0,99305	0,99324	0,99343	0,99361
2,5	0,99379	0,99396	0,99413	0,99430	0,99446	0,99461	0,99477	0,99492	0,99506	0,99520
2,6	0,99534	0,99547	0,99560	0,99573	0,99585	0,99598	0,99609	0,99621	0,99632	0,99643
2,7	0,99653	0,99664	0,99674	0,99683	0,99693	0,99702	0,99711	0,99720	0,99728	0,99736
2,8	0,99744	0,99752	0,99760	0,99767	0,99774	0,99781	0,99788	0,99795	0,99801	0,99807
2,9	0,99813	0,99819	0,99825	0,99831	0,99836	0,99841	0,99846	0,99851	0,99856	0,99861
3,0	0,99865	0,99869	0,99874	0,99878	0,99822	0,99886	0,99889	0,99893	0,99896	0,99900
3,1	0,99903	0,99906	0,99910	0,99913	0,99916	0,99918	0,99921	0,99924	0,99926	0,99929
3,2	0,99931	0,99934	0,99936	0,99938	0,99940	0,99942	0,99944	0,99946	0,99948	0,99950
3,3	0,99952	0,99953	0,99955	0,99957	0,99958	0,99960	0,99961	0,99962	0,99964	0,99965
3,4	0,99966	0,99968	0,99969	0,99970	0,99971	0,99972	0,99973	0,99974	0,99975	0,99976
3,5	0,99977									
3,6	0,99984									
3,7	0,99989									
3,8	0,99993									
3,9	0,99995									
4,0	0,99997									

TABEL II

Waardes van die Inverse Normaalverdeling

Die inverse funksie $z = \Phi^{-1}(u)$ is teen u vir $u \geqslant 0,5$ getabelleer, waar $u = \Phi(z)$ die standaard normaalverdelingsfunksie aandui. Let op dat vir $u = \Phi(z) < 0,5$ is $\Phi(-z) = 1 - \Phi(z) > 0,5$

TABLE II

Values of the Inverse Normal Distribution

Entries in the table are values of the inverse function $z = \Phi^{-1}(u)$ for $u \geqslant 0,5$, where $u = \Phi(z)$ denotes the standard normal distribution function. Note that $\Phi(-z) = 1 - \Phi(z) > 0,5$ when $u = \Phi(z) < 0,5$.

$\Phi(z)$	0,000	0,001	0,002	0,003	0,004	0,005	0,006	0,007	0,008	0,009
0,50	0,000	0,003	0,005	0,008	0,010	0,013	0,015	0,018	0,020	0,023
0,51	0,025	0,028	0,030	0,033	0,035	0,038	0,040	0,043	0,045	0,048
0,52	0,050	0,053	0,055	0,058	0,060	0,063	0,065	0,068	0,070	0,073
0,53	0,075	0,078	0,080	0,083	0,085	0,088	0,090	0,093	0,095	0,098
0,54	0,100	0,103	0,105	0,108	0,111	0,113	0,116	0,118	0,121	0,123
0,55	0,126	0,128	0,131	0,133	0,136	0,138	0,141	0,143	0,146	0,148
0,56	0,151	0,154	0,156	0,159	0,161	0,164	0,166	0,169	0,171	0,174
0,57	0,176	0,179	0,181	0,184	0,187	0,189	0,192	0,194	0,197	0,199
0,58	0,202	0,204	0,207	0,210	0,212	0,215	0,217	0,220	0,222	0,225
0,59	0,228	0,230	0,233	0,235	0,238	0,240	0,243	0,246	0,248	0,251
0,60	0,253	0,256	0,259	0,261	0,264	0,266	0,269	0,272	0,274	0,277
0,61	0,279	0,282	0,285	0,287	0,290	0,292	0,295	0,298	0,300	0,303
0,62	0,305	0,308	0,311	0,313	0,316	0,319	0,321	0,324	0,327	0,329
0,63	0,332	0,335	0,337	0,340	0,342	0,345	0,348	0,350	0,353	0,356
0,64	0,358	0,361	0,364	0,366	0,369	0,372	0,375	0,377	0,380	0,383
0,65	0,385	0,388	0,391	0,393	0,396	0,399	0,402	0,404	0,407	0,410
0,66	0,412	0,415	0,418	0,421	0,423	0,426	0,429	0,432	0,434	0,437
0,67	0,440	0,443	0,445	0,448	0,451	0,454	0,457	0,459	0,462	0,465
0,68	0,468	0,471	0,473	0,476	0,479	0,482	0,485	0,487	0,490	0,493
0,69	0,496	0,499	0,502	0,504	0,507	0,510	0,513	0,516	0,519	0,522
0,70	0,524	0,527	0,530	0,533	0,536	0,539	0,542	0,545	0,548	0,550
0,71	0,553	0,556	0,559	0,562	0,565	0,568	0,571	0,574	0,577	0,580
0,72	0,583	0,586	0,589	0,592	0,595	0,598	0,601	0,604	0,607	0,610
0,73	0,613	0,616	0,619	0,622	0,625	0,628	0,631	0,634	0,637	0,640
0,74	0,643	0,646	0,650	0,653	0,656	0,659	0,662	0,665	0,668	0,671
0,75	0,674	0,678	0,681	0,684	0,687	0,690	0,693	0,697	0,700	0,703
0,76	0,706	0,710	0,713	0,716	0,719	0,722	0,726	0,729	0,732	0,736
0,77	0,739	0,742	0,745	0,749	0,752	0,755	0,759	0,762	0,765	0,769
0,78	0,772	0,776	0,779	0,782	0,786	0,789	0,793	0,796	0,800	0,803
0,79	0,806	0,810	0,813	0,817	0,820	0,824	0,827	0,831	0,835	0,838
0,80	0,842	0,845	0,849	0,852	0,856	0,860	0,863	0,867	0,871	0,874
0,81	0,878	0,882	0,885	0,889	0,893	0,896	0,900	0,904	0,908	0,912
0,82	0,915	0,919	0,923	0,927	0,931	0,935	0,938	0,942	0,946	0,950
0,83	0,954	0,958	0,962	0,966	0,970	0,974	0,978	0,982	0,986	0,990
0,84	0,994	0,999	1,003	1,007	1,011	1,015	1,019	1,024	1,028	1,032
0,85	1,036	1,041	1,045	1,049	1,054	1,058	1,063	1,067	1,071	1,076
0,86	1,080	1,085	1,089	1,094	1,098	1,103	1,108	1,112	1,117	1,122
0,87	1,126	1,131	1,136	1,141	1,146	1,150	1,155	1,160	1,165	1,170
0,88	1,175	1,180	1,185	1,190	1,195	1,200	1,206	1,211	1,216	1,221
0,89	1,227	1,232	1,237	1,243	1,248	1,254	1,259	1,265	1,270	1,276
0,90	1 282	1,287	1,293	1,299	1,305	1,311	1,317	1,323	1,329	1,335
0,91	1,341	1,347	1,353	1,359	1,366	1,372	1,379	1,385	1,392	1,398
0,92	1,405	1,412	1,419	1,426	1,433	1,440	1,447	1,454	1,461	1,468
0,93	1,476	1,483	1,491	1,499	1,506	1,514	1,522	1,530	1,538	1,546
0,94	1,555	1,563	1,572	1,580	1,589	1,598	1,607	1,616	1,626	1,635
0,95	1,645	1,655	1,665	1,675	1,685	1,695	1,706	1,717	1,728	1,739
0,96	1,751	1,762	1,774	1,787	1,799	1,812	1,825	1,838	1,852	1,866
0,97	1,881	1,896	1,911	1,927	1,943	1,960	1,977	1,995	2,014	2,034
0,98	2,054	2,075	2,097	2,120	2,144	2,170	2,197	2,226	2,257	2,290
0,99	2,326	2,366	2,409	2,457	2,512	2,576	2,652	2,748	2,878	3,090

TABEL III

Die t-verdeling:
Boonste Waarskynlikheidspunte

$P = P(t \geqslant t_{\nu,P}) = P(t \leqslant -t_{\nu,P})$

met $t_{\nu,P} = -t_{\nu,1-P}$ sodat

$P(|t| \geqslant t_{\nu,P}) = 2P, \qquad t_{\nu,P} > 0.$

Die waardes $t_{\nu,P}$ van die t-verdeling is teen die aantal vryheidsgrade ν en die eenkantige oorskrydingswaarskynlikheid P getabelleer.

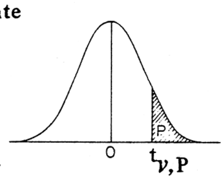


TABLE III

The t-Distribution:
Upper Probability Points

$P = P(t \geqslant t_{\nu,P}) = P(t \leqslant -t_{\nu,P})$

with $t_{\nu,P} = -t_{\nu,1-P}$ so that

$P(|t| \geqslant t_{\nu,P}) = 2P, \qquad t_{\nu,P} > 0.$

Entries in the table are the values $t_{\nu,P}$ of the t-distribution for various degrees of freedom ν and one-tailed probabilities P.

ν \ P	0,25	0,10	0,05	0,025	0,01	0,005
1	1,000	3,078	6,314	12,706	31,821	63,657
2	0,816	1,886	2,920	4,303	6,965	9,925
3	0,765	1,638	2,353	3,182	4,541	5,841
4	0,741	1,533	2,132	2,776	3,747	4,604
5	0,727	1,476	2,015	2,571	3,365	4,032
6	0,718	1,440	1,943	2,447	3,143	3,707
7	0,711	1,415	1,895	2,365	2,998	3,499
8	0,706	1,397	1,860	2,306	2,896	3,355
9	0,703	1,383	1,833	2,262	2,821	3,250
10	0,700	1,372	1,812	2,228	2,764	3,169
11	0,697	1,363	1,796	2,201	2,718	3,106
12	0,695	1,356	1,782	2,179	2,681	3,055
13	0,694	1,350	1,771	2,160	2,650	3,012
14	0,692	1,345	1,761	2,145	2,624	2,977
15	0,691	1,341	1,753	2,131	2,602	2,947
16	0,690	1,337	1,746	2,120	2,583	2,921
17	0,689	1,333	1,740	2,110	2,567	2,898
18	0,688	1,330	1,734	2,101	2,552	2,878
19	0,688	1,328	1,729	2,093	2,539	2,861
20	0,687	1,325	1,725	2,086	2,528	2,845
21	0,686	1,323	1,721	2,080	2,518	2,831
22	0,686	1,321	1,717	2,074	2,508	2,819
23	0,685	1,319	1,714	2,069	2,500	2,807
24	0,685	1,318	1,711	2,064	2,492	2,797
25	0,684	1,316	1,708	2,060	2,485	2,787
26	0,684	1,315	1,706	2,056	2,479	2,779
27	0,684	1,314	1,703	2,052	2,473	2,771
28	0,683	1,313	1,701	2,048	2,467	2,763
29	0,683	1,311	1,699	2,045	2,462	2,756
30	0,683	1,310	1,697	2,042	2,457	2,750
35	0,682	1,306	1,690	2,030	2,438	2,724
40	0,681	1,303	1,684	2,021	2,423	2,704
60	0,679	1,296	1,671	2,000	2,390	2,660
100	0,677	1,290	1,660	1,984	2,364	2,626
∞	0,675	1,282	1,645	1,960	2,326	2,576

TABEL IV

Die χ^2-verdeling:
Boonste Waarskynlikheidspunte

$P = P(\chi^2 \geqslant \chi^2_{\nu,P})$

Die waardes $\chi^2_{\nu,P}$ van die χ^2-verdeling is teen die aantal vryheidsgrade ν en die eenkantige oorskrydingswaarskynlikheid P getabelleer.

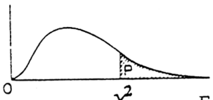


TABLE IV

The χ^2-Distribution:
Upper Probability Points

$P = P(\chi^2 \geqslant \chi^2_{\nu,P})$

Entries in the table are the values $\chi^2_{\nu,P}$ of the χ^2-distribution for various degrees of freedom ν and one-tailed probabilities P.

ν \ P	0·990	0·975	0·950	0·900	0·500	0·100	0·050	0·025	0·010	0·005
1	157088.10^{-9}	982069.10^{-9}	393214.10^{-8}	0·0157908	0·454937	2·70554	3·84146	5·02389	6·63490	7·87944
2	0·0201007	0·0506356	0·102587	0·210720	1·38629	4·60517	5·99147	7·37776	9·21034	10·5966
3	0·114832	0·215795	0·351846	0·584375	2·36597	6·25139	7·81473	9·34840	11·3449	12·8381
4	0·297110	0·484419	0·710721	1·063623	3·35670	7·77944	9·48773	11·1433	13·2767	14·8602
5	0·554300	0·831211	1·145476	1·61031	4·35146	9·23635	11·0705	12·8325	15·0863	16·7496
6	0·872085	1·237347	1·63539	2·20413	5·34812	10·6446	12·5916	14·4494	16·8119	18·5476
7	1·239043	1·68987	2·16735	2·83311	6·34581	12·0170	14·0671	16·0128	18·4753	20·2777
8	1·646482	2·17973	2·73264	3·48954	7·34412	13·3616	15·5073	17·5346	20·0902	21·9550
9	2·087912	2·70039	3·32511	4·16816	8·34283	14·6837	16·9190	19·0228	21·6660	23·5893
10	2·55821	3·24697	3·94030	4·86518	9·34182	15·9871	18·3070	20·4831	23·2093	25·1882
11	3·05347	3·81575	4·57481	5·57779	10·3410	17·2750	19·6751	21·9200	24·7250	26·7569
12	3·57056	4·40379	5·22603	6·30380	11·3403	18·5494	21·0261	23·3367	26·2170	28·2995
13	4·10691	5·00874	5·89186	7·04150	12·3398	19·8119	22·3621	24·7356	27·6883	29·8194
14	4·66043	5·62872	6·57063	7·78953	13·3393	21·0642	23·6848	26·1190	29·1413	31·3193
15	5·22935	6·26214	7·26094	8·54675	14·3389	22·3072	24·9958	27·4884	30·5779	32·8013
16	5·81221	6·90766	7·96164	9·31223	15·3385	23·5418	26·2962	28·8454	31·9999	34·2672
17	6·40776	7·56418	8·67176	10·0852	16·3381	24·7690	27·5871	30·1910	33·4087	35·7185
18	7·01491	8·23075	9·39046	10·8649	17·3379	25·9894	28·8693	31·5264	34·8053	37·1564
19	7·63273	8·90655	10·1170	11·6509	18·3376	27·2036	30·1435	32·8523	36·1908	38·5822
20	8·26040	9·59083	10·8508	12·4426	19·3374	28·4120	31·4104	34·1696	37·5662	39·9968
21	8·89720	10·28293	11·5913	13·2396	20·3372	29·6151	32·6705	35·4789	38·9321	41·4010
22	9·54249	10·9823	12·3380	14·0415	21·3370	30·8133	33·9244	36·7807	40·2894	42·7956
23	10·19567	11·6885	13·0905	14·8479	22·3369	32·0069	35·1725	38·0757	41·6384	44·1813
24	10·8564	12·4011	13·8484	15·6587	23·3367	33·1963	36·4151	39·3641	42·9798	45·5585
25	11·5240	13·1197	14·6114	16·4734	24·3366	34·3816	37·6525	40·6465	44·3141	46·9278
26	12·1981	13·8439	15·3791	17·2919	25·3364	35·5631	38·8852	41·9232	45·6417	48·2899
27	12·8786	14·5733	16·1513	18·1138	26·3363	36·7412	40·1133	43·1944	46·9630	49·6449
28	13·5648	15·3079	16·9279	18·9392	27·3363	37·9159	41·3372	44·4607	48·2782	50·9933
29	14·2565	16·0471	17·7083	19·7677	28·3362	39·0875	42·5569	45·7222	49·5879	52·3356
30	14·9535	16·7908	18·4926	20·5992	29·3360	40·2560	43·7729	46·9792	50·8922	53·6720
40	22·1643	24·4331	26·5093	29·0505	39·3354	51·8050	55·7585	59·3417	63·6907	66·7659
50	29·7067	32·3574	34·7642	37·6886	49·3349	63·1671	67·5048	71·4202	76·1539	79·4900
60	37·4848	40·4817	43·1879	46·4589	59·3347	74·3970	79·0819	83·2976	88·3794	91·9517
70	45·4418	48·7576	51·7393	55·3290	69·3344	85·5271	90·5312	95·0231	100·425	104·215
80	53·5400	57·1532	60·3915	64·2778	79·3343	96·5782	101·879	106·629	112·329	116·321
90	61·7541	65·6466	69·1260	73·2912	89·3342	107·565	113·145	118·136	124·116	128·299
100	70·0648	74·2219	77·9295	82·3581	99·3341	118·498	124·342	129·561	135·807	140·169

TABEL V

Die F-verdeling: Boonste 5% Punte

(ν_1 vryheidsgrade in die teller en ν_2 in die noemer)

TABLE V

The F-Distribution: Upper 5% Points

(ν_1 degrees of freedom in numerator and ν_2 in denominator)

ν_2	ν_1=1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	161	200	216	225	230	234	237	239	241	242	244	246	248	249	250	251	252	253	254
2	18,5	19,0	19,2	19,2	19,3	19,3	19,4	19,4	19,4	19,4	19,4	19,4	19,4	19,5	19,5	19,5	19,5	19,5	19,5
3	10,1	9,55	9,28	9,12	9,01	8,94	8,89	8,85	8,81	8,79	8,74	8,70	8,66	8,64	8,62	8,59	8,57	8,55	8,53
4	7,71	6,94	6,59	6,39	6,26	6,16	6,09	6,04	6,00	5,96	5,91	5,86	5,80	5,77	5,75	5,72	5,69	5,66	5,63
5	6,61	5,79	5,41	5,19	5,05	4,95	4,88	4,82	4,77	4,74	4,68	4,62	4,56	4,53	4,50	4,46	4,43	4,40	4,36
6	5,99	5,14	4,76	4,53	4,39	4,28	4,21	4,15	4,10	4,06	4,00	3,94	3,87	3,84	3,81	3,77	3,74	3,70	3,67
7	5,59	4,74	4,35	4,12	3,97	3,87	3,79	3,73	3,68	3,64	3,57	3,51	3,44	3,41	3,38	3,34	3,30	3,27	3,23
8	5,32	4,46	4,07	3,84	3,69	3,58	3,50	3,44	3,39	3,35	3,28	3,22	3,15	3,12	3,08	3,04	3,01	2,97	2,93
9	5,12	4,26	3,86	3,63	3,48	3,37	3,29	3,23	3,18	3,14	3,07	3,01	2,94	2,90	2,86	2,83	2,79	2,75	2,71
10	4,96	4,10	3,71	3,48	3,33	3,22	3,14	3,07	3,02	2,98	2,91	2,85	2,77	2,74	2,70	2,66	2,62	2,58	2,54
11	4,84	3,98	3,59	3,36	3,20	3,09	3,01	2,95	2,90	2,85	2,79	2,72	2,65	2,61	2,57	2,53	2,49	2,45	2,40
12	4,75	3,89	3,49	3,26	3,11	3,00	2,91	2,85	2,80	2,75	2,69	2,62	2,54	2,51	2,47	2,43	2,38	2,34	2,30
13	4,67	3,81	3,41	3,18	3,03	2,92	2,83	2,77	2,71	2,67	2,60	2,53	2,46	2,42	2,38	2,34	2,30	2,25	2,21
14	4,60	3,74	3,34	3,11	2,96	2,85	2,76	2,70	2,65	2,60	2,53	2,46	2,39	2,35	2,31	2,27	2,22	2,18	2,13
15	4,54	3,68	3,29	3,06	2,90	2,79	2,71	2,64	2,59	2,54	2,48	2,40	2,33	2,29	2,25	2,20	2,16	2,11	2,07
16	4,49	3,63	3,24	3,01	2,85	2,74	2,66	2,59	2,54	2,49	2,42	2,35	2,28	2,24	2,19	2,15	2,11	2,06	2,01
17	4,45	3,59	3,20	2,96	2,81	2,70	2,61	2,55	2,49	2,45	2,38	2,31	2,23	2,19	2,15	2,10	2,06	2,01	1,96
18	4,41	3,55	3,16	2,93	2,77	2,66	2,58	2,51	2,46	2,41	2,34	2,27	2,19	2,15	2,11	2,06	2,02	1,97	1,92
19	4,38	3,52	3,13	2,90	2,74	2,63	2,54	2,48	2,42	2,38	2,31	2,23	2,16	2,11	2,07	2,03	1,98	1,93	1,88
20	4,35	3,49	3,10	2,87	2,71	2,60	2,51	2,45	2,39	2,35	2,28	2,20	2,12	2,08	2,04	1,99	1,95	1,90	1,84
21	4,32	3,47	3,07	2,84	2,68	2,57	2,49	2,42	2,37	2,32	2,25	2,18	2,10	2,05	2,01	1,96	1,92	1,87	1,81
22	4,30	3,44	3,05	2,82	2,66	2,55	2,46	2,40	2,34	2,30	2,23	2,15	2,07	2,03	1,98	1,94	1,89	1,84	1,78
23	4,28	3,42	3,03	2,80	2,64	2,53	2,44	2,37	2,32	2,27	2,20	2,13	2,05	2,01	1,96	1,91	1,86	1,81	1,76
24	4,26	3,40	3,01	2,78	2,62	2,51	2,42	2,36	2,30	2,25	2,18	2,11	2,03	1,98	1,94	1,89	1,84	1,79	1,73
25	4,24	3,39	2,99	2,76	2,60	2,49	2,40	2,34	2,28	2,24	2,16	2,09	2,01	1,96	1,92	1,87	1,82	1,77	1,71
28	4,20	3,34	2,95	2,71	2,56	2,45	2,36	2,29	2,24	2,19	2,12	2,04	1,96	1,91	1,87	1,82	1,77	1,71	1,65
30	4,17	3,32	2,92	2,69	2,53	2,42	2,33	2,27	2,21	2,16	2,09	2,01	1,93	1,89	1,84	1,79	1,74	1,68	1,62
34	4,13	3,28	2,88	2,65	2,49	2,38	2,29	2,23	2,17	2,12	2,05	1,97	1,89	1,84	1,80	1,75	1,69	1,63	1,57
40	4,08	3,23	2,84	2,61	2,45	2,34	2,25	2,18	2,12	2,08	2,00	1,92	1,84	1,79	1,74	1,69	1,64	1,58	1,51
48	4,04	3,19	2,80	2,57	2,41	2,29	2,21	2,14	2,08	2,03	1,96	1,88	1,79	1,75	1,70	1,64	1,59	1,52	1,45
60	4,00	3,15	2,76	2,53	2,37	2,25	2,17	2,10	2,04	1,99	1,92	1,84	1,75	1,70	1,65	1,59	1,53	1,47	1,39
80	3,96	3,11	2,72	2,49	2,33	2,21	2,13	2,06	2,00	1,95	1,88	1,79	1,70	1,65	1,60	1,54	1,48	1,41	1,32
120	3,92	3,07	2,68	2,45	2,29	2,18	2,09	2,02	1,96	1,91	1,83	1,75	1,66	1,61	1,55	1,50	1,43	1,35	1,25
∞	3,84	3,00	2,60	2,37	2,21	2,10	2,01	1,94	1,88	1,83	1,75	1,67	1,57	1,52	1,46	1,39	1,32	1,22	1,00

TABEL VI

Die F-verdeling: Boonste 2,5% Punte

(ν_1 vryheidsgrade in die teller en ν_2 in die noemer)

TABLE VI

The F-Distribution: Upper 2,5% Points

(ν_1 degrees of freedom in numerator and ν_2 in denominator)

ν_2	$\nu_1=1$	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	648	800	864	900	922	937	948	957	963	969	977	985	993	997	1001	1006	1010	1014	1018
2	38,5	39,0	39,2	39,2	39,3	39,3	39,4	39,4	39,4	39,4	39,4	39,4	39,4	39,5	39,5	39,5	39,5	39,5	39,5
3	17,4	16,0	15,4	15,1	14,9	14,7	14,6	14,5	14,5	14,4	14,3	14,3	14,2	14,1	14,1	14,0	14,0	13,9	13,9
4	12,2	10,6	9,98	9,60	9,36	9,20	9,07	8,98	8,90	8,84	8,75	8,66	8,56	8,51	8,46	8,41	8,36	8,31	8,26
5	10,0	8,43	7,76	7,39	7,15	6,98	6,85	6,76	6,68	6,62	6,52	6,43	6,33	6,28	6,23	6,18	6,12	6,07	6,02
6	8,81	7,26	6,60	6,23	5,99	5,82	5,70	5,60	5,52	5,46	5,37	5,27	5,17	5,12	5,07	5,01	4,96	4,90	4,85
7	8,07	6,54	5,89	5,52	5,29	5,12	4,99	4,90	4,82	4,76	4,67	4,57	4,47	4,42	4,36	4,31	4,25	4,20	4,14
8	7,57	6,06	5,42	5,05	4,82	4,65	4,53	4,43	4,36	4,30	4,20	4,10	4,00	3,95	3,89	3,84	3,78	3,73	3,67
9	7,21	5,71	5,08	4,72	4,48	4,32	4,20	4,10	4,03	3,96	3,87	3,77	3,67	3,61	3,56	3,51	3,45	3,39	3,33
10	6,94	5,46	4,83	4,47	4,24	4,07	3,95	3,85	3,78	3,72	3,62	3,52	3,42	3,37	3,31	3,26	3,20	3,14	3,08
11	6,72	5,26	4,63	4,28	4,04	3,88	3,76	3,66	3,59	3,53	3,43	3,33	3,23	3,17	3,12	3,06	3,00	2,94	2,88
12	6,55	5,10	4,47	4,12	3,89	3,73	3,61	3,51	3,44	3,37	3,28	3,18	3,07	3,02	2,96	2,91	2,85	2,79	2,72
13	6,41	4,97	4,35	4,00	3,77	3,60	3,48	3,39	3,31	3,25	3,15	3,05	2,95	2,89	2,84	2,78	2,72	2,66	2,60
14	6,30	4,86	4,24	3,89	3,66	3,50	3,38	3,29	3,21	3,15	3,05	2,95	2,84	2,79	2,73	2,67	2,61	2,55	2,49
15	6,20	4,77	4,15	3,80	3,58	3,41	3,29	3,20	3,12	3,06	2,96	2,86	2,76	2,70	2,64	2,58	2,52	2,46	2,40
16	6,12	4,69	4,08	3,73	3,50	3,34	3,22	3,12	3,05	2,99	2,89	2,79	2,68	2,63	2,57	2,51	2,45	2,38	2,32
17	6,04	4,62	4,01	3,66	3,44	3,28	3,16	3,06	2,98	2,92	2,82	2,72	2,62	2,56	2,50	2,44	2,38	2,32	2,25
18	5,98	4,56	3,95	3,61	3,38	3,22	3,10	3,01	2,93	2,87	2,77	2,67	2,56	2,50	2,44	2,38	2,32	2,26	2,19
19	5,92	4,51	3,90	3,56	3,33	3,17	3,05	2,96	2,88	2,82	2,72	2,62	2,51	2,45	2,39	2,33	2,27	2,20	2,13
20	5,87	4,46	3,86	3,51	3,29	3,13	3,01	2,91	2,84	2,77	2,68	2,57	2,46	2,41	2,35	2,29	2,22	2,16	2,09
21	5,83	4,42	3,82	3,48	3,25	3,09	2,97	2,87	2,80	2,73	2,64	2,53	2,42	2,37	2,31	2,25	2,18	2,11	2,04
22	5,79	4,38	3,78	3,44	3,22	3,05	2,93	2,84	2,76	2,70	2,60	2,50	2,39	2,33	2,27	2,21	2,14	2,08	2,00
23	5,75	4,35	3,75	3,41	3,18	3,02	2,90	2,81	2,73	2,67	2,57	2,47	2,36	2,30	2,24	2,18	2,11	2,04	1,97
24	5,72	4,32	3,72	3,38	3,15	2,99	2,87	2,78	2,70	2,64	2,54	2,44	2,33	2,27	2,21	2,15	2,08	2,01	1,94
25	5,69	4,29	3,69	3,35	3,13	2,97	2,85	2,75	2,68	2,61	2,51	2,41	2,30	2,24	2,18	2,12	2,05	1,98	1,91
28	5,61	4,22	3,63	3,29	3,06	2,90	2,78	2,69	2,61	2,55	2,45	2,34	2,23	2,17	2,11	2,05	1,98	1,91	1,83
30	5,57	4,18	3,59	3,25	3,03	2,87	2,75	2,65	2,57	2,51	2,41	2,31	2,20	2,14	2,07	2,01	1,94	1,87	1,79
34	5,50	4,12	3,53	3,19	2,97	2,81	2,69	2,59	2,52	2,45	2,35	2,25	2,13	2,07	2,01	1,95	1,88	1,80	1,72
40	5,42	4,05	3,46	3,13	2,90	2,74	2,62	2,53	2,45	2,39	2,29	2,18	2,07	2,01	1,94	1,88	1,80	1,72	1,64
48	5,35	3,99	3,40	3,07	2,84	2,69	2,56	2,47	2,39	2,33	2,23	2,12	2,01	1,94	1,88	1,81	1,73	1,65	1,56
60	5,29	3,93	3,34	3,01	2,79	2,63	2,51	2,41	2,33	2,27	2,17	2,06	1,94	1,88	1,82	1,74	1,67	1,58	1,48
80	5,22	3,86	3,28	2,95	2,73	2,57	2,45	2,35	2,28	2,21	2,11	2,00	1,88	1,82	1,75	1,68	1,60	1,51	1,40
120	5,15	3,80	3,23	2,89	2,67	2,52	2,39	2,30	2,22	2,16	2,05	1,94	1,82	1,76	1,69	1,61	1,53	1,43	1,31
∞	5,02	3,69	3,12	2,79	2,57	2,41	2,29	2,19	2,11	2,05	1,94	1,83	1,71	1,64	1,57	1,48	1,39	1,27	1,00

TABEL IX

Die Produkmoment-korrelasiekoëffisiënt: Boonste Kritieke Waardes (vir ρ=0)

TABLE IX

The Product Moment Correlation Coefficient: Upper Critical Values (for ρ=0)

n = aantal pare waarnemings

n = number of pairs of observations

n	Betekenispeil vir eenkantige toets / Significance level for one-tailed test 0,25	0,10	0,05	0,025	0,01	0,005
3	0,7071	0,9511	0,9877	0,9969	0,9995	0,9999
4	0,5000	0,8000	0,9000	0,9500	0,9800	0,9900
5	0,4040	0,6870	0,8054	0,8783	0,9343	0,9587
6	0,3473	0,6084	0,7293	0,8114	0,8822	0,9172
7	0,3091	0,5509	0,6694	0,7545	0,8329	0,8745
8	0,2811	0,5067	0,6215	0,7067	0,7887	0,8343
9	0,2596	0,4716	0,5822	0,6664	0,7498	0,7977
10	0,2423	0,4428	0,5494	0,6319	0,7155	0,7646
11	0,2281	0,4187	0,5214	0,6021	0,6851	0,7348
12	0,2161	0,3981	0,4973	0,5760	0,6581	0,7079
13	0,2058	0,3802	0,4762	0,5529	0,6339	0,6835
14	0,1968	0,3646	0,4575	0,5324	0,6120	0,6614
15	0,1890	0,3507	0,4409	0,5140	0,5923	0,6411
16	0,1820	0,3383	0,4259	0,4973	0,5742	0,6226
17	0,1757	0,3271	0,4124	0,4821	0,5577	0,6055
18	0,1700	0,3170	0,4000	0,4683	0,5425	0,5897
19	0,1649	0,3077	0,3887	0,4555	0,5285	0,5751
20	0,1602	0,2992	0,3783	0,4438	0,5155	0,5614
21	0,1558	0,2914	0,3687	0,4329	0,5034	0,5487
22	0,1518	0,2841	0,3598	0,4227	0,4921	0,5368
23	0,1481	0,2774	0,3515	0,4132	0,4815	0,5256
24	0,1447	0,2711	0,3438	0,4044	0,4716	0,5151
25	0,1415	0,2653	0,3365	0,3961	0,4622	0,5052
26	0,1384	0,2598	0,3297	0,3882	0,4534	0,4958
27	0,1356	0,2546	0,3233	0,3809	0,4451	0,4896
28	0,1330	0,2497	0,3172	0,3739	0,4372	0,4785
29	0,1305	0,2451	0,3115	0,3673	0,4297	0,4705
30	0,1281	0,2407	0,3061	0,3610	0,4226	0,4629
31	0,1258	0,2366	0,3009	0,3550	0,4158	0,4556
32	0,1237	0,2327	0,2960	0,3494	0,4093	0,4487
35	0,1179	0,2220	0,2826	0,3338	0,3916	0,4296
40	0,1098	0,2070	0,2638	0,3120	0,3665	0,4026
45	0,1032	0,1947	0,2483	0,2940	0,3457	0,3801
50	0,0976	0,1843	0,2353	0,2787	0,3281	0,3610
60	0,0888	0,1678	0,2144	0,2542	0,2997	0,3301
70	0,0820	0,1550	0,1982	0,2352	0,2776	0,3060
80	0,0765	0,1448	0,1852	0,2199	0,2597	0,2864
90	0,0720	0,1364	0,1745	0,2072	0,2449	0,2702
100	0,0682	0,1292	0,1654	0,1966	0,2324	0,2565

TABEL X

Die z-transformasie vir die Korrelasiekoëffisiënt

Die getransformeerde waardes

$$z = \tanh^{-1} r = ½ \log_e \frac{1+r}{1-r}$$

is teen die korrelasiekoëffisiënt r getabelleer.

TABLE X

The z-Transformation for the Correlation Coefficient

Entries in the table are the transformed values

$$z = \tanh^{-1} r = ½ \log_e \frac{1+r}{1-r}$$

for various values of the correlation coefficient r.

r	0,00	0,01	0,02	0,03	0,04	0,05	0,06	0,07	0,08	0,09
0,0	0,0000	0,0100	0,0200	0,0300	0,0400	0,0500	0,0601	0,0701	0,0802	0,0902
0,1	0,1003	0,1104	0,1206	0,1307	0,1409	0,1511	0,1614	0,1717	0,1820	0,1923
0,2	0,2027	0,2132	0,2237	0,2342	0,2448	0,2554	0,2661	0,2769	0,2877	0,2986
0,3	0,3095	0,3205	0,3316	0,3428	0,3541	0,3654	0,3769	0,3884	0,4001	0,4118
0,4	0,4236	0,4356	0,4477	0,4599	0,4722	0,4847	0,4973	0,5101	0,5230	0,5361
0,5	0,5493	0,5627	0,5763	0,5901	0,6042	0,6184	0,6328	0,6475	0,6625	0,6777
0,6	0,6931	0,7089	0,7250	0,7414	0,7582	0,7753	0,7928	0,8107	0,8291	0,8480
0,7	0,8673	0,8872	0,9076	0,9287	0,9505	0,9730	0,9962	1,0203	1,0454	1,0714
0,8	1,0986	1,1270	1,1568	1,1881	1,2212	1,2562	1,2933	1,3331	1,3758	1,4219
0,9	1,4722	1,5275	1,5890	1,6584	1,7380	1,8318	1,9459	2,0923	2,2976	2,6466

r	0,000	0,001	0,002	0,003	0,004	0,005	0,006	0,007	0,008	0,009
0,90	1,4722	1,4775	1,4828	1,4882	1,4937	1,4992	1,5047	1,5103	1,5160	1,5217
0,91	1,5275	1,5334	1,5393	1,5453	1,5513	1,5574	1,5636	1,5698	1,5762	1,5826
0,92	1,5890	1,5956	1,6022	1,6089	1,6157	1,6226	1,6296	1,6366	1,6438	1,6510
0,93	1,6584	1,6658	1,6734	1,6811	1,6888	1,6967	1,7047	1,7129	1,7211	1,7295
0,94	1,7380	1,7467	1,7555	1,7645	1,7736	1,7828	1,7923	1,8019	1,8117	1,8216
0,95	1,8318	1,8421	1,8527	1,8635	1,8745	1,8857	1,8972	1,9090	1,9210	1,9333
0,96	1,9459	1,9588	1,9721	1,9857	1,9996	2,0139	2,0287	2,0439	2,0595	2,0756
0,97	2,0923	2,1095	2,1273	2,1457	2,1649	2,1847	2,2054	2,2269	2,2494	2,2729
0,98	2,2976	2,3235	2,3507	2,3796	2,4101	2,4427	2,4774	2,5147	2,5550	2,5987
0,99	2,6466	2,6996	2,7587	2,8257	2,9031	2,9945	3,1063	3,2504	3,4534	3,8002

Table A. Percentage points for the distribution of B_1
Lower percentage point $=$ – (tabulated upper percentage point)

Size of sample	Percentage points	Size of sample	Percentage points
n	5%	n	5%
25	0, 711	200	0, 280
30	0, 662	250	0, 251
35	0, 621	300	0, 230
40	0, 587	350	0, 213
45	0, 558	400	0, 200
50	0, 534	450	0, 188
		500	0, 179
60	0, 492	550	0, 171
70	0, 459	600	0, 163
80	0, 432	650	0, 157
90	0, 409	700	0, 151
100	0, 389	750	0, 146
		800	0, 142
125	0, 350	850	0, 138
150	0, 321	900	0, 134
175	0, 298	950	0, 130
200	0, 280	1000	0, 127

Table B. Percentage points of the distribution of B_2

Size of sample n	Percentage points	
	Upper 5%	Lower 5%
50	3, 99	2, 15
75	3, 87	2, 27
100	3, 77	2, 35
125	3, 71	2, 40
150	3, 65	2, 45
200	3, 57	2, 51
250	3, 52	2, 55
300	3, 47	2, 59
350	3, 44	2, 62
400	3, 41	2, 64
450	3, 39	2, 66
500	3, 37	2, 67
550	3, 35	2, 69
600	3, 34	2, 70
650	3, 33	2, 71
700	3, 31	2, 72
800	3, 29	2, 74
900	3, 28	2, 75
1000	3, 26	2, 76

Table C. Percentage points for the distribution of $A = \dfrac{\text{mean deviation}}{\text{standard deviation}}$

Size of sample n	$n-1$	Percentage points			
		Upper 5%	Upper 10%	Lower 10%	Lower 5%
11	10	0,9073	0,8899	0,7409	0,7153
16	15	0,8884	0,8733	0,7452	0,7236
21	20	0,8768	0,8631	0,7495	0,7304
26	25	0,8686	0,8570	0,7530	0,7360
31	30	0,8625	0,8511	0,7559	0,7404
36	35	0,8578	0,8468	0,7583	0,7440
41	40	0,8540	0,8436	0,7604	0,7470
46	45	0,8508	0,8409	0,7621	0,7496
51	50	0,8481	0,8385	0,7636	0,7518
61	60	0,8434	0,8349	0,7662	0,7554
71	70	0,8403	0,8321	0,7683	0,7583
81	80	0,8376	0,8298	0,7700	0,7607
91	90	0,8353	0,8279	0,7714	0,7626
101	100	0,8344	0,8264	0,7726	0,7644

Table D

Tabel D

The hypergeometric probability distribution: $P\ (X \leq x)$ for $N = 12$

Die hipergeometriese verdeling: $P\ (X \leq x)$ vir $N = 12$

n	k	x	P	n	k	x	P	n	k	x	P
1	1	0	0,917	4	4	0	0,141	6	2	0	0,227
1	1	1	1,000	4	4	1	0,594	6	2	1	0,773
				4	4	2	0,933	6	2	2	1,000
2	1	0	0,833	4	4	3	0,998				
2	1	1	1,000	4	4	4	1,000	6	3	0	0,091
								6	3	1	0,500
2	2	0	0,682	5	1	0	0,583	6	3	2	0,909
2	2	1	0,985	5	1	1	1,000	6	3	3	1,000
2	2	2	1,000								
				5	2	0	0,318	6	4	0	0,030
3	1	0	0,750	5	2	1	0,848	6	4	1	0,273
3	1	1	1,000	5	2	2	1,000	6	4	2	0,727
								6	4	3	0,970
3	2	0	0,545	5	3	0	0,159	6	4	4	1,000
3	2	1	0,955	5	3	1	0,636				
3	2	2	1,000	5	3	2	0,955	6	5	0	0,008
				5	3	3	1,000	6	5	1	0,121
3	3	0	0,382					6	5	2	0,500
3	3	1	0,873	5	4	0	0,071	6	5	3	0,879
3	3	2	0,995	5	4	1	0,424	6	5	4	0,992
3	3	3	1,000	5	4	2	0,848	6	5	5	1,000
				5	4	3	0,990				
4	1	0	0,667	5	4	4	1,000	6	6	0	0,001
4	1	1	1,000					6	6	1	0,040
				5	5	0	0,027	6	6	2	0,284
4	2	0	0,424	5	5	1	0,247	6	6	3	0,716
4	2	1	0,909	5	5	2	0,689	6	6	4	0,960
4	2	2	1,000	5	5	3	0,955	6	6	5	0,999
				5	5	4	0,999	6	6	6	1,000
4	3	0	0,255	5	5	5	1,000				
4	3	1	0,764								
4	3	2	0,982	6	1	0	0,500				
4	3	3	1,000	6	1	1	1,000				

Table E

Upper 5% percentage points of the ratio, $S^2_{\max}/S^2_{\min}$

υ	$k = 2$	3	4	5	6
2	39, 0	87, 5	142	202	266
3	15, 4	27, 8	39, 2	50, 7	62, 0
4	9, 60	15, 5	20, 6	25, 2	29, 5
5	7, 15	10, 8	13, 7	16, 3	18, 7
6	5, 82	8, 38	10, 4	12, 1	13, 7
7	4, 99	6, 94	8, 44	9, 70	10, 8
8	4, 43	6, 00	7, 18	8, 12	9, 03
9	4, 03	5, 34	6, 31	7, 11	7, 80
10	3, 72	4, 85	5, 67	6, 34	6, 92
12	3, 28	4, 16	4, 79	5, 30	5, 72
15	2, 86	3, 54	4, 01	4, 37	4, 68
20	2, 46	2, 95	3, 29	3, 54	3, 76
30	2, 07	2, 40	2, 61	2, 78	2, 91
60	1, 67	1, 85	1, 96	2, 04	2, 11
∞	1, 00	1, 00	1, 00	1, 00	1, 00

k = number of samples

υ = degrees of freedom for each sample variance