PYC3704 (2017)

Psychological Research Study Notes

IMPORTANT: Read through your UNISA study guide first!

- **Get an overview of the module and then study each Topic individually**
- **Use this guide in conjunction with the UNISA guide- it is NOT a substitute**
- **If you have previous question papers- PLEASE DON'T RELY ON THESE, you need to UNDERSTAND the content of this module if you want to pass and carry on to complete your Honours!**
- **PLEASE NOTE THE Zp and Zc test statistics are NO longer part of your syllabus, I have however included them in my notes to assist you when working through the past exam papers (prior to 2012).**
- **So you only need to know the following test statistics: Zx Tx Tc Td, Pearson's r and Xp^2. Refer to the flow chart on p.177 in the UNISA guide.**

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Dear UNISA Student

I understand that this is a **difficult** module and many students close their study guides about half way through the second topic! However, I know that if you work through this guide and dedicate the time and energy to this module not only will you **pass** but if you put in the hours you can achieve a *DISTINCTION*! Which is not an easy task for a third year module…

So take a deep breath and work systematically throughout the semester so you don't end up trying to come to terms with tricky concepts a week before you write your exam.

The UNISA guide can be very confusing and the language used is quite complex- so I am going to try to relate all the difficult concepts back to things you would use in your daily life so you can easily memorise and most importantly UNDERSTAND what is going on in the strange but interesting world of statistics.

Many students have not done maths beyond standard 6 or 8 (or completed school many years ago!) and get thrown when they see all the formulae in the study guide. Please note that you only need a **very basic level of maths** to do the calculations (your calculator will do all the work if you know how to use it). The best news is there are usually only about **5-6 calculations** in the **exam** so you can leave them out if you are really not comfortable doing calculations and still do really well :) Please see the list of formula on page 36 that you NEED to know and use in the exam!

If you spot any errors or typos in this guide please email me: taryn.elise.herselman@gmail.com so I can correct future versions.

Good luck and may the force be with you!!

1. WHAT TO MEMORISE FOR YOUR EXAM:

1. The **decision rule** (Unisa Study Guide p.83, second paragraph) - ***MANY*** of the exam questions are a restatement of the decision rule so make sure you know it well!
2. The **meaning of the various symbols**, e.g. p (p-value), s (standard deviation), µ (mean) etc. Use the table on page 161 (Appendix C) as a guide and add any extra symbols from Topic 3-6 that you come across. You will see a chart that I have started for you on the next page.
3. The **Appendix F table** (**Flow Chart**) at the back of your UNISA Study Guide. P.177 Make a **big, colourful poster** and stick it up on your wall. You can also refer to the flow chart in my notes on **page 37** which gives a bit more detail on how to choose the correct statistic

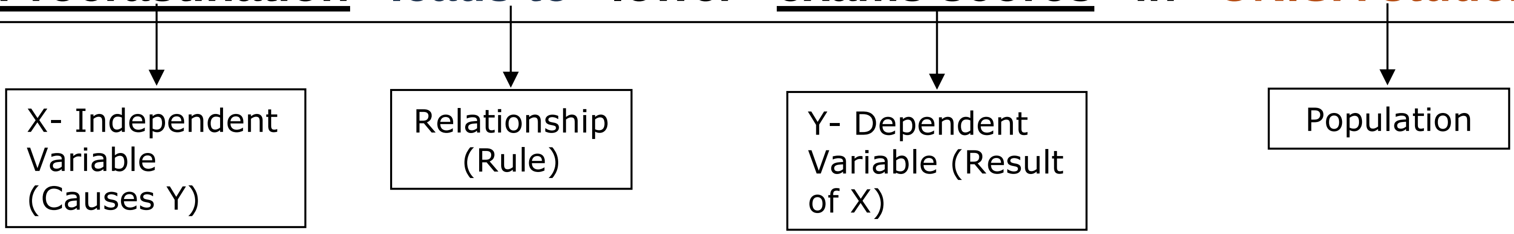
2. TABLE OF THE SYMBOLS USED IN THIS MODULE

	POPULATIONS (Hypothesis) **PARAMETER**	SAMPLES **STATISTIC**	TEST STATISTICS
ARITHMETIC MEAN	μ	$\bar{x}$	
STANDARD DEVIATION	σ	s	
VARIANCE	σ^2	s^2 $(s=\sqrt{s^2})$	
STD ERROR OF THE MEAN Also called: Standard deviation of the sampling distribution of the mean OR Standard deviation of the sample means	$\sigma_{\bar{x}} = (\sigma/\sqrt{n})$	$s_{\bar{x}} = (s/\sqrt{n})$	
MEAN OF ALL MEANS Also called: Mean of sampling distribution of the mean This is equal to the **mean under H_0** the Null Hypothesis	$\mu_{\bar{x}}$	-	
Z score for means	-	$z_{\bar{x}}$	
Difference between scores	D	d	
Pearson's R	ρ (Rho looks like a small p)	r	
Squared correlation Can be used as indication of **effect size**		r^2	
Proportions	P	p	
P-value (probability value associated with a test statistic)	-	p	
Level of significance (alpha) Set by the researcher at the start of the project	-	α	
Beta Refers to the probability of making a type II error	-	β	
Z-test / T-test (independent and dependent groups) / Chi- squared *(See the flow chart on p.36 for the full list)	-	-	T_c/ T_d / $Z_{\bar{x}}/Xp^2$
Statistical Hypotheses (Null and Alternative) which ALWAYS refer to a population never a sample!	H_0 and H_1	-	

3. BASIC OVERVIEW OF THE STEPS IN RESEARCH

Step 1: Hypothesis - a relation between constructs specifically targeted for testing.
- a rule that associated values of one variable with values of another variable

H_1: Procrastination leads to lower exams scores in UNISA students



Step 2: Create an OPERATIONAL HYPOTHESIS
- Measure the constructs (two or more) - HOW??- Be specific!
- Define the population (general/ universal rule)
- Decide what design you will use, e.g. Groups.
- Your measure (e.g. questionnaire) must be reliable and valid

Step 3: Translate the Research Hypothesis into a STATISTICAL HYPOTHESIS

1. Null Hypothesis/ H_o : e.g. H_o: $\mu = 300$
2. Alternative Hypothesis / H_1: e.g. H_1: $\mu \neq 300$ (Non directional / two-tailed hypothesis) OR
   Alternative Hypothesis / H_1: e.g. H_1 $\mu > 300$ (Directional / One-tailed hypothesis)

Step 4: Test your Null Hypothesis using the data you get from your SAMPLE to see if the relationship exists

Step 5: CHOOSE between the Null and Alternative Hypothesis and then draw a conclusion regarding your Research / Operational Hypothesis (i.e. accept OR reject H_1)

4. TOPIC 1: OVERVIEW OF THE MAIN CONCEPTS

GOAL of Research in PYC → **Test THEORIES about HUMAN BEHAVIOUR**

DEF: **Network** of postulated theoretical relations between **CONSTRUCTS / CONCEPTS / VARIABLES**

THEORIES

HYPOTHESIS: Part of the theory that is tested
- **Those relations between constructs that are targeted for testing**
- **ALWAYS refers to a POPULATION**
- **Gives us a RULE- about the relationship**

SAMPLES: Represent the POPULATION
- **INFERENTIAL STATISTICS: Infer numerical properties of populations FROM sample data**

RESEARCH DESIGNS:

1. **GROUPS DESIGN: Compare 2 or more POPULATION groups. E.g. Male and Female student IQ scores**
2. **CORRELATIONAL DESIGN: Look for a relationship between two or more variables in a single sample. E.g. does stress affect your diet?**

CONSTRUCTS

Def: building blocks of theories

Construct 1 → Construct 2
↕ ↕
Construct 3 → Construct 4

- Constructs can be abstract/ hypothetical, theoretical E.g. creativity, depression, etc
- Created to EXPLAIN certain observations
- 2 STEPS: From a hypothetical to an **OPERATIONAL DEFINITION**

  STEP 1: Specify the OBSERVABLE behaviour: **E.g.** anxiety- sweaty palms, avoiding stress

  STEP 2: Formulate a SCORE from the observable behaviour

VS

VARIABLES

- Can have different values E.g. height, IQ, stress
- Can **VARY** in quantity/ quality
- We get **dependent (Y)** and **(X) independent variables**
- To remember: Kisses (XXX) **cause** pregnancy (Y- lady with her arms up)

Levels of Measurement

1. Nominal (categorical)
2. Ordinal (rank ordered)
   - 1 and 2 can be grouped into "categorical"
3. Interval, e.g. IQ
4. Ratio, e.g. Score- 0%-100%
   - 3 and 4 can be grouped into "continuous"

*Interval and Ratio can be REDUCED to nominal, e.g. Low IQ / High IQ

2 TYPES OF VARIABLES:

1. **MANIFEST**: behavioural instances, indicators, referents, etc (you can SEE them!)
2. **LATENT**: hidden variable, intervening, hypothetical, factor (they are HIDDEN!)

INFORMATION BOX: **Levels of Measurement**

CATEGORICAL: Nominal Scale – Discrete (distinct – either pregnant, or not pregnant)
Mutually exclusive (yes, or no i.e. yes, I am male or no, I am not male)
Exhaustive (includes **all** possible categories)
E.g. Gender; Yes/No questions

Ordinal Scale - Same as nominal but are also **ranked** in order of importance (strongly agree, agree, neutral, disagree, strongly disagree)
Although categories are ranked the interval is not equal, there is no measured degree of difference between strongly agree and agree or disagree and strongly disagree. Often used to measure preferences, behaviour, attitudes and opinions. Includes Likert scales and semantic differential scales.
NOT IMPORTANT FOR THIS MODULE!

CONTINUOUS: Interval Scale - Same characteristics as nominal but the interval scale **can** measure the interval between two points. Numbers are meaningful as numbers not just as categories (i.e. thermometer). **This** scale does not have an absolute zero point, zero is determined arbitrarily (intelligence tests)
E.g. IQ scores; anxiety scores; exam results.

Ratio Scale - Highest level of measurement: includes all of the above but **has an absolute zero point.**
E.g. Measures weight, length and time.

Note:
* Nominal and ordinal measurement – have a **limited number** of categories
* Interval and ratio scales can be **reduced** to nominal or ordinal scales i.e. exam results could be the individual results of each candidate or could just be pass or fail (an ordinance measure)

You need to be able to **IDENTIFY** which scale the variable has been measured on, **NOT give a definition** of them

4. TOPIC 1: THEORY, HYPOTHESIS AND RESEARCH DESIGNS

This is usually the first question in the exam: What is the BASIC **GOAL** OF RESEARCH?
Answer: To **test theories** about **HUMAN BEHAVIOUR**
Remember we are studying to become a psychologist so all the examples and exam questions will be dealing with some aspect of human behaviour, i.e. memory and IQ, gender and aggression, childhood trauma and current psychological disorders, etc.

So how do we go about testing theories?
Well it is usually not possible to test a whole theory so we pull out an aspect of a theory that we want to test- called a **HYPOTHESIS**. This hypothesis is the part of a theory that we are going to test. It contains similar key aspects to a theory- such as RELATIONSHIPS and CONSTRUCTS.

What is a construct?
These are certain groupings of behaviours that scientist have observed and given specific labels to describe the behaviour- e.g. STRESS, IQ, ANXIETY, CREATIVITY. Please note that you cannot observe (i.e. touch, smell, taste, see, etc) these concepts!!
They are HYPOTHETICAL or ABSTRACT in nature.
Unobservable or abstract constructs such as anxiety can be measured through their **observable behaviours** such as sweaty palms, increased heart rate, etc.
This is done by giving the concept an operational definition:

The operational definition of constructs

If you want to carry out a research project then you need to be able to MEASURE your constructs! There are two steps you need to do in order to measure your constructs.
Step 1: Specify the observable instances of the construct
Step 2: Formulate a SCORE from your observable, i.e. IQ of 100 (this tell you how much of a particular construct the person has)
So there are two types of definitions – 1. Theoretical - Dictionary definition – construct defined in terms of other constructs
2. Operational - Defined in terms of observable instances
Necessary for measurement
Defines what a researcher must DO to measure a construct

What are variables?

There are different ways to look at variables.

1. **Manifest vs Latent:**

MANIFEST VARIABLES: E.G. Exam Score
Behaviour instances
Indicators
Referents
Observable consequences
Observable implications

LATENT VARIABLES: E.G. Anxiety
Hidden variable
Intervening variable
Hypothetical variable
Factor

2. **Dependent vs Independent**

Dependent: If I am dependent it means that I NEED you. So if I am a young child I depend on my mother. We depend on air to live, etc.
The symbol for a dependent variable is **Y**.

Independent: If I am independent it means that I don't need you, I can survive without you. It is also the 'cause' of your dependent variable.
The symbol for an independent variable is **X**.

What are populations?

This is the actual group that we want to study and be able to generalise our findings to. For example: **all** female nurses between the ages of 25 and 45, or **all** school going children aged eight in rural areas.

Why do we work with samples instead of populations?

Most research organisations do not have the budget or the resources to test a whole population, so we draw a sample from the population and get our scores from them. We then INFER back to our population using our sample results, if there is a small change that our sample scores are due to ERROR. This is actually why we need to work with **statistic**s in research, so we can actually calculate how much error we have in our sample!

1.4.1 Representing the structure of a theory
Psychological theories should be scientific- i.e. observable and testable
Scientific theories must specify what observations, made under particular conditions, could disprove this theory.

Construct 1 → Construct 2
↕ ↕
Construct 3 → Construct 4

1.4.2 The Hypothesis as a relationship between two variables

DEFINITIONS:

1. **Hypothesis** – a postulated relation (or absence of a relationship) between two or more constructs that is specifically targeted for testing in a particular research project.
   The nature of the relation is a ***rule*** that associates values of one variable with values of another variable. (The correct definition is often worded like this in the exam!)
2. **Variable** – anything that can take on different values i.e. age, IQ, height, weight, etc - as people can be different ages, intelligence, anxiety etc
3. **Dependent Variable** – the variable that is influenced or changed (symbol is: Y)
4. **Independent Variable** – the variable that does the influencing (symbol is: X)
5. **Intervening Variable** – a variable that is the effect of one variable and the cause of another

NOTE: All the unknown variables can be lumped together to be called ONE unknown variable. (U)
When testing a hypothesis we cannot state that a rule exists without taking into account that more than just one independent variable might be influencing the dependent variable. E.g. gender (the independent variable) influences "attitude to AIDS" (the dependent variable) in the women and men have a different "attitude to AIDS". Attitude to AIDS is, however dependent on other variables as well, **not** just gender.

Apart from X, Y is also influenced by U. So to overcome this problem we take all the male scores and get and average and all the female scores and get an average. If we assume the average U effect on Y is **zero** we can use this as a basis for working out the other scores. This is easier said than done as it is never possible to test an entire population (I.e. all the employees everywhere). Because it is unlikely that we will ever *know* the exact average score we tend to hypothesise that something is 'more likely' to happen or one gender would be 'more positive' than the other, simply because we cannot know the exact score.

1.4.2 The Hypothesis as a statement which is widely true

The postulated relation is claimed to be as generally (widely) true as possible i.e. the relation holds true for each person in the population in a wide range of situations. A hypothesis provides a rule that associates values of one variable with those of another, and also includes the population that relates to the relation.

1.4.3 Operational forms of the Hypothesis

Theoretical hypothesis – the variables cannot be directly observed
Research or Operational hypothesis – variables are measurable and can be directly observed. The variables imply how they can be measured.

1.4.5 Comparisons of group designs with correlation designs

Group design	Correlation Design
The researcher can define a population of subjects for each of the values of the independent variable. The researcher has control over the size of the sample, selects a random sample of the desired size from the population and plans how the two can be **compared**.	A single sample is selected from the research population and it is then noted how many belong to each category i.e. male and female. It could be that there are NO females in the sample… this is typical of a correlation design.

5. TOPIC 2: PROBABILITY

5.1 OVERVIEW OF THE RESEARCH PROCESS: WHERE DOES PROBABILITY FIT IN?

THEORY →

Ultimately we want to Prove / Disprove
Accept / Don't accept the theory

↓

HYPOTHESIS →

Test for a **relationship**- identify the Dependent Variable (Y) and the Independent Variable (X)

Translate your research hypothesis into your statistical hypothesis: H_0 and H_1.

↓

POPULATION (e.g. 10 000 students)

↓

Inferential statistics →

Can we infer from our sample back to our population??? We need to look at **Sampling ERROR** first before we can decide- this is the reason we use **PROBABILITY**

SAMPLE (e.g. 100 students)

↓

TESTED →

Results → Scores, e.g. Means/ Averages
E.g. score 72% for research exam

↓

PROBABILITY →

(p-value)

A test statistic, e.g. t_c, t_d, z_x, t_x, etc will give you an associated p-value. You **DON'T** calculate it!!!
Note the p-value is $0 \leq p \leq 1$ * can NEVER be negative or more than 1
We can also use Normal Distributions and the Z-Table to get p-values.

DEFINITIONS:

1. Probability – a measure of uncertainty
   – the likelihood of an event (something happening) occurring where various outcomes are possible. i.e. if you flip a coin (event) what is the probability it will land on heads (outcome)

Probability is measured by dividing the number of times the event occurs by the number of possible outcomes i.e. you flip a coin once you can have heads (1 outcome) or tails (2 outcomes) so the probability of the event occurring is 1 divided by 2

FORMULA: $P(E) = \frac{\text{Number of favourable events}}{\text{Number of possible outcomes}}$ Where (E) is the Event

EXAMPLE: So the probability of a coin landing heads up is:

$P(\text{Heads}) = \frac{1}{2}$

This only works when there is a specific number of events and a specific number of possible outcomes.

Where we do not know the frequency (f) of events or possible outcomes we use a different formula:

$$P(E) = \frac{\text{Number of observations of E}}{\text{Number of times the experiment was performed}} = \frac{f(E)}{N}$$

This is called **RELATIVE FREQUENCY (another term for probability!)**

Statistical experiment – broad term in statistics to cover everything from counting the number of times a coin lands heads up to predicting trends in the economy, or the probability of possible causes of a disease. The more times you carry out the same statistical experiment the closer you will come to the **theoretical probability** of your results, as opposed to the **relative frequency** which is an approximation of the event occurring.

Events are **independent** if the result of one has no bearing on the result of the next. i.e. your last flip of the coin will not determine your next flip of the coin.
Events are **mutually exclusive** if either one or the other can occur, but not both. i.e. you can either get heads or tails with one coin, it cannot land both sides up!
Sample space – all possible outcomes of a statistical experiment. Denoted as S. Also called a population.

i.e. S = {heads, tails} as there are only two possible outcomes of one flip of a coin or
S = {(heads, tails) (heads, heads) (tails, heads) (tails, tails) for flipping the coin twice

The law of large numbers – If an experiment is done repeatedly, and the results are independent of each other, the observed proportion (frequency) of favourable occurrences of an event will approach its theoretical probability.

If you flip a coin 100 times it is unlikely that you will get heads 50 times but if you flip it a million times you will be much closer to getting heads exactly half the time.

Some characteristics and rules of probability

- P- value – the probability value (the chances of an event happening). The outcome that we're **interested in** is called an EVENT. An event is a specific outcome (like choosing a girl out of a group of students) when there are many possible outcomes (e.g. 700 students, with 300 girls and 400 boys)
- P-values can be expressed as a percentage, fraction or decimal (proportions are better).
- A p-value represents a proportion (proportion of outcomes supporting an event.)

- Proportion – a decimal number between 0 and 1
- A p-value range is $0 \leq p \leq 1$ (Between 0 and 1; included 0 and 1)
- If p = 0 – the event definitely will NOT happen; If p = 1 – the event definitely WILL happen
- The probability of an even NOT occurring: 1 – P(E)
- The sum of all probability in a sample space (also called the area under the curve) = 1

S: All possible outcome of a statistical experiment = **sample space** (or the POPULATION). E.g. S = [heads, tails].

Sum of S (sample space) = 1 so if we know the probability of an event occurring we can work out the probability of an event NOT occurring. [1 – P (E)]

See the example of Zanier cards in the U.S.G. pg. 32

RULES FOR COMBINING PROBABLITY: when dealing with independent events **(know these for the exam!)**:

1. **Additive Rule –** P (A or B) = P (A) + P (B) simple rule used when events are mutually exclusive and signalled by the word **OR**. Either A or B can happen but not both.
   E.g. what is the probability that I will draw an ACE or a JACK? There are 4 aces in a deck and 4 Jacks.
   P (ACE) or P (Jack) = 4/52 + 4/52 = 8/52 = 0.15 (there is a 15% chance of drawing either an ace or a Jack)
   P (A or B) = P (A) + P(B) – P(A and B) – use if the events are **NOT mutually exclusive** P(A and B) takes into account the possible overlap in probabilities.
   See example [p (ace or heart] in U.S.G. Top of pg. 35

2. **Multiplicative Rule –** P (A and B) = P (A) x P (B) used to determine the product of two or more probabilities and is indicated by the word AND. i.e. the probability of A and B occurring.

3. **Conditional Probability –** P (A | B) the probability of event A occurring depends on event B also occurring.

The Probability Model

This indicates the chance of a possible outcome of an event, rather than choosing one outcome and testing that. So:

Either heads or tails has an equal chance of occurring: P (heads) = P (tails) = ½ = 0.5- NB: this is the <u>THEORETICAL PROBABLITY</u> of this event occurring!

Example 2: Normal die have 6 possible outcomes so the chance of each one occurring is the same:
P (1) = P (2) = P (3) = P (4) = P (5) = P (6) = 1/6

UNISA Study Unit 2.3 Probability distributions and the normal curve

How we go from normal distributions to probability?

Work through the example in the UNISA guide from p. 46

A. Sample: 36 kids - each kid has a score out of 15 e.g. 9/15

B. What is the probability of getting a score of 9/15?
4 kids got this score
P (9/15) = f/p = 4/ 36 = 0.11 = 11% of getting a score of 9/15 on the memory test!

C. What is the probability of getting a score of 9/15 or MORE, i.e. 10/15, 11/15, etc?
Add up ALL the probabilities for each score above 9/15
= 4/36 + 3/36 + 2/36 + 1/36 = 10/36 = 0.277 = 0.28 = 28% of getting a score of 9/15 or MORE
We call this **CUMULATIVE PROBABILITY**

D. If you plot the 36 kids score on a bar graph you will see it makes a bell-shaped curve (p.48)

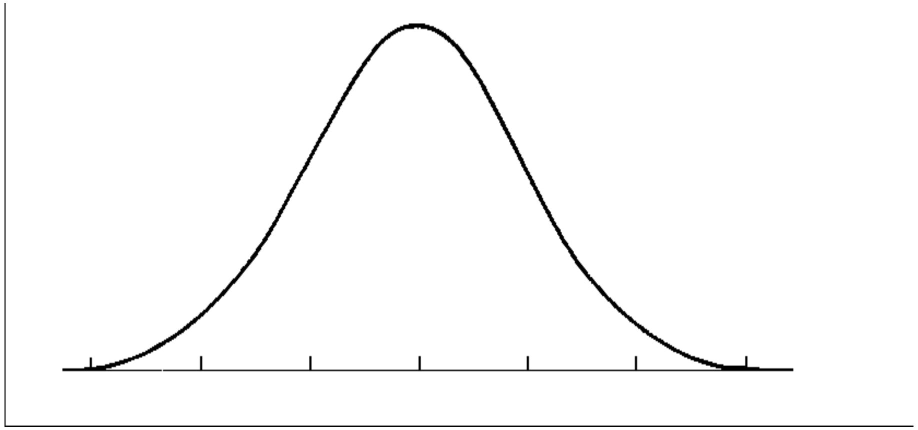
Cumulative probability is the probability of a number of possible events falling into a certain category i.e. probability of something equal to **or** greater than a certain point.

The data can be interpreted as 'under a curve' if a curve is drawn over the top of the histogram.
Continuous variable – such as age it can be measured in years, months, days, hours, minutes and seconds so the data is continuous. (As opposed to discrete variables which are measured in whole numbers)

Continuous variables depend on an infinite number of possibilities so: P (number of 30 year olds)/infinity = 0
Thus the probability of any value in a continuous variable is 0. So the normal rule for probability does not work!

The Normal distribution – was constructed specifically to deal with continuous variables

Real data rarely offers exact sameness of the lower and higher end scores with the middle score always the highest. Statisticians however, do say that it is a very good **abstract model** for distribution and that 'normally' the majority clusters around the centre with a certain amount of tapering at the start and finish. This is why this is called 'normal' distribution and is represented by the 'normal' curve.



The Normal Curve

The more data included in the sample population the smoother the curve will be based on the law of large numbers. Most normal curves are not as smooth as the one illustrated as the data is not perfectly symmetrical.

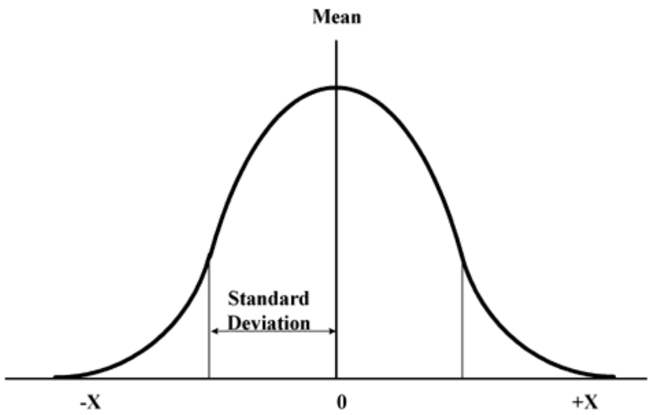
Most psychological and educational data are distributed approximately normally so that the normal curve can be used as a theoretical model of interpreting the distribution of that data.

The normal curve is very useful in practice as many kinds of statistical tests can be derived from normal distributions and many psychological tests work if the data is approx. normally distributed as well as with wide deviations.

The family of normal curves

2 variables: mean (μ) and standard deviation (σ), the rest of the terms are constants

All normal curves are **bell shaped** distributions but the height and spread depend on the **mean** and standard **deviation**



Gaussian ("Normal") Probability Distribution

Figure 1

Taken from: http://www.aaas.org/spp/dser/03_Areas/beyond/perspectives/Miller_James_Paper_gauss/fig1.gif

4 key properties

1. They are **bell shaped**, most observations occur at the midpoint of the curve
2. They are **symmetrical**
3. They are **continuous**, theoretically there are an infinite number of values so the curve is smooth
4. The curves are **asymptotic**: the two tails NEVER touch the horizontal axis as there is always some probability of a more extreme result. (Most common exam question)

Anything to the left of the mean is –ve and anything to the right is +ve. We only have to work out the values to the right as, by logic, we can work out that the values to the left are the same only negative.

- Total area under the curve gives a probability of -∞ and +∞ and is equal to + 1

** See Appendix D in the UNISA Study Guide

The **standard normal distribution** is a normal distribution with a **mean of 0** and a standard deviation of 1. Normal distributions can be *converted* to a Standard Normal Distribution using the **z-score formula** as follows:

Formula:

$$Z = \frac{X - \mu}{\sigma}$$

X = raw score/ individual score
µ = population mean
σ = standard deviation of the population

The measures on the x axis of the Normal Curve are called **z-scores**
Z-scores can be derived from any data provided we know the mean and standard deviation of the scores.

When dealing with samples or test data, rather than populations the formula becomes

$$Z = \frac{X - \bar{X}}{S}$$

X = raw score/ individual score
$\bar{x}$-bar = sample mean
s = standard deviation of the sample

Z-scores – always reflect the number of **standard deviations** that a particular score lies above or below the mean.
All distributions of z-scores have a mean of 0 and a standard deviation of 1- AS THEY ARE STANDARD NORMAL DISTRIBUTIONS!
Can be used to compare an individual across different distributions, each with a different mean and a different deviation

BENEFIT: Standardised distribution allows us to compare variables with different mean, standard deviation and scores expressed in differing original units

1. **EXAMPLE: How to calculate a z-score:**
   1. Student A scores 56% on a cognitive psychology exam (X)
   2. The class average is 52 % (X)
   3. The standard deviation is 4 (s)

Step 1: Write out your Formula:

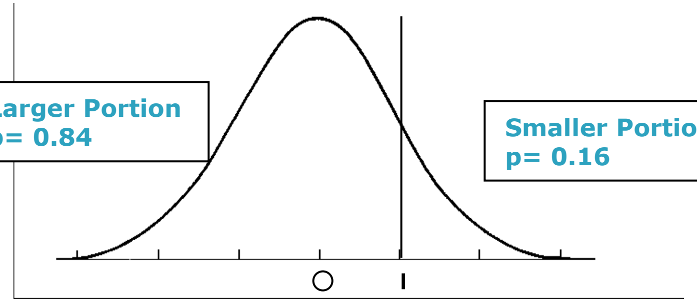
$$Z = \frac{X - \bar{X}}{S}$$

Step 2: Fill in the correct values and do the calculation:

$$Z = \frac{56-52}{4}$$

$$Z = \frac{4}{4}$$

$$Z = 1$$ (A z-score of 1 = 1 standard deviation above the mean)



Step 3: Plot your **z score** on you X-axis and see where it cuts the curve
Step 4: Go to your **z-tables** and look up your z-score with its smaller and larger portion p-values
Write the values under your heading
Step 5: Look at the question: Do they want to know the probability of getting her score or **HIGHER** then use the **Smaller Portion** value.
Do they want to know the probability of getting her score or **LOWER**? Then use the **Larger Portion** p-value.

1. EXAMPLE: How to calculate a z-score:

1. Student A scores 56% on a cognitive psychology exam (X)
2. The class average is 65 % (X)
3. The standard deviation is 6 (s)

Step 1: Write out your Formula:

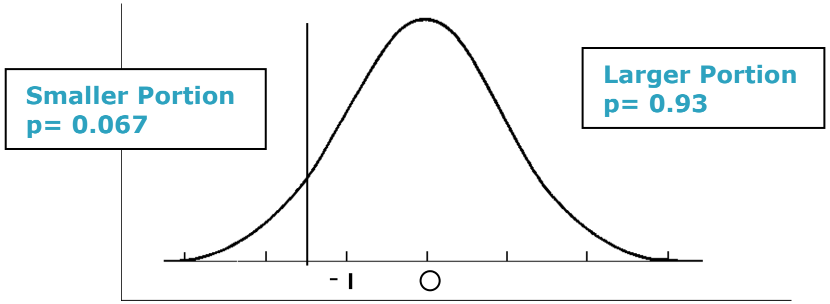
$$Z = \frac{X - \bar{X}}{S}$$

Step 2: Fill in the correct values and do the calculation:

$$Z = \frac{56-65}{6}$$

$$Z = \frac{-9}{6}$$

$$Z = -1.5$$



Step 3: Plot your **z score** on your X-axis and see where it cuts the curve

Step 4: Go to your **z-tables** find your z-score and your smaller and larger portion p-values
Write the values under your heading
NB: If your z-score is NEGATIVE use the positive z-score value!!

Step 5: Look at the question: Do they want to know the probability of getting her score or **HIGHER** then use the **Larger Portion** value.
Do they want to know the probability of getting her score or **LOWER**? Then use the **Smaller Portion** p-value.

TOPIC 2: Z-SCORE SUMMARY

1. Any **NORMAL DISTRIBUTION** can be transformed into a **STANDARD NORMAL DISTRIBUTION** (Z-Score)

2. Using this Formula

   $$Z = \frac{X - \mu}{\sigma}$$ **– Population values**

   X = variable
   μ = population mean
   σ = standard deviation of the population

   $$Z = \frac{X - \bar{X}}{S}$$ **– Sample values**

   $$\frac{\text{Raw score- Average Score}}{\text{Standard Deviation}}$$

3. Once you have a Z-score look it up in your **Z-TABLES**
4. **Find your SMALLER PORTION and LARGER PORTION VALUES (P-VALUES)**
5. **Plot them on your Standard Normal Curve**
6. **Advantage of using z-scores:** Compare variables with different means, standard deviations and units of measurement

* A Z-Score tells you how many standard deviations YOUR score lies above or below the μ mean (0).

INFORMATION BOX

Frequency distribution – table or graph indicating how observations are distributed

Frequency distribution table:

- Column 1: ordered list of all possible scores or categories
- Column 2: number of times each score occurs – tally marks used to help counting but not included in final table
- Column 3: total frequency
- Sum of the frequencies should be the same as the number of cases in the sample
- Categories should be mutually exclusive
- Sufficient categories so that each case can be classified into one of the available categories

Cumulative frequency – number of scores below or above a certain frequency
Frequency graph – X axis: categories or scores, Y axis: frequency
Bar Chart – frequency distribution of categorical data (bars do not touch)
Histogram – frequency distribution of successive scores or class intervals (numerical data and bars are touching)
Frequency polygon – points are joined with straight lines

Study Unit 2.3.1 Sampling and sampling distributions- Overview

WHAT ARE SAMPLING DISTRIBUTIONS?

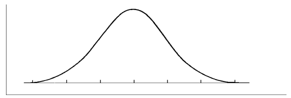
Sampling distributions are needed in order to estimate POPULATION values- remember that we NEVER actually know the population value (e.g. mean score) so we rely on sampling distributions as our BEST GUESS for population values

SAMPLING ERROR: **The difference between the SAMPLE results and the result you would have gotten if you had tested the entire POPULATION**

*** Your sample is your best GUESS about the population value**

MEAN OF ALL MEANS:

- **Is identical to the mean (μ) of ALL the individual score in the population.**
- **It is the mean (average) of all the SAMPLE means**
- **The distribution of these MEANS is NORMAL**



- **It is your best ESTIMATE of the (unknown) POPULATION mean**

SAMPLING DISTRIBUTION OF THE MEAN

($\mu_{\bar{x}}$)

STANDARD ERROR:

- **A measure of sampling error**
- **Tells you the variability of the scores**
- **Also called 'Standard Deviation of the sampling distribution of the mean'**
- **Standard Error Formula:**

Population: $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

Sample: $S_{\bar{x}} = \frac{s}{\sqrt{n}}$

1. NOTE:
If you add all the SAMPLE MEANS ($\bar{X}$s) this will give you the mean of all means ($\mu_{\bar{x}}$) which equals the POPULATION MEAN (μ)

THIS IS WHAT A SAMPLING DISTRIBUTION LOOKS LIKE.
Read study unit 2.4 in your UNISA guide, p.57

2. SAMPLING ERROR:
1. **Sample Size**
2. **Nature of the variable- i.e. how much they vary!**

2. STANDARD ERROR:
($\sigma_{\bar{x}} = \sigma / \sqrt{n}$)
Tells us how much the SAMPLE MEANS deviate from the mean of the sampling distribution ($\mu_{\bar{x}}$)

UNISA STUDENTS
(Attitude to HIV)
⇩
Population
⇩
Mean-μ
⇩
E.g. Average score μ = 110

Sample of UNISA Students
n= 500
X= 95

Possible sample error!

Sample of UNISA
n= 500
X= 110

Sample of UNISA
n= 500
X= 115

Sample of UNISA
n= 500
X= 110

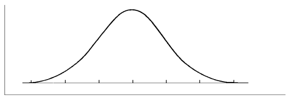
Sample of UNISA
n= 500
X= 100

Sample of UNISA
n= 500
X= 105

Sample of UNISA
n= 500
X= 110

Sample of UNISA
n= 500
X= 110

3. NB: SAMPLING DISTRIBUTIONS ARE NORMAL DISTRIBUTIONS



So you can use the Z-tables once you have calculated your Z-statistic

4. TO CALCULATE A Z-TEST STATISTIC FOR A SAMPLE MEAN $\bar{X}$

$$Z_{\bar{x}} = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}}$$

Remember: $\sigma_{\bar{x}} = \sigma / \sqrt{n}$ (The standard error of the mean)
SYMBOLS:
σ = Population Standard Deviation
n = Sample Size
$\bar{X}$ = Sample mean (single sample)
$\mu_{\bar{x}}$ = Population Mean under H_0 (The mean of all means)

TOPIC 2: Study Unit 2.4 Sampling distributions and the central limit theorem

DEFINITIONS:

1. Sampling – using a relatively small number of cases to draw conclusions about a much larger group.
2. Population – the large group you wish to study
3. Sample – the group actually involved in research

Results from the sample (data / findings) are used to **generalise results** to the larger population (remember we are dealing with INFERENTIAL STATISTICS).
This saves the researcher time and money – results are almost as accurate as those when the complete population is studied but at a fraction of the cost.
The Law of diminishing returns – once a certain number of cases have been studied, each successive case will not add much more to the understanding gained.

Random Sampling is most commonly used as it ensures each individual in a population has an equal chance of being selected or every possible sample of a particular size has the same probability of being selected.

- Random samples – can never be sure that they are completely representative, we can only hope!
- **Thus the notions of sampling and error are intimately connected**
- Summary values for **populations are called parameters** denoted by Greek symbols – they are usually unknown
- Summary values for **samples are called statistics** denoted by Roman symbols
- See the table on page 4 of this study guide for examples of parameters and statistics.
- 2 separate samples will always have slight differences as they are made up of unique individuals, different scores, different means etc
- **Because a sample is randomly chosen from a larger population, any statistic such as the value of the sample mean, will vary from sample to sample**

Sampling Distributions

The ability **to predict the characteristics** of a sample is based on the concept of the *distribution of sample statistics.*
Sampling distribution of a statistic – the set of all possible values of the statistic when all possible samples of a fixed size are taken from the population. Sampling distribution refers to the variation of a hypothetical set of all possible samples (can be an infinite number of samples). See the example in U.S.G. pg. 57/58

Central limit theorem

Definition: The **central limit theorem** states that when an **infinite** number of **successive random samples** are taken from a population, the distribution of **sample means** calculated for each sample will become **approximately normally distributed** with mean **μ** and standard deviation **σ / √ n** as the **sample size (n)** becomes **larger, irrespective of the shape of the population distribution**.

Standard error of the mean – the average amount that the sample means deviate from the mean μ_x of the sampling distribution.
Allows us to see how much error can be expected between X and μ in the estimate of our sample population compared to the complete population. See the example in U.S.G. on pg 54

Determining z-statistics for the sample mean
All information relating to z-scores for an **individual** also applies to a z-statistic for a **sample mean**.

Formula:

$$z_x = \frac{\bar{X} - \mu_x}{\sigma}$$

Sample estimate – used ONLY when we do **not** have knowledge of the population parameter σ, and is obtained by dividing with (n-1) instead of n (sample size)

Using the sample estimate the formula for a t-statistic is:

$$s_x = s/\sqrt{n}$$

and $$t_x = \frac{(\bar{X} - \mu_x)}{s_x}$$

See the example in U.S.G. on pg 55/56 and the formula sheet on pg 35

1. Def: **SAMPLING DISTRIBUTIONS**: A HYPOTHETICAL dispersion of a SET of sample statistics (e.g. the mean)

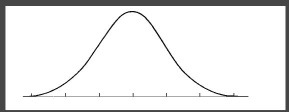
Samples of a certain size are drawn an infinite number of times from a POPULATION and the MEAN of each sample is determined

The distribution of these SAMPLE MEANS is known as THE SAMPLING DISTRIBUTION OF THE MEAN ($\mu_{\bar{X}}$)

3. Why Is This Important?
The **central limit theorem** is the foundation for all tests of **means**. It provides a set of simple rules for determining the mean, variance, and shape of a distribution of sample means.

Distributions of sample means are used in all hypothesis tests with means.

The Central Limit Theorem



2. Def: **CENTRAL LIMIT THEOREM**:
Sampling Distributions will approach a NORMAL DISTRIBUTION as the sample size (n) INCREASES, regardless of the form of the POPULATION distribution
* can be used if the population distribution is NOT normal or unknown

E.g.: Population

Sample
n=20

Sample
n= 200

4. USE: Allows us to use the KNOW PROPERTIES of the Standard normal curve for our distribution of sample means
= so we can use the z-tables

5. SAMPLES: Mean = $\bar{X}$

Standard Deviation = S

SAMPLING DISTRIBUTIONS: Mean = μ_x

Standard Error (Standard Deviation of the sample mean) = $\frac{\sigma}{\sqrt{n}}$

TOPIC 3: GENERAL PRINCIPIALS OF STATISTICAL HYPOTHESIS TESTING

Study Unit 3.1
The translation of the research hypothesis into statistical hypotheses

Come up with a theory to be tested
Refine the hypothesis to create a research/operational hypothesis

- How will the constructs be measured
- What is the research population
- What design will be used to test the relation
- What is the nature or rule of the relation

Translate the operational hypothesis into statistical hypotheses to test, using sample observations, whether the relation actually exists (see p.75 TABLE 3.1: Directional and non-directional hypotheses)

See example U.S.G. pg. 72/73 (Study Unit 3.1.2)
Research hypothesis always translates into 2 mutually exclusive hypotheses:

H_0 – null hypothesis (symbol always =)
H_1 – alternative hypothesis (symbols are < > ≠)

If the hypothesis does not state greater than or less than it must be ≠

It is conventional to state the null hypothesis first – not a law though!

< or > is a directional or one tailed test

≠ is a non-directional or two tailed test

The object is to try to reject Ho in favour of H_1 based on sample observations

Study Unit 3.2
Obtaining Sample data and sample results

Hypotheses

Ho: $\mu = 100$
H1: $\mu > 100$

We must:

- Obtain a random sample of UNISA students
- Apply an intelligence test to each student in the sample
- Calculate a value for the sample mean

We need not obtain a sample of 'others' as we know their mean is = 100

The calculated mean of the random sample is the first important result.

Common sense tells us:

- To assess if the sample mean can lead to rejection of the Ho
- To decide if the sample result looks important from a psych point of view
- That statistical data may not be psychologically important

I.e. if H1 = 104 it is greater than Ho which was equal to 100, but not significantly so statistically we can reject Ho but not necessarily from a psychological point of view.

The sample statistic needs to be an estimator of the sample mean and sample proportion and an estimator of the corresponding population parameter, the population mean and the population parameter.

3.2.2 The calculation of the probability of the sample result under the null hypothesis

We need to continue with hypothesis testing in order to decide if we can reject the null hypothesis. We assume that: (when we do z-tests or t-tests)

- Population distributions are normal
- Population distributions have the same standard deviation

See figure 3.1 on page 71, it shows that:

- We select one sample only which is the 'solid line' frequency distribution.
- We could select an infinite number of such samples
- We can represent these samples by a dotted line to indicate they are not actually selected by the researcher, but that we can imagine that they have been
- The mean could be calculated for each sample – in actual research only the mean for the selected sample is calculated

The sampling distribution of the means is represented as a solid line normal curve because we can *derive* this distribution precisely from the mean values. Finally the result of the actual research can be viewed under this sample distribution.
See figure 3.1!

This is called the one-tailed or directional p-value of the sample result

The smaller the p-value the easier it is to reject the null hypothesis as it means that the gap between the population mean and the sample mean is bigger i.e. if the sample mean was 109 the p-value would be smaller than it is at 104.

REMEMBER: we always test the null hypothesis against the alternative hypothesis

The **smaller** the p-value under Ho the **larger** the p-value under H1, thus a small p-value under Ho leads us to rejects Ho in favour of H1.

3.2.2.1 The non-directional or two-tailed test

This type of test is carried out when the H1 is either greater OR smaller than the Ho, but not equal to! See Table 3.1 in your U.S.G. on pg. 75.

The two-tailed p-value is exactly twice the size of the one tailed p-value. (See page 81 U.S.G) So if you need to work out a one directional value, divide the two directional p-value by 2 and if you need to work out a two directional, multiply the p-value by 2.

NB **p-value** indicates the likelihood of a particular result occurring under the **null hypothesis**. I.e. what is the probability (p-value) of the mean of a sample being equal to 100?

3.2.2.2 The Test Statistic

The z-statistic is known as a test statistic. For example: Z_x = z-test for a single mean

Researchers must decide the appropriate test statistic to use- use your **Flow Chart** (appendix E) at the back of your study guide to help you!

Test statistic: a variable with a known, theoretical probability distribution and for which a computer program exists to calculate the probabilities associated with a given test statistic values

See Appendix E for test statistic symbols

Study Unit 3.3
Making a decision regarding the null and alternative hypotheses

3.3.1 Making a decision regarding the null and alternative hypothesis

Question: How small does a p-value have to be to reject the null hypothesis?
Answer: A "cut off" p-value is decided prior to conducting research. The "cut off" p-value is called the **significance level** of the statistical test procedure and is often set at 0.05 or 0.01

Significance level is represented by alpha - α
The size of α-value depends on the willingness to risk a type I error versus a type II error

Decision rule for the null hypothesis: If the p-value of the sample result is < α, reject the null hypothesis. If the p-value is NOT < α, do not reject the null hypothesis (we are **not accepting** the Ho, just not rejecting it!!!) See your U.S.G. e.g., on pg. 83

3.3.2 Making a decision regarding the research hypothesis

After making the decisions about the statistical hypothesis, the researcher must make a decision about the research hypothesis. The research hypothesis can either be confirmed (the null hypothesis is rejected) or NOT CONFIRMED (the null hypothesis is not rejected. We do not show the research hypothesis (H1) to be false, just not confirmed.

3.3.3 The probability of a type I and type II error and the power of a statistical test

Type I error – mistakenly rejecting the Ho when it is true

We never know for sure if the null hypothesis is true or false but we can show the *probability* that a Type I error has occurred based on the p-value.

Setting the α-level prior to research protects us against the probability of making a Type I error by setting the maximum probability of a type I error that we are willing to risk.

Type II error – not rejecting Ho when Ho is false and H1 is true

β – The risk we are prepared to take to make a Type II error

We cannot calculate the β in advance because H1 does not specify the specific mean for a population distribution.

Generally the smaller α is the larger β is so if we want to avoid Type I errors we set α to a small value such as 0.01 or if we want to avoid Type II errors we set α to possibly 0.20 or even 0.30

The power of a statistical test is the probability of rejecting Ho when in fact it is false and H1 is true. The power of a statistical test is the test's ability to detect a significant result. Indicated as 1 – β. Because we cannot calculate β we cannot calculate the power of a test.

So to increase the power of a test without being able to calculate β we could
- Increase the sample size
- Decrease error such as sampling error, measurement error, error due to external variables etc. This results in a smaller standard error of the sampling distribution of the mean. See pg. 86

The larger the sample size the smaller the p-value so statistically significant results can be derived even when psychologically the results are insignificant. See examples Pg. 87.

Topic 3: BEFORE YOU CAN APPLY THE DECISION RULE

ALTERNATIVE HYPOTHESIS AND P-VALUES

- **They must match- i.e. BOTH must be one or two tailed!!**
- **You changed the p-value to match the Alternative Hypothesis, you CAN'T change H_1.**

Z-TABLES:

Give **DIRECTIONAL / ONE-TAILED p-values** (Remember that a **z**ebra only has **ONE** tail and can only walk in one **DIRECTION**!)

If: *Alternative hypothesis* is NON-DIRECTIONAL (look for words like 'differ' in your research hypothesis)

e.g. H_1: $\mu \neq 110$

Then you have to take the p-value (that you got from your z-tables) and **times it by two** (x 2) so that you get a **TWO-TAILED p-value**

ONLY THEN CAN YOU APPLY YOUR DECISION RULE!

COMPUTERS:

Give NON**-DIRECTIONAL / TWO-TAILED p-values** (Remember that a computer can do **TWO** or more things at a time!)

If: *Alternative hypothesis* is DIRECTIONAL (look for words like, 'more' or 'less' in your research hypotheses)

e.g. H_1: $\mu < 110$

Or

H_1: $\mu > 110$

Then you have to take the p-value (that you got from your computer) and **divide it by two** (÷ 2) so that you get a **ONE-TAILED p-value**

ONLY THEN CAN YOU APPLY YOUR DECISION RULE!

Topic 3 Summary: THE DECISION RULE (p. 82/83 UNISA Study Guide)

*If the p-value (which you get from the z-tables or a computer) is **SMALLER** than the α - level of significance then **REJECT H_o** and **accept** H_1.*

If not, do NOT (<u>FAIL TO</u>) reject H_o and reject H_1

(See the UNISA STUDY GUIDE page 83)

Note:

1. NB: We never use the term 'ACCEPT' for the Null hypothesis (H_o), either REJECT or FAIL TO REJECT
2. Alpha is set in **advance** by the researcher (usually at 0.01 or 0.05)
3. We **test H_o** NOT H_1 using sample data – we try to reject H_o so that therefore H_1 (Alternative hypothesis) is accepted / true
4. If H_1 is a **TWO TAILED / NON DIRECTIONAL**- you have to **DOUBLE** your p-values (x 2) BEFORE applying the Decision Rule. – Don't forget this; you <u>will</u> get tested in the exam!
   REMEMBER that the z-tables give you ONE-TAILED/ DIRECTIONAL p-values and a computer gives you a TWO-TAILED / NON DIRECTIONAL p-value. – Your p-value must MATCH YOUR H_1 – directional (< / >) or non-directional (≠)
5. We calculate a **test statistic** such as Zp, tx etc to *test* our statistical hypothesis. The **P-VALUE** that comes from your test statistic tells us the **extent** to which the *observed relationship* between the variable **deviates** from what may be expected by **chance** (i.e. sampling error).

Topic 3: TYPE 1 AND TYPE 2 ERRORS

Type 1 Error

Def: Rejecting the Null Hypothesis (Ho) when it is actually TRUE

* We can NEVER know if such an error has occurred- ONLY the PROBABILITY that is has occurred

WHY?

Because we never know what the TRUE population value is, e.g. mean

* **P-VALUE:** gives us the **PROBABILITY** that we have made a Type 1 Error

*** TO AVOID A TYPE 1 ERROR**
Set α (alpha) to a SMALL value, e.g. 0.01 (1%)

Type 2 Error

Def: Accepting the Null Hypothesis (Ho) when it is actually FALSE

* We can NEVER know if such an error has occurred

WHY?

Because we never know what the TRUE population value is, e.g. mean

* **BETA β:** gives us the **PROBABILITY** that we have made a Type 2 Error, BUT we <u>can't actually calculate</u> β so we have to rely on α – the level of significance

*** TO AVOID A TYPE 2 ERROR**
Set α (alpha) to a LARGE value, e.g. 0.20 (20%)

TOPIC 3: SUMMARY OF THE STEPS TO FOLLOW

10 STEPS WHEN DOING PSYCHOLOGICAL RESEARCH

STEP 1:	**THEORY**	**About some aspect of human behaviour**
STEP 2:	**HYPOTHESIS**	**Research Hypothesis. E.g. Female student perform better in PYC304 than male students**
STEP 3:	**OPERATIONAL HYPOTHESIS**	**You need to be able to MEASURE your constructs / variables**
STEP 4:	**STATISTICAL HYPOTHESIS**	**H_0: μ= 110 (Will always state there is NO relationship)** **H_1: μ > 110 (Make a note, is it directional or non-directional)**
STEP 5:	**DRAW A SAMPLE**	**Use as large a sample as possible to reduce sampling error E.g. 100 UNISA students**
STEP 6:	**GET TEST RESULTS (DATA)**	**Assume they are normally distributed scores, e.g. Exam marks**
STEP 7:	**FACE VALUE STEP**	**Is your sample result possible under H1: look at your sample value- does is SUPPORT your research hypothesis? If yes carry on, if NO then you CANNOT REJECT Ho and would stop at this point.**
STEP 8:	**CHOOSE A TEST STATISTIC**	**Use your flow chart to decide which one to use, eg. Zx, tc, etc**
STEP 9:	**APPLY THE DECISION RULE**	**Check if your p-value and H1 match (directional or non directional) if not then x2 or ÷2 (p-value) and then compare the p-value to α and reject or fail to reject Ho**
STEP 10:	**ACCEPT OR REJECT H_1**	**Draw a final conclusion about your Research Hypothesis**

TOPIC 3: EXAMPLE 1 UNISA STUDY GUIDE PAGE 93

A psychologist hypothesises that **sleep deprivation (X)** affects **cognitive performance (Y)** **negatively**. He selects a sample of **16 students** randomly and deprives them of sleep for 10 hours over and above the normal 14 hours during which they are awake in a day. He then measures each student's performance on a computer game which requires cognitive skill to perform. It is also known that the general **population** of students has a **mean score of 1200** for this particular computer game, with a **standard deviation of 200**. Suppose it is found that the mean score for the sample is 1050 and that the **level of significance is set at 0.05.**

Step 1: Read through the research scenario carefully.
Step 2: Identify what SAMPLE and POPULATION values have been given.
Step 3: Write out your statistical hypothesis.

SAMPLE	POPULATION
n= 16	**μ= 1200 (Ux- the mean of the sampling distribution)**
$\bar{X}$= 1050	**σ= 200**

α = 0.05 (level of significance)

Null hypothesis: $H_0: \mu = 1200$
Alternative hypothesis: $H_1: \mu < 1200$

Standard Error: $\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = 200 / \sqrt{16} = 200/4 = 50$

Z-statistics = $z = \frac{\bar{X} - \mu_{\bar{X}}}{\sigma_{\bar{X}}} = 1050 - 1200 / 50 = -3.00$ **/ p = 0.0013 (UNISA Study guide p. 94)**

Decision Rule: Reject Ho, Sleep deprivation has a negative effect on cognitive performance.

TOPIC 4: $t_{\bar{x}}$- STATISTICAL TEST FOR MEANS: SINGLE SAMPLE GROUPS DESIGN

GROUPS DESIGN

*(Comparing to or more **population** groups)*

↓

*Y-VARIABLE (DEPENDENT) is measured on an **INTERVAL OR RATIO SCALE** (CONTINUOUS) so you can calculate a MEAN SCORE (μ)*

↓

*Only **ONE SAMPLE** is selected*

(You are given the mean score for the second group)

↓

σ** (Population Standard Deviation) is **NOT known

Use: $t_{\bar{x}}$

Formula: $t_{\bar{x}} = \frac{\bar{x} - \mu_x}{S_{\bar{x}}}$

Remember to calculate standard error:

$S_{\bar{x}} = \frac{s}{\sqrt{n}}$

*You need to use s (**sample** standard deviation) as you don't know the population standard deviation!*

NOTE:
t-test p-values are given by a **computer** and are therefore always NON-DIRECTIONAL/ TWO TAILED
Therefore: you must take the p-value and DIVIDE BY 2 (÷ 2) IF you have a ONE-TAILED / DIRECTIONAL ALTERNATIVE HYPOTHESIS

TOPIC 5: t_c, AND t_d TEST STATISTICS: TWO SAMPLE GROUPS DESIGN

ASSUMPTIONS:

1. Normal distributions for both **population** groups
2. Equal Variance (σ^2) for both **population** groups

Always:
Set α in advance

*1. **GROUPS DESIGN** (Comparing to or more population groups)*

↓

If…

***2. Y-VARIABLE (DEPENDENT)** is measured*

↓

*3. on a **RATIO or INTERVAL SCALE***

E.g. IQ, height, stress levels, etc

↓

And…

*4. The statistical hypotheses are about **Population Means (μ)***

E.g.: H_0: μ=104 / H_1: μ › 104

↓

And you…

*5. Select **TWO SAMPLES***

If they are…

6. Independent Groups

There is no systematic relationship between the composition of one sample and the other

↓

7. Use t_c

6. Dependent Groups

Are they matched? Pre-test/ Post-test, or on age, IQ, etc

↓

7. Use t_d

TOPIC 5: T-TEST FOR THE DIFFERENCE BETWEEN TWO MEANS OF TWO DEPENDENT GROUPS ($t_{\bar{d}}$)

DEPENDENT SAMPLES: **One sample is MATCHED with regards to some CHARACTERISTIC to a particular subject in the other sample.**

E.g. set of correlated scores, rank ordering (high to low or low to high scores) in one group influences rank ordering in the other group.

So if you test one group of students at the beginning and again at the end of the semester the marks would be DEPENDENT, i.e. tied / matched to a particular student.

- **See page 117 in the U.S.G for more examples**

With dependent samples, you would calculate a DIFFERENCE SCORE:
$\bar{d} = X_2 - X_1$ (SUBTRACT THE SECOND SET OF RESULTS OR SAMPLE MEAN FROM THE FIRST SET OF RESULTS OR SAMPLE MEAN)

The statistical hypotheses (for MEANS) for dependent groups look like this:

H0: D = 0

H1: D › 0 (or less 0 than or not equal to 0)

$\bar{D}$ = population mean of “difference” scores

$S_{\bar{d}}$ = this is the sample standard deviation of the “difference” scores.
To calculate $S_{\bar{d}}$ subtract the standard deviations of your two sample results.

- **See U.S.G. p. 112 for an example.**

TOPIC 6: CORRELATIONAL DESIGNS- look for relationships between variables

1. CORRELATIONAL DESIGN

*2. BOTH THE **DEPENDENT VARIABLE** (**Y**) and **INDEPENDENT VARIABLE** (**X**) ARE…*

3. **CONTINUOUS VARIABLES** (Interval / Ratio)

4. **USE:** Pearson's correlation coefficient t_r

Note: t_r**:**

- Can never be less than -1 or larger than 1
- r = 1 (Perfect **positive** linear relationship
- r= 0 (NO **linear** relationship)
- r = -1 (Perfect **negative** linear relationship)

SCATTER PLOT

- The data is displayed in on a SCATTER PLOT
- You plot the data on the x and y axis
- If the dots form a straight line then there is a linear relationship

Statistical hypothesis for t_r

H_o: ρ = 0 (No relationship exists)

H_1: ρ ≠ 0 (A linear relationship exists)
H_1: ρ › 0 (A positive relationship exists)
H_1: ρ ‹ 0 (A negative relationship exists)

3. **CATEGORICAL VARIABLES** (Nominal only)

4. **USE:** Pearson's chi-squared test X^2p

Note: X^2p:

- Can never be less than 0
- Can only be NON-DIRECTIONAL (H_1) - so you don't divide your computer calculated p-value by 2!

CONTINGENCY TABLE

- The data is displayed in a CONTINGENCY TABLE
- You may be asked to calculate the expected cell frequency
- The expected cell frequency should be 5 or MORE before you would carry on to calculate X^2p
- You use the contingency table to compare the OBSERVED and EXPECTED frequency distribution
- A large X^2p will lead to a statistically significant result (i.e. reject H_o and accept H_1)
- FORMULA:

$$X^2p = \Sigma \frac{(0_{II} - E_{Ii})}{E_I}$$

10. ADDITIONAL NOTES: DEGREES OF FREEDOM

In many statistical problems we are required to determine the degrees of freedom. This refers to a positive whole number that indicates the lack of restrictions in our calculations. The degree of freedom is the number of values in a calculation that we can vary.

Group Design: The T-Test (Refer to UNISA guide p.103)

Degrees of freedom: The t-distribution refers to a range of possible distributions determined by the degrees of freedom implied in the RESEARCH DESIGN.
In the t-test, the degrees of freedom is the **sum of the persons in both groups minus 1**.
In a t-test for ONE sample, the degrees of freedom will be: n-1.
In a t-test for TWO samples, degrees of freedom will be: $n_1 + n_2 - 2$
You also need to determine the degrees of freedom (df) to be able to find the relevant p-value if you are calculating the statistic manually. *Usually a computer program will calculate it for you!*
Given the alpha level, the df, and the t-value, you can look the t-value up in a standard table of significance (available as an appendix in the back of most statistics texts) to determine whether the t-value is large enough to be significant. If it is, you can conclude that the difference between the means for the two groups is different (even given the variability).

Correlation Design: **Pearson's R: to compute the degrees of freedom (df).**

For Pearson's r the df is simply equal to N-2.
You also have to check if it's a one-tailed or two-tailed test.
With these three pieces of information
1. - the significance level (alpha = .05)
2. - degrees of freedom (e.g. df = 18),
3. -and type of test (e.g. two-tailed) -- you can now test the significance of the correlation.

For Chi square:

The value of Xp2 together with the degrees of freedom can be used to look up the p-value.

Degrees of freedom = (r-1) (k-1)
r= number of rows
k = number of columns in your contingency table

Before we can proceed we need to know how many degrees of freedom we have. When a comparison is made between one sample and another, a simple rule is that the degrees of freedom equal (number of columns minus one) x (number of rows minus one) not counting the totals for rows or columns. For our data this gives E.g. (r-1) (k-1) = (2-1) x (3-1) = 1 x 2 = 2

EFFECT SIZE

What is the effect size of a particular outcome and how do you calculate it?

Effect size is a simple way of quantifying (measuring) the difference between two groups.

*It has many advantages over the use of tests of statistical significance alone. Effect size emphasises the **size of the difference** rather than confusing this with sample size.*

Effect size, power of a statistical test and sample size are all interrelated.

You can calculate one if you have the values for the other 2.

T-test (UNISA p.88)

Formula = Effect size = d = mean difference / standard deviation

d = 0.2	Small effect.
d = 0.5	Medium effect
d= 0.8	Large effect

Pearson's r: (UNISA p.139)

r-squared (r^2=): how much variance your variables have in common

r^2=0.01	Small effect.
r^2= 0.09	Medium effect
r^2= 0.25	Large effect

E.g. = r = 0.65, r^2= 0.42 = 42% of the variance is shared by x and y. (Large effect!)

11. APPENDIX A: LIST OF FORMULAE

I have not included the Z-TABLES as they are given in the UNISA study guide. The list of formulae that **will be provided** in the **exam** is given below. You do not need to memorize these equations but you should be able to **recognize** which one is to be used in a particular research scenario.

Note that the formulae for the calculation of **probability (f/p),** the **mean**, the **standard error** and the **z-score** are not given.

So you need to memorise them!!! Although they give you this list of formulae they do **not** make you do these test statistic calculations in the exam! The examination will actually require very **few calculations:** usually **probability, a z-score and standard error calculation are asked**.

$$t_c = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$$t_{\bar{X}} = \frac{\left(\bar{X} - \mu_{\bar{X}}\right)}{s_{\bar{X}}}$$

$$z_p = \frac{\left(p - P_o\right)}{\sqrt{P\left(1 - P_o\right)/n}}$$

$$z_c = \frac{\left(p_1 - p_2\right)}{\sqrt{\frac{p_1\left(1 - p_1\right)}{n_1} + \frac{p_2\left(1 - p_2\right)}{n_2}}}$$

$$\chi_p^2 = \sum_{ij} \frac{\left(O_{ij} - E_{ij}\right)^2}{E_{ij}}$$

$$z_{\bar{X}} = \frac{\left(\bar{X} - \mu_{\bar{X}}\right)}{\frac{\sigma}{\sqrt{n}}}$$

$$t_{\bar{d}} = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2} - 2r\,s_1 s_2}} = \frac{(\bar{d} - D)}{\frac{s_{\bar{d}}}{\sqrt{n}}}$$

$$r = \frac{\operatorname{cov}(X,Y)}{\sqrt{\operatorname{var}(X)\operatorname{var}(Y)}} = \frac{N\sum XY - \left(\sum X\right)\left(\sum Y\right)}{\sqrt{\left[N\sum X^2 - \left(\sum X\right)^2\right]\left[N\sum Y^2 - \left(\sum Y\right)^2\right]}}$$

$$t_r = \frac{r\sqrt{N - 2}}{\sqrt{1 - r^2}}$$

12. LIST OF FORMULA YOU WILL USE IN THE EXAM

1. Probability: P(event) = f/p (also know the multiplicative and additive rules)

2. Z-score: $z = x - \bar{x} / s$ (raw score minus the mean score divided by standard deviation)

3. Standard error: Population: $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

Sample: $S_{\bar{x}} = \frac{s}{\sqrt{n}}$

13. FLOW CHART FROM THE 2011 UNISA STUDY GUIDE