# Tutorial Letter 203/2/2018

# Electromagnetism and Heat
# PHY1506

## Semester 2

## Department of Physics

**Solutions to Assignment 03**

Bar code

## Question 1

(a) We have

$$\begin{aligned} F &= q_0 v B \sin\phi, \quad B = \frac{\mu_0 I}{2\pi r} \Rightarrow F = q_0 v\left(\frac{\mu_0 I}{2\pi r}\right) \\ F &= 5.6\times 10^{-6}\,\mathrm{C}\times 280\,\mathrm{m/s}\left(\frac{4\pi\times 10^{-7}\,\mathrm{Tm/A}\times 3\,\mathrm{A}\sin 90^\circ}{2\pi(0.050\,\mathrm{m})}\right) \\ &= \mathbf{2.2\times 10^{-8}\,N} \end{aligned}$$

(b) We first write,

$$\begin{aligned} L &= \frac{n\ell\Phi}{I}, \quad Q = BA\cos\phi, \quad B = \mu_0 n I \Rightarrow L = \frac{n\ell(\mu_0 n I A\cos\phi)}{I} \\ L &= \mu_0 n^2 \ell A \cos 0^\circ = \mu_0 n^2 \ell A. \\ &= 4\pi\times 10^{-7}\,\mathrm{Tm/A}(6500\,\mathrm{turn/m})^2(8\times 10^{-2}\,\mathrm{m})(5\times 10^{-5}\,\mathrm{m}^2) \\ &= \mathbf{0.2124\,Tm^2/A} \end{aligned}$$

The emf is then given by

$$\mathcal{E} = -L\frac{\Delta I}{\Delta t} = 0.2124\times\frac{1.5}{2-0} = -0.1593\,\mathrm{V}$$

## Question 2

Let $I_1$, $I_2$ and $I_3$ be the currents flowing through 5Ω, 10Ω and 10Ω. Then applying Kirchhoff's law to the left loop we get,

$$5\Omega I_1 + 10\Omega I_3 + 2\,V - 10\,V = 0 \tag{1}$$

Similarly, for the right loop we have,

$$10\Omega I_2 + 10\Omega I_3 + 2\,V - 15\,V = 0. \tag{2}$$

At the junction we have,

$$I_1 + I_2 = I_3. \tag{3}$$

The substitution of equation (3) into equations (1) and (2), results in

$$\begin{aligned} 15 I_1 + 10 I_2 &= 8 \\ 10 I_1 + 20 I_2 &= 13, \end{aligned}$$

which gives $I_1 = 0.15\,\mathrm{A}$ and $I_2 = 0.575\,\mathrm{A}$. Then $I_3 = I_1 + I_2 = 0.725\,\mathrm{A}$. The voltages are

$$\begin{aligned} V_1 &= R_1\times I_1 = (0.15)(5\Omega) = 0.75\,V \\ V_2 &= R_2\times I_2 = (0.575)(10\Omega) = 5.75\,V \\ V_1 &= R_1\times I_1 = (0.725)(10\Omega) = 7.25\,V. \end{aligned}$$

## Question 3

We recall that the amount of charge that flows through at conduct in a time interval is given by

$$\Delta Q = I\Delta t, \tag{4}$$

where the current is related to the emf $\mathcal{E}$ through

$$I = \frac{\mathcal{E}}{R}. \tag{5}$$

Equation (4) becomes

$$\Delta Q = \frac{\mathcal{E}}{R}\Delta t. \tag{6}$$

With the emf given by

$$\mathcal{E} = -N\left(\frac{\Delta\Phi}{\Delta t}\right) = -N\left(\frac{BA\cos\phi - BA\cos\phi_0}{\Delta t}\right),$$

equation (6) becomes,

$$\Delta Q = \left[\frac{-N\left(\frac{BA\cos\phi - BA\cos\phi_0}{\Delta t}\right)}{R}\right]\Delta t = \frac{-NBA(cos\phi - \cos\phi_0)}{R}.$$

Solving this equation for the magnetic field $B$, we obtain

$$B = \frac{{}_R\Delta Q}{NA(\cos\phi - \cos\phi_0)} = \frac{-140\times(8.5\times10^{-5})}{50\times(1.3\times10^{-3})\times(\cos 90 - \cos 0)} = 0.16\,\mathrm{T}$$

## Question 4

Each of the four wires makes a contribution to the net magnetic field $B$ at the center of the square. The magnitude of each wireâĂŹs magnetic field is given by

$$B = \frac{\mu_0 I}{2\pi r},$$

where $r$ is the radial distance from the wire to the center of the square. For all four wires, the radial distance $r$ is half of the length $s$ of one of the sides of the square ($r = s/2$). In order to add up the four magnetic fields, we must determine the direction of each one. Using the Right-Hand Rule, we can find that at the center of the square, the currents $I_1 = 3.9\,\mathrm{A}$, $I_1 = 8.5\,\mathrm{A}$ and $I_1 = 4.6\,\mathrm{A}$ all produce fields pointing into the page, while the magnetic field $\mathbf{B}$ of the unknown current $I$ points out of the page. Therefore, the magnitude $B_{\mathrm{net}}$ of the net field is given by

$$B_{\mathrm{net}} = B_1 + B_2 + B_3 - B.$$

Knowing the expression of each magnetic field, we obtain

$$B_{\mathrm{net}} = \frac{\mu_0 I_1}{2\pi r} + \frac{\mu_0 I_2}{2\pi r} + \frac{\mu_0 I_3}{2\pi r} - \frac{\mu_0 I}{2\pi r} = \frac{\mu_0}{2\pi r}\left(I_1 + I_2 + I_3 - I\right).$$

Solving for the unknown current $I$, yields

$$I = I_1 + I_2 + I_3 - \frac{2\pi r B_{\text{net}}}{\mu_0}.$$

Since $s = 0.050\,\text{m}$, $r = 0.025\,\text{m}$. We then get

$$I = 3.9 + 8.5 + 4.6 - \frac{2\pi(0.025)(64 \times 10^{-6})}{4\pi \times 10^{-7}} = 9\,\text{A}$$

## Question 5

(a) The electric field is

$$E_y = E_0 \cos(kx - \omega t),$$

where $E_0 = 20.0\,\text{V/m}$ and $k = 6.28 \times 10^8\,\text{m}^{-1}$. The wavelength is

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{6.28 \times 10^8\,\text{m}^{-1}} = 10^{-8}\,\text{m} = 10.0\,\text{nm}.$$

(b) The frequency is

$$f = \frac{c}{\lambda} = \frac{3.0 \times 10^8\,\text{m/s}}{10^{-8}\,\text{m}} = 3.0 \times 10^{16}\,\text{Hz}$$

(c) the magnetic field amplitude is

$$B_0 = \frac{E_0}{c} = \frac{20}{3.0 \times 10^8} = 6.67 \times 10^{-8}\,\text{T}$$

## Question 6

(a) The impedance of the circuit is

$$\begin{aligned} Z &= \sqrt{R^2 + (X_L - X_C)^2} \\ &= \sqrt{(25\Omega)^2 + \left[2\pi(60\,\text{Hz})(0.10\,\text{H}) - \frac{1}{2\pi(60\,\text{Hz})(100 \times 10^{-6}\,\text{F})}\right]} \\ &= \sqrt{25^2 + (37.7 - 26.53)^2} = 27.4\,\Omega. \end{aligned}$$

The rms voltage is

$$\mathcal{E}_{\text{rms}} = I_{\text{rms}} Z = 2.5 \times 27.4 = 68.5\,\text{V}.$$

(b) The phase angle is

$$\phi = \tan^{-1}\left[\frac{X_L - X_C}{R}\right] = \tan^{-1}\left[\frac{11.173}{25}\right] = 24^\circ$$

(c) The average power is

$$\begin{aligned} P_R = I_{\text{rms}}\mathcal{E}_{\text{rms}}\cos\phi &= \frac{\mathcal{E}_{\text{rms}}^2}{R}\cos\phi \\ &= \frac{(68.5)^2}{25}\cos(24^\circ) = 170W \end{aligned}$$

**END**