Tutorial Letter 203/1/2016

Electromagnetism and Heat
PHY1506

Semester 1

Department of Physics

Solutions to Assignment 03

Bar code

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Solutions to compulsory Assignment 03

Question 1

Find first the equivalent resistor for the resistors in parallel, which are $R_1 = 60\,\Omega$, $R_2 = 60\,\Omega$ and $R_3 = 45\,\Omega$. Therefore

$$\begin{aligned}
\frac{1}{R_{\text{eq1}}} &= \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \\
&= \frac{1}{60\,\Omega} + \frac{1}{60\,\Omega} + \frac{1}{45\,\Omega} \\
&= \frac{2}{60\,\Omega} + \frac{1}{45\,\Omega} \\
&= \frac{90+60}{60\times 45} = \frac{150}{2700} \\
R_{\text{eq1}} &= \frac{2700}{150} = 18\,\Omega.
\end{aligned}$$

This equivalent resistor is now is in series with $R_4 = 42\,\Omega$. Therefore, $R_{\text{eq2}} = 18\,\Omega + 42\,\Omega = 60\,\Omega$. Finally, R_{eq2} and $R_5 = 40\,\Omega$ are in parallel.

$$\begin{aligned}
\frac{1}{R_{\text{eq}}} &= \frac{1}{60\,\Omega} + \frac{1}{40\,\Omega} \\
&= \frac{40+60}{240} \\
R_{\text{eq}} &= \mathbf{24\,\Omega}.
\end{aligned}$$

Question 2

Because the $4\,\Omega$ resistor is grounded at both ends, the potential difference across this resistor is zero. That is, no current flows through the $4\,\Omega$ resistor, and the negative terminals of both batteries are at zero potential. To determine the current in the $2\,\Omega$ resistor, we apply Kirchhoff's loop law. We assume that current I flows clockwise through the $2\,\Omega$ resistor. Starting from the lower-left corner, the sum of the potential differences across various elements in the circuit is

$$\begin{aligned}
+9 - 2I - 3 &= 0 \\
I &= \frac{6\,\text{A}}{2} \\
&= \mathbf{3\,A}
\end{aligned}$$

Question 3

The current through the 10 cm-segment is

$$\begin{aligned} I &= \frac{\varepsilon}{R} \\ &= \frac{15\,\mathrm{V}}{3\,\Omega} \\ &= 5\,\mathrm{A}, \end{aligned}$$

which is flowing down. The force on this wire, given by the right-hand rule, is to the right and perpendicular to the current and the magnetic field. The magnitude of the force is

$$\begin{aligned} F &= ILB = 5\,\mathrm{A} \times (0.1\,\mathrm{m}) \times 50\,\mathrm{mT} \\ &= 0.025\,\mathrm{N}. \end{aligned}$$

Therefore, $\vec{F} = (0.025\,\mathrm{N},\mathrm{right})$.

Question 4

The Biot-Savart law for the magnetic field of a current segment $\Delta\vec{s}$ is

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{I\Delta\vec{s}\times\hat{r}}{r^2},$$

where the unit vector $\hat{r}$ points from current segment Δs to the point, a distance r away, at which we want to evaluate the field. For the two linear segments of the wire, $\Delta\vec{s}$ is in the same direction as $\hat{r}$, so $\Delta\vec{s}\times\hat{r} = 0$. For the curved segment $\Delta\vec{s}$ and $\hat{r}$ are always perpendicular, such that $\Delta\vec{s}\times\hat{r} = \Delta s$. The above equation becomes

$$B = \frac{\mu_0}{4\pi}\frac{I\Delta s}{r^2}.$$

Now we are ready to sum the magnetic field of all the segments at point P. For all segments on the arc, the distance to point P is $r = R$. The superposition of the fields is

$$\begin{aligned} B &= \frac{\mu_0}{4\pi}\frac{I}{R}\int_{\mathrm{arc}} ds \\ &= \frac{\mu_0}{4\pi}\frac{IL}{R^2} \\ &= \frac{\mu_0 I\theta}{4\pi R}, \end{aligned}$$

where $L = R\theta$.

Question 5

The induced emf of the coil is

$$\varepsilon = N\left|\frac{d\Phi}{dt}\right|,$$

where $\Phi = \vec{A}.\vec{B}$. Since $\vec{B}$ is parallel to $\vec{A}$ and $A = \pi r^2$, one obtains

$$\begin{aligned}
\varepsilon &= N\pi r^2\left|\frac{\Delta B}{\Delta t}\right| \\
&= 10^3 \times \pi(0.0050\,\mathrm{m})^2\left(\frac{0.20\,\mathrm{T}}{10\times 10^{-3}\,\mathrm{s}}\right) \\
&= \mathbf{1.6\,V}
\end{aligned}$$

Question 6

(a) For a point on the axis, $r = 0\,\mathrm{m}$, so $E = 0\,\mathrm{V/m}$.
(b) For a point 2.0 cm from the axis, we have

$$\begin{aligned}
E &= \frac{r}{2}\left(\frac{dB}{dt}\right) \\
&= \left(\frac{0.020\,\mathrm{m}}{2}\right)(4.0\,\mathrm{T/s}) \\
&= 40\,m\mathrm{V/m}
\end{aligned}$$

Question 7

Since the field is perpendicular to the plane of the loop, $\vec{A}$ is parallel to $\vec{B}$ and $\Phi = AB$. The emf is

$$\begin{aligned}
\varepsilon &= \left|\frac{d\Phi}{dt}\right| = A\left|\frac{dB}{dt}\right| \\
&= (0.2\,\mathrm{m})^2|4-4t| \\
&= 0.4\,\mathrm{m}^2|4-4t| = 1.6|1-t| \\
I &= \frac{\varepsilon}{R} = \frac{1.6|1-t|}{0.10} \\
&= 16(1-t) \\
t &= 1\,\mathrm{s},\quad I = 16\,\mathrm{A} \\
t &= 0\,\mathrm{s},\quad I = 0\,\mathrm{A} \\
t &= 2\,\mathrm{s},\quad I = -16\,\mathrm{A}
\end{aligned}$$

Question 8

The electric field is in the direction of the positive y-axis and the magnetic field is in the direction of the negative z-axis. Substituting into the Lorentz force law, one gets

$$\begin{aligned}
\vec{F}_{\text{net}} &= q(\vec{E} + \vec{v} \times \vec{B}) \\
&= 1.6 \times 10^{-19}[-1.0 \times 10^{6}\,\hat{\imath} + 1.0 \times 10^{7}\,\hat{\imath}(-0.10\hat{k})] \\
&= -1.6 \times 10^{-13}\,\hat{\imath} + 1.6 \times 10^{-12}\,\hat{\imath}(-0.10\hat{k}) \\
&= -1.6 \times 10^{-13}\,\hat{\imath} - 1.6 \times 10^{-13}\,\hat{\imath} \times \hat{k} \\
&= 1.6 \times 10^{-13}(-\hat{\imath} + \hat{\jmath})\,\text{N}.
\end{aligned}$$

Thus

$$\begin{aligned}
F_{\text{net}} &= \sqrt{(1.6)^2 + (1.6)^2} \times 10^{-13} \\
&= \mathbf{2.26 \times 10^{-13}\,N} \\
\theta &= \tan^{-1}\left(\frac{1.6 \times 10^{-13}}{1.6 \times 10^{-13}}\right) \\
&= \mathbf{45^\circ}
\end{aligned}$$

Question 9

We have that $V_R = IR$ and $V_C = IX_C$, where

$$\begin{aligned}
X_C &= \frac{1}{\omega C} = \frac{1}{2\pi f C} \\
&= \frac{1}{2\pi(1.0 \times 10^4)(80 \times 10^{-9})} \\
&= 199\,\Omega.
\end{aligned}$$

The peak current is

$$\begin{aligned}
I &= \frac{\varepsilon_0}{\sqrt{R^2 + X_C^2}} \\
&= \frac{10\,\text{V}}{\sqrt{(199\,\Omega)^2 + (150\,\Omega)^2}} \\
&= 0.\mathbf{0401}\,\text{A}.
\end{aligned}$$

Thus,

$$V_R = 0.0401\,\text{A} \times 150\,\Omega = \mathbf{6.0\,V}$$
$$V_C = 0.0401\,\text{A} \times 199\,\Omega = \mathbf{8.0\,V}$$

Question 10

(a) The impedance of the circuit is given by

$$Z = \sqrt{R^2 + (X_L - X_C)^2}.$$

For a frequency of $3000\,\mathrm{H_z}$, we have

$$\begin{aligned} X_L &= 2\pi f L = 3000\,\mathrm{H_z} \times 2\pi \times 3 \times 10^{-3}\,\mathrm{H} \\ &= 62.20\,\Omega \\ X_C &= \frac{1}{2\pi \times 3000\,\mathrm{H_z} \times 480 \times 10^{-9}\,\mathrm{F}} \\ &= 110.52\,\Omega. \end{aligned}$$

Therefore, the impedance is

$$\begin{aligned} Z &= \sqrt{(50\,\Omega)^2 + (62.20\,\Omega - 110.52\,\Omega)^2} \\ &= 69.53\,\Omega. \end{aligned}$$

The peak current is

$$\begin{aligned} I &= \frac{\varepsilon_0}{Z} \\ &= \frac{5\,\mathrm{V}}{69.53\,\Omega} \\ &= \mathbf{0.072\,A = 72\,mA}, \end{aligned}$$

and the phase angle is

$$\begin{aligned} \phi &= \tan^{-1}\left(\frac{X_L - X_C}{R}\right) \\ &= \tan^{-1}\left(\frac{-48.32}{50}\right) \\ &= \mathbf{-4.4^\circ}. \end{aligned}$$

(b) For $f = 4000\,\mathrm{H_z}$, we obtain $Z = 50\,\Omega$ and $I = 0.10\,\mathrm{A}$, which gives $\phi = 0.0^\circ$.
(c) For $f = 5000\,\mathrm{H_z}$, we obtain $Z = 62\,\Omega$ and $I = 0.08\,\mathrm{A}$, which gives $\phi = 37.0^\circ$.

END

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