Tutorial Letter 201/1/2016

Electromagnetism and Heat
PHY1506

Semester 1

Department of Physics

Solutions to Assignment 01

Bar code

Solutions to compulsory Assignment 01

Question 1

We know that

$$pV = nRT$$
$$p = \frac{nRT}{V}.$$

Then

$$\begin{aligned} p &= p_f \\ &= \frac{6000 \times 8.314 \times 350}{89} \\ &= 196.17 \times 10^3 \, \text{Pa}. \end{aligned}$$

Since V is constant, we have that

$$\begin{aligned} \frac{p_f}{T_f} = \frac{p_i}{T_i} \Rightarrow p_i &= \frac{p_f T_i}{T_f} \\ &= \frac{196.17 \times 10^3 \times 270}{350} \\ &= 0.15 \times 10^6 \, \text{Pa} \\ &= \mathbf{0.15 \, MPa} \end{aligned}$$

Answer: Option [1]

Question 2

Again we use

$$\begin{aligned} pV = nRT \Rightarrow n &= \frac{pV}{RT} \\ &= \frac{3.3 \times 3.2 \times 10^{-3}}{8.314 \times 330} \\ &= 0.385 \, \text{moles} \end{aligned}$$

$$\begin{aligned} n = \frac{M}{M\text{mol}} \Rightarrow M &= n \times M_{\text{mol}} \\ &= 0.385 \times 20.2 \\ &= \mathbf{7.9 \times 10^{-3} \, kg} \end{aligned}$$

Answer: Option [3]

Question 3

The mass density of the gas is calculated as follows

$$\begin{aligned} pV = nRT \Rightarrow V &= \frac{nRT}{p} \\ &= \frac{0.02\,\text{moles} \times 0.0821\,\text{L atm/mol K} \times 290}{1.5\,\text{atm}} \\ &= 0.3175\,\text{L} \end{aligned}$$

$$\begin{aligned} M_{\text{mol}}(N) &= 14\,\text{g/mol} \Rightarrow M_{\text{mol}}(N_2) = 28\,\text{g/mol} \\ m &= n \times M_{\text{mol}} \\ &= 0.02 \times 28 \\ &= 0.56\,\text{g} \end{aligned}$$

The mass density will be

$$\begin{aligned} \rho &= \frac{m}{V} \\ &= \frac{0.56\,\text{g}}{0.3175\,\text{L}} \\ &= \frac{0.56 \times 10^3\,\text{kg}}{0.3175 \times 10^3\,\text{m}^3} \\ &= 1.7638\,\text{kg/m}^3 \\ &\simeq \mathbf{1.8\,kg/m^3} \end{aligned}$$

Answer: Option [3]

Question 4

The mass of the hot air inside the balloon is calculated as follows

$$\begin{aligned} n &= \frac{pV}{RT} \\ &= \frac{2 \times 10^3 \times 1.01 \times 10^5}{8.314 \times (120 + 273)} \\ &= 0.618 \times 10^5\,\text{moles} \end{aligned}$$

$$\begin{aligned} M &= n \times M_{\text{mol}} \\ &= 28.8 \times 10^{-3} \times 0.618 \times 10^5 \\ &= \mathbf{1780\,kg} \end{aligned}$$

Answer: Option [4]

Question 5

We calculate the number of moles of water as

$$\begin{aligned} n &= \frac{pV}{RT} \\ &= \frac{8 \times 10^5 \times 4}{8.314 \times (600 + 273)} \\ &= 0.00441 \times 10^5 \\ &= \mathbf{441\,moles} \end{aligned}$$

Answer: Option [2]

Question 6

Since the temperature is constant, we use

$$\begin{aligned} \frac{p_i}{V_i} = \frac{p_f}{V_f} \Rightarrow p_f &= \frac{1}{6} p_i \\ &= 107.5\,\text{Pa}. \end{aligned}$$

The work done is given by

$$\begin{aligned} W &= -p_i V_i \ln\left(\frac{V_f}{V_i}\right) = -p_f V_f \ln\left(\frac{V_f}{V_i}\right) \\ &= -645 \times 0.1 \ln\left(\frac{0.6}{0.1}\right) \\ &= -1.156 \times 10^2 \\ &\simeq -1.2 \times 10^2 \\ &\simeq \mathbf{-120\,J} \end{aligned}$$

Note that there is expansion, then the work done is negative regardless whether there is a negative sign or not.
Answer: Option [5]

Question 7

We use the following equations

$$W = nC_V \Delta T \quad \text{or} \quad \Delta E_{\text{th}} = nC_V \Delta T$$

$$\frac{C_p}{C_V} = 1.67(\text{monoatomic gas}) \tag{1}$$

$$C_p - C_V = R \tag{2}$$

From equation (1), one gets

$$\begin{aligned} C_p &= 1.67C_V \Rightarrow 1.67C_V - C_V = 8.314 \\ 0.67C_V &= 8.314 \\ C_V &= \frac{8.314}{0.67} = 12.409 \end{aligned}$$

Finally we obtain

$$\begin{aligned} n &= \frac{\Delta E_{\text{th}}}{C_V \Delta T} \\ &= \frac{831}{12.409 \times 50} \\ &= \mathbf{1.3394\,moles} \end{aligned}$$

Answer: Option [4]

Question 8

The heat flowing into the gas during the two-step process is given by

$$\begin{aligned} Q &= nC_p\Delta T - nC_V\Delta T = n\Delta T(C_p - C_V) \\ &= 3 \times 20(6.9 - 4.9) \\ &= \mathbf{120\,Cal} \end{aligned}$$

Answer: Option [2]

Question 9

We use the following equation

$$\begin{aligned} p_f V_f^{\gamma} &= p_i V_i^{\gamma} \\ p_f &= p_i \left(\frac{V_i}{V_f}\right)^{\gamma} \\ &= \left(\frac{10}{10.4}\right)^{1.67} \\ &= 0.9366\,\text{atm} \end{aligned}$$

Also we know that for any process, one has

$$\begin{aligned}
\frac{p_iV_i}{T_i} &= \frac{p_fV_f}{T_f} \\
T_f &= T_i\frac{p_fV_f}{p_iV_i} \\
&= \frac{273 \times 0.9366 \times 10.4}{10} \\
&= 265.9\,\text{K} \\
t &= 265.9 - 273 \\
&= \mathbf{-7.1\,^\circ C}
\end{aligned}$$

Answer: Option [2]

Question 10

The adiabatic constant of the gas is calculated as

$$\begin{aligned}
C_V &= \frac{Q_V}{n\Delta T} \quad \text{and} \quad C_p = \frac{Q_p}{n\Delta T} \\
\frac{C_p}{C_V} &= \frac{Q_p}{Q_V} \\
&= \frac{900\,\text{kJ}}{800\,\text{kJ}} \\
&= 1.125 \\
&\simeq \mathbf{1.13}
\end{aligned}$$

Answer: Option [1]

Question 11

We have $m(\text{He}) = 4u = 4(1.66\times10^{-27}\,\text{kg}) = 6.64\times10^{-27}\,\text{kg}$ and $m(\text{O}_2) = 32u = 32(1.66\times10^{-27}\,\text{kg}) = 53.12 \times 10^{-27}\,\text{kg}$ The ratio of the speed of He to the speed of O_2 is

$$\begin{aligned}
\frac{v_{\text{rms}}(\text{He})}{v_{\text{rms}}(\text{O}_2)} = \frac{\sqrt{\frac{3k_BT}{m(\text{He})}}}{\sqrt{\frac{3k_BT}{m(\text{O}_2)}}} = \sqrt{\frac{m(\text{O}_2)}{m(\text{He})}} &= \sqrt{\frac{53.12 \times 10^{-27}\,\text{kg}}{6.64 \times 10^{-27}\,\text{kg}}} \\
&= 2.83
\end{aligned}$$

Answer: Option [3]

Question 12

We know that

$$\begin{aligned}
v &= \sqrt{\frac{3k_BT}{m}} \Rightarrow v^2 = \frac{3K_BT}{m} \\
m &= \frac{3K_BT}{v^2} \\
&= \frac{3 \times 1.38 \times 10^{-23}(100 + 273)}{(500)^2} \\
&= 61.77 \times 10^{-27}\,\text{kg}
\end{aligned}$$

For $T = 473\,\text{K}$, the speed becomes

$$\begin{aligned}
v &= \sqrt{\frac{3 \times 1.38 \times 10^{-23} \times 473}{61.77 \times 10^{-27}}} \\
&= 5.630 \times 10^2\,\text{m/s} \\
&= \mathbf{563\,m/s}
\end{aligned}$$

Answer: Option [2]

Question 13

The internal energy is given by

$$\varepsilon = \frac{3}{2}k_BT$$

Therefore,

$$\varepsilon_1 = \frac{3}{2}k_BT_1 \quad \text{and} \quad \varepsilon_2 = \frac{3}{2}k_BT_2.$$

Then

$$\begin{aligned}
\frac{\varepsilon_2}{\varepsilon_1} &= \frac{\frac{3}{2}k_BT_2}{\frac{3}{2}k_BT_1} \\
\frac{\varepsilon_2}{\varepsilon_1} &= \frac{T_2}{T_1} \\
&= \frac{373}{323} \\
&= 1.15 \\
\varepsilon_2 &= 1.15\varepsilon_1 \\
\varepsilon_2 &= \mathbf{1.15K}
\end{aligned}$$

Answer: Option [4]

Question 14

Again we start by using

$$pV = nRT, \quad n = \frac{N}{N_A} \tag{3}$$

$$n = \frac{2 \times 10^{23}}{6.022 \times 10^{23}} = 0.3321 \, \text{moles}$$

The temperature can be obtained from the thermal speed equation

$$v_{\text{rms}} = \sqrt{\frac{3k_BT}{m}} \Rightarrow v_{\text{rms}}^2 = \frac{3k_BT}{m}$$

$$T = \frac{mV_{\text{rms}}^2}{3k_B}. \tag{4}$$

Introducing equation (4) into equation (3) and using $V = r^3$, one obtains the pressure given by

$$\begin{aligned} p &= \frac{nRmv_{\text{rms}}^2}{3k_Br^3} \\ &= \frac{0.3321 \times 8.314 \times 3.4 \times 10^{-27} \times (200)^2}{3 \times 1.38 \times 10^{-23} \times (0.2)^3} \\ &= 1.1338 \times 10^3 \, \text{Pa} \\ &= \mathbf{1.13 \, kPa}. \end{aligned}$$

Answer: Option [5]

Question 15

We first write the collision cross section as

$$\sigma = 4\pi r^2$$
$$r^2 = \frac{\sigma}{4\pi}.$$

We know that the mean free path is given by

$$\lambda = \frac{1}{4\pi\sqrt{2}(N/V)r^2},$$

where

$$\frac{N}{V} = \frac{p}{k_BT}.$$

Then the mean free path becomes

$$\lambda = \frac{k_BT}{\sqrt{2}p\sigma},$$

which gives

$$\lambda = \frac{1.38 \times 10^{-23} \times 300}{\sqrt{2} \times 10^{5} \times 2 \times 10^{-20}} \;=\; \mathbf{1.46 \times 10^{-6}\,m}.$$

Answer: Option [4]

Question 16

The efficiency of the engine is given by

$$\begin{aligned}
\eta &= \frac{W_{\text{out}}}{Q_H} = \frac{Q_H - Q_C}{Q_H} = 1 - \frac{Q_C}{Q_H} \\
&= 1 - \frac{700}{1300} \\
&= \frac{600}{1300} \\
&= 46.15\% \\
&\simeq \mathbf{46}\%
\end{aligned}$$

Answer: Option [3]

Question 17

We calculate the heat is discharged to the lower temperature reservoir

$$\begin{aligned}
Q_H &= \frac{W_{\text{out}}}{\eta} \\
&= \frac{2500}{0.3} \\
Q_C &= Q_H - W_{\text{out}} \\
&= \frac{2500}{0.3} - 2500 \\
&= \mathbf{5833{,}33\,J}
\end{aligned}$$

Answer: Option [2]

Question 18

The work that must be done on the refrigerator in order to remove 250 J of heat from the interior is obtained as follows

$$\begin{aligned}
K &= \frac{Q_C}{W_{\text{in}}} = 4.2 \\
W_{\text{in}} &= \frac{Q_C}{K} = \frac{250}{4.2} \\
&= 59.52\,\text{J} \\
&\simeq \mathbf{60\,J}.
\end{aligned}$$

Answer: Option [1]

Question 19

The temperature of the colder reservoir is

$$\begin{aligned}
\eta &= \frac{Q_H - Q_C}{Q_H} = \frac{T_H - T_C}{T_H} \\
\eta T_H &= T_H - T_C \\
T_C &= T_H(1-\eta) \\
&= 233\left(1 - \frac{8}{10}\right) \\
&= 233 \times 0.2 \\
&= 46.6\,\mathrm{J} \\
&\simeq \mathbf{47\,J}.
\end{aligned}$$

Answer: Option [5]

Question 20

The power requirement for the heat pump under these operating conditions is

$$\begin{aligned}
\eta &= 1 - \frac{T_C}{T_H} \\
&= 1 - \frac{229}{293} \\
&= 21.84 \\
P_{\text{in}} &= \eta \times P_{\text{out}} \\
&= 0.2184 \times 34 \times 10^3\,\mathrm{J/s} \\
&= 7425.6\,J/s \\
&= \mathbf{7425.6\,W}
\end{aligned}$$

7425.6 W is closest to 7500 W.
Answer: Option [4]

END