SECOND SEMESTER 2017, SOLUTIONS (PHY1506)

Question 1

(a) The problem is solved as follows:
First run:

$$\begin{aligned} m_{\text{alc}} &= 0.2\,\text{kg},\ t_{\text{alc}} = -10°\text{C} \\ m_{\text{wat}} &= 0.3\,\text{kg},\ t_{\text{wat}} = 20°\text{C} \\ t_{\text{final}} &= 5° \end{aligned}$$

Second run:

$$\begin{aligned} m_{\text{alc}} &= 0.2\,\text{kg},\ t_{\text{alc}} = -10°\text{C} \\ m_{\text{wat}} &= 0.5\,\text{kg},\ t_{\text{wat}} = 20°\text{C} \\ t_{\text{final}} &= 10°\text{C} \end{aligned}$$

For the two runs, we the calorimetric equation as

$$\begin{aligned} m_{\text{alc}}c_{\text{alc}}\Delta t_{\text{alc}} + Q_f + m_{\text{wat}}c_{\text{wat}}\Delta t_{\text{wat}} &= 0 \\ 0.2c_{\text{alc}}[5-(-10)] + Q_f + 0.3\times 4190(5-20) &= 0 \\ 3c_{\text{alc}} + Q_f - 18855 &= 0 \\ 0.2c_{\text{cal}}[10-(-10)] + Q_f - 20950 &= 0 \\ 4c_{\text{alc}} + Q_f - 20950 &= 0. \end{aligned}$$

(i) To find c_{alc} and Q_f, we solve the following equations

$$3c_{\text{alc}} + Q_f - 18855 = 0 \quad (1)$$
$$4c_{\text{alc}} + Q_f - 20950 = 0. \quad (2)$$

If we multiple (1) by -1, and solve for c_{calc}, we obtain

$$c_{\text{alc}} = 20950 - 18855 = \mathbf{2095}\,\text{J}/(\text{kg}\cdot\text{K}).$$

(ii) Substituting this value into (1) [or into (2)], gives

$$3\times 2095 + Q_f - 18855 = 0 \Rightarrow \mathbf{Q_f} = \mathbf{12570}\,\text{J}.$$

The latent heat of fusion is then given by

$$L_f = \frac{Q_f}{m} = \frac{12570\,\text{J}}{0.2\,\text{kg}} = \mathbf{62850}\,\text{J/kg} = \mathbf{6.3\times 10^4}\,\text{J/kg}.$$

(b) Ideal gas: An ideal gas is a gas of noninteracting atoms.

Question 2

(a) We use the following equations

$$W = nC_V\Delta T \quad \text{or} \quad \Delta E_{\text{th}} = nC_V\Delta T$$

$$\frac{C_p}{C_V} = 1.67 \quad \text{(monoatomic gas)} \tag{3}$$

$$C_p - C_V = R \tag{4}$$

From equation (3), one gets

$$\begin{aligned} C_p &= 1.67C_V \Rightarrow 1.67C_V - C_V = 8.314 \\ 0.67C_V &= 8.314 \\ C_V &= \frac{8.314}{0.67} = 12.409 \end{aligned}$$

Finally we obtain

$$\begin{aligned} n &= \frac{\Delta E_{\text{th}}}{C_V\Delta T} \\ &= \frac{831}{12.409 \times 50} \\ &= \mathbf{1.3394\,moles} \end{aligned}$$

(b) Specific heat capacity: The specific heat capacity is the amount of energy that raises the temperature of 1kg of a substance by 1 K.

Question 3

(a) We first write the collision cross section as

$$\sigma = 4\pi r^2$$
$$r^2 = \frac{\sigma}{4\pi}.$$

We know that the mean free path is given by

$$\lambda = \frac{1}{4\pi\sqrt{2}(N/V)r^2},$$

where

$$\frac{N}{V} = \frac{p}{k_B T}.$$

Then the mean free path becomes

$$\lambda = \frac{k_B T}{\sqrt{2}p\sigma},$$

which gives

$$\lambda = \frac{1.38 \times 10^{-23} \times 300}{\sqrt{2} \times 10^5 \times 2 \times 10^{-20}} \quad = \quad \mathbf{1.46 \times 10^{-6}\,m}.$$

(b) The temperature of the colder reservoir is

$$\begin{aligned}
\eta &= \frac{Q_H - Q_C}{Q_H} = \frac{T_H - T_C}{T_H} \\
\eta T_H &= T_H - T_C \\
T_C &= T_H(1-\eta) \\
&= 233\left(1 - \frac{8}{10}\right) \\
&= 233 \times 0.2 \\
&= 46.6\,\mathrm{J} \\
&\simeq \mathbf{47\,K}.
\end{aligned}$$

Question 4

(a) We have

$$\begin{aligned}
F &= q_0 v B \sin\phi, \quad B = \frac{\mu_0 I}{2\pi r} \Rightarrow F = q_0 v\left(\frac{\mu_0 I}{2\pi r}\right) \\
F &= 5.6 \times 10^{-6}\,\mathrm{C} \times 280\,\mathrm{m/s}\left(\frac{4\pi \times 10^{-7}\,\mathrm{Tm/A} \times 3\,\mathrm{A}\sin 90^\circ}{2\pi(0.050\,\mathrm{m})}\right) \\
&= \mathbf{2.2 \times 10^{-8}\,N}
\end{aligned}$$

(b) We first write,

$$\begin{aligned}
L &= \frac{n\ell\Phi}{I}, \quad Q = BA\cos\phi, \quad B = \mu_0 n I \Rightarrow L = \frac{n\ell(\mu_0 n I A\cos\phi)}{I} \\
L &= \mu_0 n^2 \ell A \cos 0^\circ = \mu_0 n^2 \ell A. \\
&= 4\pi \times 10^{-7}\,\mathrm{Tm/A}(6500\,\mathrm{turn/m})^2(8 \times 10^{-2}\,\mathrm{m})(5 \times 10^{-5}\,\mathrm{m}^2) \\
&= \mathbf{0.2124\,Tm^2/A}
\end{aligned}$$

The emf is then given by

$$\xi = -L\frac{\Delta I}{\Delta t} = 0.2124 \times \frac{1.5}{0.2} = -1.6 \times 10^{-3}\,\mathrm{V}$$

Question 5

(a) We start by reminding that when resistors in parallel are connected to the same voltage. Therefore,

$$V_8 = V_{16}, \quad V_9 = V_{18}, \quad I_{16} = \frac{V_{16}}{R_{16}} = \frac{V_8}{R_{16}} = \frac{I_8 \times R_8}{R_{16}} = \frac{0.5 \times 8}{16} = 0.25\,\mathrm{A}.$$

We can see that

$$I_8 + I_{16} = I_{20} \Rightarrow I_{20} = 0.5 + 0.25 = 0.75\,\mathrm{A}.$$

To determine the currents from through R_9 and R_{18}, we need first need the current in the whole branch. Therefore, we obtain the equivalent resistors of R_8, R_{16}, R_{20}, and R_9, R_{18}

$$\begin{aligned}\frac{1}{8}+\frac{1}{16} &= \frac{3}{16} \Rightarrow R_{81620} = \frac{16}{3}+20 = \frac{76}{3}\,\Omega.\\ \frac{1}{R_{918}} &= \frac{1}{9}+\frac{1}{18} \Rightarrow R_{918} = 6\,\Omega.\end{aligned}$$

From here we have that $V_{81620} = V_{918}$.

$$\begin{aligned}I_{918} &= \frac{V_{918}}{R_{918}} = \frac{V_{81620}}{R_{918}} = \frac{\frac{76}{3}\times 0.75}{6} = 3.2\,\mathrm{A}\\ V_9 &= V_{18} \Rightarrow 9I_9 = 18I_{18}, \quad I_9 + I_{18} = 3.2\\ I_9 &= \mathbf{2.14}\,\mathrm{A}\\ I_{18} &= \mathbf{1.06}\,\mathrm{A}\end{aligned}$$

(b) The total current flowing in the circuit is $I = I_{20} + I_{918} = 0.75 + 3.2 = \mathbf{3.95\,A}$

(c) To obtain the voltage we need the equivalent resistance of the circuit, which is given by

$$\begin{aligned}\frac{1}{R} &= \frac{1}{\frac{76}{3}}+\frac{1}{6} = \frac{3}{76}+\frac{1}{6} \Rightarrow R = \frac{456}{94} = 4.85\,\Omega\\ V &= RI = 4.85\times 3.95 = \mathbf{19.16\,V}\end{aligned}$$

Question 6

The inductance X_L and capacitance X_C are

$$\begin{aligned}\omega &= 2\pi f = 2\pi\times 3000\,\mathrm{Hz} = 18.85\times 10^3\,\mathrm{rad/s}\\ X_L &= \omega L = (18.85\times 10^3\,\mathrm{rad/s})(3.3\times 10^{-3}\,\mathrm{H}) = 62.20\,\Omega\\ X_C &= \frac{1}{\omega f} = \frac{1}{(18.85\times 10^3\,\mathrm{rad/s})(0.48\times 10^{-6})} = 110.52\,\Omega.\\ Z &= \sqrt{R^2+(X_L-X_C)^2} = \sqrt{50^2+(62.20-110.52)^2} = 69.53\,\Omega.\\ I &= \frac{\xi_0}{Z} = \frac{5}{69.53} = 0.072\,A = 72\,\mathrm{mA}.\end{aligned}$$

At resonance,

$$\begin{aligned}X_L &= X_C \Rightarrow Z = R\\ I &= \frac{\xi_0}{R} = \frac{5}{50} = 0.1\,A\\ f_0 &= \frac{\omega_0}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} = \frac{1}{2\pi}\sqrt{\frac{1}{(3.3\times 10^{-3})(0.48\times 10^{-6})}}\\ &= 3999\,\mathrm{Hz} \simeq 4000\,\mathrm{Hz}\end{aligned}$$