FIRST SEMESTER 2017, SOLUTIONS (PHY1506)

Question 1

(a) The temperature is obtained following

$$\begin{aligned}
P_aV_a &= nRT_a \Rightarrow T_a = \frac{P_aV_a}{nR} = \frac{3\times10^4\times0.2}{7.5\times8.31} = 96.27\,\mathrm{K} = -176.73^\circ\,\mathrm{C} \\
\frac{P_aV_a}{T_a} &= \frac{P_bV_b}{T_b} \Rightarrow T_b = T_a\left(\frac{P_bV_b}{P_aV_a}\right) = 96.27\left(\frac{5\times10^4\times0.6}{3\times0.2}\right) = 481.35\,\mathrm{K} = 208.35^\circ\,\mathrm{C} \\
\frac{P_b}{T_b} &= \frac{P_c}{T_c} \Rightarrow T_c = \frac{P_cT_b}{P_b} = \frac{3\times481.35}{5} = 288.81\,\mathrm{K} = 120^\circ\,\mathrm{C}
\end{aligned}$$

(b) The work done will be given by the triangle abc. That is

$$W = \frac{1}{2}ab\times bc = \frac{1}{2}\times0.4\times2\times10^4 = 4\times10^3\,\mathrm{J} = 4\,\mathrm{KJ}$$

(c) Since on part bc of the cycle the volume is constant, we have

$$Q_V = Q_{bc} = nC_V\Delta T = 7.5\times\frac{5}{2}R(288.81-481.35) = -30\,\mathrm{KJ}$$

This heat leaves the gas due to the negative sign.

(d) The change in the thermal energy is given by

$$\Delta_{\mathrm{th}} = Q_{bc} - W_{bc} = -30 - 0 = -30\,\mathrm{KJ}$$

(e) The change in the thermal energy during the entire cycle will be

$$\Delta E_{\mathrm{th}} = \Delta E_{ab} + \Delta E_{bc} = Q_{ab} - W_{ab} + Q_{bc} - W_{bc}$$

Question 2

(a) The field lines at Q_1 and Q_3 are outgoing, therefore these charges are repulsive (**positive charges**). The field lines at Q_2 are incoming, it is an attractive charge (**negative charge**).

(b) We write the calorimetric equation as

$$M_mC_m(t_m-t_f) + M_wC_w(t_w-t_f) = 0 \Rightarrow M_mC_m(t_m-t_f) = M_wC_w(t_f-t_w)$$

$$C_m = \frac{M_wC_w(t_f-t_w)}{M_mC_m(t_m-t_f)} = \frac{0.45\times4186(40-15)}{0.4(120-40)} = 1471.61\,\mathrm{J/Kg^\circ C}$$

(c) This is due to the latent heat.

Question 3

(a) The potentials are calculated as

$$
\begin{aligned}
U_A &= \frac{Kq_1}{r_1} + \frac{Kq_2}{r_2} = \frac{(9\times 10^9)(2\times 10^{-6})}{0.4} + \frac{(9\times 10^9)(4\times 10^{-6})}{0.8} \\
&= 45\times 10^3 + 45\times 10^3 = 9\times 10^4\,\mathrm{V} \\
U_B &= \frac{(9\times 10^9)(4\times 10^{-6})}{0.4} + \frac{(9\times 10^9)(2\times 10^{-6})}{0.8} \\
&= 9\times 10^4 + 22.5\times 10^3 = 1.125\times 10^5\,\mathrm{V}
\end{aligned}
$$

(b) We calculate the heat is discharged to the lower temperature reservoir as follows

$$
\begin{aligned}
Q_H &= \frac{W_{\text{out}}}{\eta} \\
Q_C &= Q_H - W_{\text{out}} = \frac{2500}{0.3} - 2500 = 5833,3\,\text{Joules}
\end{aligned}
$$

Question 4

(a) (i) This equation represents the Bio-Savart law

(ii) r: distance from the charge to the field, μ: permeability constant, v: charge speed, θ: angle between $\vec{r}$ and $\vec{v}$

(b) An electromagnetic induction is a creation of a current by a changing magnetic field

(c) Faraday's law: an emf is induced around a closed loop if the magnetic flux through the loop changes, and its direction is such as to drive an induced current in the direction given by the Lenz's law

Question 5

(a) The resistors 4Ω and 12Ω are in parallel

$$\frac{1}{R_{412}} = \frac{1}{4} + \frac{1}{12} \Rightarrow R_{412} = 3\Omega.$$

R_{412} and 5Ω are in series,

$$R_{(412)5} = 3\Omega + 5\Omega = 8\Omega.$$

$R_{(412)5}$ and 24Ω are in parallel,

$$\frac{1}{R_{(412)(524)}} = \frac{1}{8} + \frac{1}{24} = 6\Omega.$$

(b) According to Kirchoff's law thew current flowing in the circuit is obtained as

$$\sum V_i = 0 \Rightarrow 3I + 6I - 6\,\mathrm{V} + 3\,\mathrm{V} = 0 \Rightarrow 3I + 6I = 6\,\mathrm{V} - 3\,\mathrm{V} \Rightarrow I = \frac{6\,\mathrm{V} - 3\,\mathrm{V}}{6\,\Omega + 3\,\Omega} = 1\,\mathrm{A}.$$

Therefore

$$V_3 = R_3 I = 3 \times 1 = 3\,\text{V}, \quad V_6 = R_6 I = 6 \times 1 = 6\,\text{V}.$$

$24\,\Omega$ and $8\,\Omega$ are equivalent and in parallel, therefore they have the same voltage

$$I_8 = \frac{6}{8} = \frac{3}{4}\,\text{A}, \quad I_{24} = \frac{6}{24} = \frac{1}{4}\,\text{A}.$$

$4\,\Omega$ and $12\,\Omega$ are in parallel, and have $V = \frac{9}{4}\,\text{V}$. Their currents are

$$I_4 = \frac{9/4}{4} = \frac{9}{16}\,\text{A}, \quad I_{12} = \frac{9/4}{12} = \frac{9}{48}\,\text{A}.$$

$$V_5 = \frac{3}{4} \times 5 = \frac{15}{4}\,\text{V}.$$

Resistor (Ω)	Potential diff. (V)	Current (A)
3	3	1
4	9/4	9/16
5	15/4	4/3
12	9/4	9/48
24	6	1/4

Question 6

(a) At resonance, we have

$$\begin{aligned} \omega &= \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(1.0 \times 10^{-3})(1.0 \times 10^{-6})}} = 3.24 \times 10^4\,\text{rad/s} \\ f &= \frac{\omega}{2\pi} = \frac{3.24 \times 10^4\,\text{rad/s}}{2\pi} = 5 \times 20^3\,\text{H}_\text{Z} \end{aligned}$$

(b) We have that

$$Z = \sqrt{R^2 + (X_L - X_C)^2}.$$

At resonance, $X_L = X_C$. Therefore,

$$\begin{aligned} Z &= R, I = \frac{\xi_0}{R} = \frac{\xi(t=0)}{R} = \frac{10\,\text{V}}{10\,\Omega} = 1.0\,\text{A} \\ V_R &= IR = 10\,\text{V} \\ V_C &= IX_C = \frac{I}{\omega C} = \frac{1}{(3.24 \times 10^4)(1.0 \times 10^{-6})} = 32\,\text{V} \end{aligned}$$