PHY1506
Semester 1, 2015
Assignment 03 MEMO

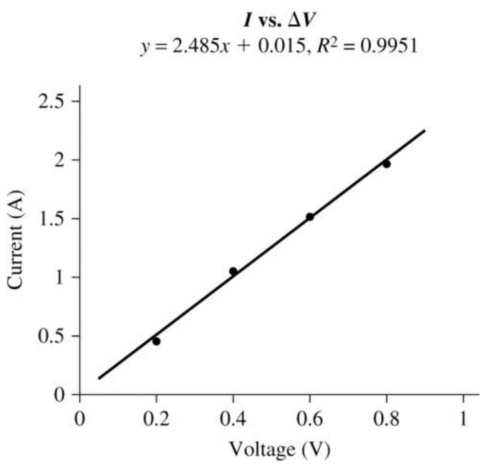
30

CURRENT AND RESISTANCE

30.39. Model: We will not initially assume the material is ohmic; this is what we are trying to find out.

Visualize: If an object (resistor) is ohmic then a graph of I vs. ΔV will be linear and the slope will be $1/R$. The cross- sectional area $A = (1.0\times10^{-3}\ \text{m})\times(0.50\times10^{-3}\ \text{m}) = 0.50\times10^{-6}\ \text{m}^2$.

Solve:



We see from the graph that the linear fit is excellent and the slope is 2.485 A/V. The intercept is also very small, which agrees with our hypothesis that the two variables are directly proportional (and linearly related), so we proceed to compute the conductivity. From $R = \rho L/A$ and $\sigma = 1/\rho$ we get

$$\text{slope} = \frac{1}{R} = \frac{\sigma A}{L}$$

Solve for σ:

$$\sigma = \frac{(\text{slope})(L)}{A} = \frac{(2.485\ \text{A/V})(45\times10^{-3}\ \text{m})}{0.50\times10^{-6}\ \text{m}^2} = 2.2\times10^{5}\ \Omega^{-1}\text{m}^{-1}$$

Assess: The material is ohmic, but not a particularly great conductor since the conductivity is a couple of orders less than the metal conductors in the table.

Open Rubric

PHY1506
Semester 1, 2015
Assignment 03 MEMO

30.47. **Model:** We assume that Ohm's law applies to the situation.

$$I = \frac{\Delta V}{R}$$

We also use Equation 30.22, which gives R in terms of ρ, L, and A.

$$R = \frac{\rho L}{A}$$

Visualize: We are given that $\Delta V = 9.0\text{ V}$, $L = 0.050\text{ m}$, $I = 230\ \mu\text{A}$, and $A = \pi r^2 = \pi(d/2)^2 = \pi(1.5\text{ mm}/2)^2 = 1.77\times10^{-6}\text{ m}^2$.

Solve: Combine the two previous equations.

$$\rho = \frac{RA}{L} = \frac{\Delta V}{I}\frac{A}{L} = \frac{(9.0\text{ V})(1.77\times10^{-6}\text{ m}^2)}{(230\times10^{-6}\text{A})(0.050\text{ m})} = 1.4\ \Omega\cdot\text{m}$$

Assess: This resistivity is close to the value for blood (1.6 Ω · m) found in references.

31

FUNDAMENTALS OF CIRCUITS

31.51. **Solve:** Let the units guide you. For the incandescent bulb, the life-cycle cost p_{bulb} is

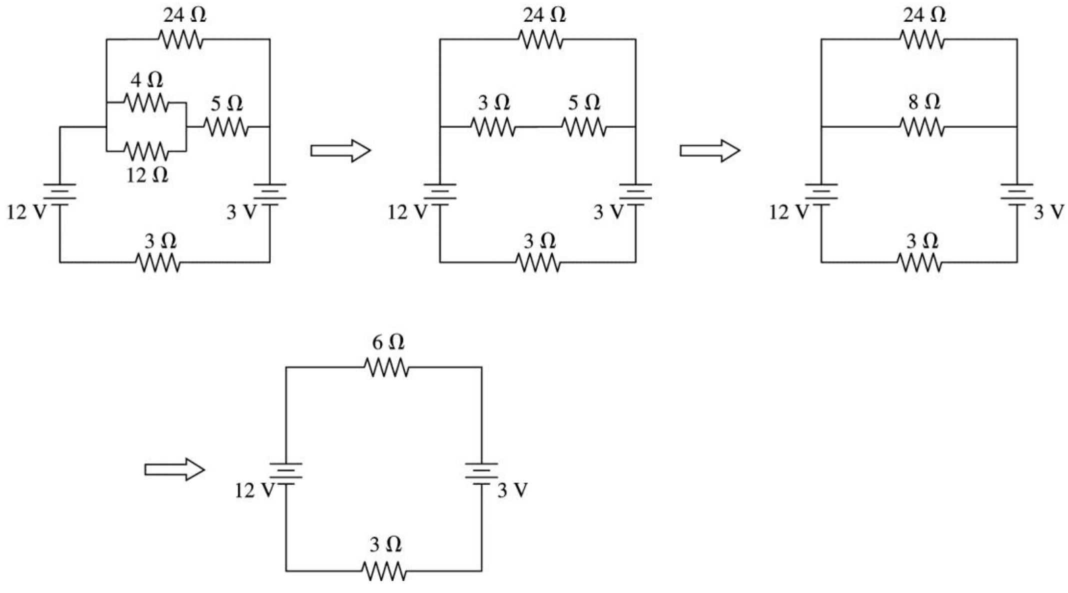
$$p_{\text{bulb}} = \$0.50 + (0.060\ \text{kW})(0.10\ \$/\text{kWh})(10{,}000\ \text{h}) = \$60.50$$

This will give 1,000 hours, so the cost for 10,000 hours is $65.00. For the fluorescent tube, the cost for 10,000 hours is

$$p_{\text{tube}} = \$5 + (0.015\ \text{kW})(0.10\ \$/\text{kW h})(10{,}000\ \text{h}) = \$20$$

Assess: The lifetime cost of the fluorescent bulb is one-third that of the incandescent bulb.

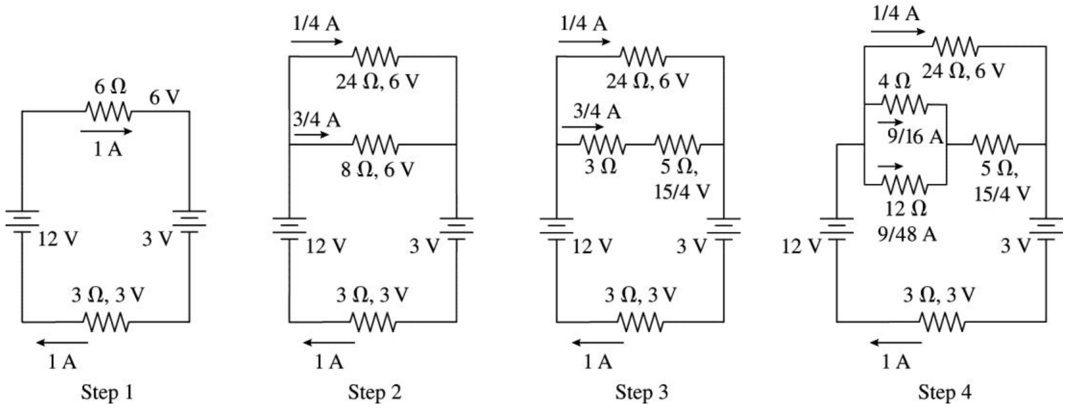
31.61. **Model:** The batteries and the connecting wires are ideal.
Visualize:



The figure shows how to simplify the circuit in Figure P31.61 using the laws of series and parallel resistances. Having reduced the circuit to a single equivalent resistance, we will reverse the procedure and "build up" the circuit using the loop law and the junction law to find the current and potential difference of each resistor.
Solve: From the last circuit in the figure and from Kirchhoff's loop law,

$$I = \frac{12\ \text{V} - 3\ \text{V}}{6\ \Omega + 3\ \Omega} = 1\ \text{A}$$

Thus, the current through the batteries is 1 A. As we rebuild the circuit, we note that series resistors *must* have the same current *I* and that parallel resistors *must* have the same potential difference.



In Step 1 of the above figure, both resistors must have the same 1 A current. We use Ohm's law to find

$$\Delta V_3 = (1\text{ A})(3\ \Omega) = 3\text{ V} \qquad \Delta V_{6\text{eq}} = 6\text{ V}$$

As a check we sum the voltages around the circuit starting at the lower-left corner: $12\text{V} - 6\text{ V} - 3\text{ V} - 3\text{ V} = 0\text{ V}$, as required. In Step 2, the $6\ \Omega$ equivalent resistor is returned to the $24\ \Omega$ and $8\ \Omega$ resistors in parallel. The two resistors must have the same potential difference $\Delta V = 6\text{ V}$. From Ohm's law,

$$I_{8\text{eq}} = \frac{6\text{ V}}{8\ \Omega} = \frac{3}{4}\text{ A} \qquad I_{24} = \frac{6\text{ V}}{24\ \Omega} = \frac{1}{4}\text{ A}$$

As a check, $3/4\text{ A} + 1/4\text{ A} = 1\text{ A}$ which was the current *I* of the $6\ \Omega$ equivalaent resistor. In Step 3, the $8\ \Omega$ equivalent resistor is returned to the $3\ \Omega$ and $5\ \Omega$ resistors in series, so the two resistors must have the same current of 3/4 A. We use Ohm's law to find

$$\Delta V_{3\text{eq}} = \left(\frac{3}{4}\text{ A}\right)(3\ \Omega) = \frac{9}{4}\text{ V} \qquad \Delta V_5 = \left(\frac{3}{4}\text{ A}\right)(5\ \Omega) = \frac{15}{4}\text{ V}$$

As a check, $9/4\text{ V} + 15/4\text{ V} = 24/4\text{ V} = 6\text{ V}$, which was ΔV of the $8\ \Omega$ equivalent resistor. In Step 4, the $3\ \Omega$ equivalent resistor is returned to $4\ \Omega$ and $12\ \Omega$ resistors in parallel, so the two must have the same potential difference $\Delta V = 9/4\text{ V}$. From Ohm's law,

$$I_4 = \frac{9/4\text{ V}}{4\ \Omega} = \frac{9}{16}\text{ A} \qquad I_{12} = \frac{9/4\text{ V}}{12\ \Omega} = \frac{9}{48}\text{ A}$$

As a check, $9/16\text{ A} + 9/48\text{ A} = 3/4\text{ A}$, which was the same as the current through the $3\ \Omega$ equivalent resistor. The results are summarized in the table below.

Resistor	Potential difference (V)	Current (A)
24 Ω	6	1/4
3 Ω	3	1
5 Ω	15/4	3/4
4 Ω	9/4	9/16
12 Ω	9/4	9/48

PHY1506
Semester 1, 2015
Assignment 03 MEMO

32

THE MAGNETIC FIELD

32.34. **Model:** Assume that the magnetic field is uniform over the 10 cm length of the wire. Force on top and bottom pieces will cancel.

Visualize: The figure shows a 10-cm-segment of a circuit in a region where the magnetic field is directed into the page.

Solve: The current through the 10-cm-segment is

$$I = \frac{\varepsilon}{R} = \frac{15\text{ V}}{3\ \Omega} = 5\text{ A}$$

and is flowing *down*. The force on this wire, given by the right-hand rule, is to the right and perpendicular to the current and the magnetic field. The magnitude of the force is

$$F = ILB = (5\text{ A})(0.10\text{ m})(50\text{ mT}) = 0.025\text{ N}$$

Thus $\vec{F} = (0.025\text{ N, right})$.

32.46. **Model:** Use the Biot-Savart law for a current-carrying segment.

Solve: The Biot-Savart law (Equation 32.6) for the magnetic field of a current segment $\Delta\vec{s}$ is

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{I\Delta\vec{s} \times \hat{r}}{r^2}$$

where the unit vector $\hat{r}$ points from current segment Δs to the point, a distance r away, at which we want to evaluate the field. For the two linear segments of the wire, $\Delta\vec{s}$ is in the same direction as $\hat{r}$, so $\Delta\vec{s} \times \hat{r} = 0$. For the curved segment, $\Delta\vec{s}$ and $\hat{r}$ are always perpendicular, so $\Delta\vec{s} \times \hat{r} = \Delta s$. Thus

$$B = \frac{\mu_0}{4\pi}\frac{I\Delta s}{r^2}$$

Now we are ready to sum the magnetic field of all the segments at point P. For all segments on the arc, the distance to point P is $r = R$. The superposition of the fields is

$$B = \frac{\mu_0}{4\pi}\frac{I}{R^2}\int_{\text{arc}} ds = \frac{\mu_0}{4\pi}\frac{IL}{R^2} = \frac{\mu_0 I\theta}{4\pi R}$$

where $L = R\theta$ is the length of the arc

33

ELECTROMAGNETIC INDUCTION

33.25. **Visualize:** Please refer to Figure EX33.25. This is a simple *LR* circuit if the resistors in parallel are treated as an equivalent resistor in series with the inductor.
Solve: We can find the equivalent resistance from the time constant since we know the inductance. We have

$$\tau = \frac{L}{R_{eq}} \quad \Rightarrow \quad R_{eq} = \frac{L}{\tau} = \frac{7.5\times10^{-3}\ \text{H}}{25\times10^{-6}\ \text{s}} = 300\ \Omega$$

The equivalent resistance is the parallel addition of the unknown resistor *R* and $500\ \Omega$. We have

$$\frac{1}{R_{eq}} = \frac{1}{500\ \Omega} + \frac{1}{R} \quad \Rightarrow \quad R = \frac{(500\ \Omega)(300\ \Omega)}{500\ \Omega - 300\ \Omega} = 750\ \Omega$$

33.49. **Model:** Assume that the magnetic field is uniform in the region of the loop.
Visualize: Please refer to Figure P33.49. The rotating semicircle will change the area of the loop and therefore the flux through the loop. This changing flux will produce an induced emf and corresponding current in the bulb.

Solve:
(a) The spinning semicircle has a normal to the surface that changes in time, so while the magnetic field is constant, the area is changing. The flux through in the lower portion of the circuit does not change and will not contribute to the emf. Only the flux in the part of the loop containing the rotating semicircle will change. The flux associated with the semicircle is

$$\Phi = \vec{A}\cdot\vec{B} = BA = BA\cos\theta = BA\cos(2\pi ft)$$

where $\theta = 2\pi ft$ is the angle between the normal of the rotating semicircle and the magnetic field and *A* is the area of the semicircle. The induced current from the induced emf is given by Faraday's law. We have

$$I = \frac{\mathcal{E}}{R} = \frac{1}{R}\left|\frac{d\Phi}{dt}\right| = \frac{1}{R}\left|\frac{d}{dt}BA\cos(2\pi ft)\right| = \frac{B}{R}\frac{\pi r^2}{2}2\pi f\sin(2\pi ft)$$

$$= \frac{2(0.20\ \text{T})\pi^2(0.050\ \text{m})^2}{2(1.0\ \Omega)} f\sin(2\pi ft) = (4.9\times10^{-3}) f\sin(2\pi ft)\ \text{A}$$

where the frequency *f* is in Hz.

(b) We can now solve for the frequency necessary to achieve a certain current. From our study of DC circuits we know how power relates to resistance:

$$P = I^2 R \quad \Rightarrow \quad I = \sqrt{P/R} = \sqrt{(4.0\ \text{W})/(1.0\ \Omega)} = 2.0\ \text{A}$$

The maximum of the sine function is $+1$, so the maximum current is

$$I_{\text{max}} = 4.9 \times 10^{-3} f \quad \text{A s} = 2.0\ \text{A} \quad \Rightarrow \quad f = \frac{2.0\ \text{A}}{4.9 \times 10^{-3}\ \text{A s}} = 4.1 \times 10^{2}\ \text{Hz}$$

Assess: This is not a reasonable frequency to obtain by hand

PHY1506
Semester 1, 2015
Assignment 03 MEMO

34

ELECTROMAGNETIC FIELDS AND WAVES

34.49. **Model:** Radio waves are electromagnetic waves. Assume that the transmitter unit radiates in all directions.
Solve: The transmitting unit radiates energy in all directions at the rate of 250 mJ per second. From Equation 34.36, the signal intensity at a distance of 42 m is

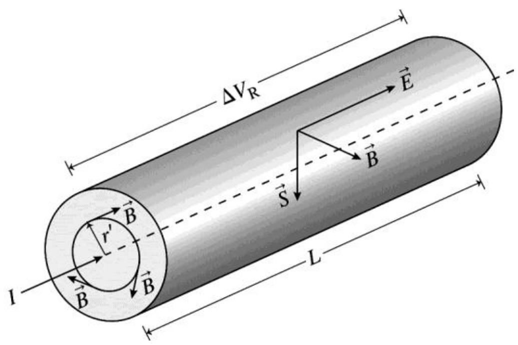
$$I = \frac{P}{A} = \frac{P}{4\pi r^2} = \frac{250\times10^{-3}\text{ W}}{4\pi(42\text{ m})^2} = 1.13\times10^{-5}\text{ W/m}^2$$

Using Equation 34.36 again,

$$I = \frac{c\varepsilon_0}{2}E_0^2 \Rightarrow E_0 = \sqrt{\frac{2I}{c\varepsilon_0}} = \sqrt{\frac{2(1.13\times10^{-5}\text{ W/m}^2)}{(3.0\times10^{8}\text{ m/s})(8.85\times10^{-12}\text{ C}^2\text{/N m}^2)}} = 0.092\ \frac{\text{V}}{\text{m}}$$

A few steps before 42 m, the field strength was 0.100 V/m and the door opened. The manufacturer's claims are correct.

34.63. **Model:** Use Ampere's law and assume that the current through the resistor is uniform.
Visualize:



Solve:
(a) The electric field E in the resistor is

$$E = \frac{\Delta V_R}{L} = \frac{IR}{L} = E_{\text{surface}}$$

For the magnetic field, we use the Ampere-Maxwell law:

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{\text{through}} = \mu_0 I \Rightarrow B(2\pi r) = \mu_0 I \Rightarrow B = \frac{\mu_0 I}{2\pi r} = B_{\text{surface}}$$

(b) The Poynting vector $\vec{S} = \mu_0^{-1}\vec{E}\times\vec{B}$ is directed into the curved surface. That is, into the wire. The magnitude is

$$S = \frac{1}{\mu_0}EB = \frac{1}{\mu_0}\frac{IR}{L}\frac{\mu_0 I}{2\pi r} = \frac{I^2 R}{2\pi r L}$$

(c) The flux over the surface of the resistor is

$$\int \vec{S}\cdot d\vec{A} = \int_{\text{faces}} \vec{S}\cdot d\vec{A} + \int_{\text{wall}} \vec{S}\cdot d\vec{A} = 0 - SA_{\text{wall}} = \frac{I^2 R}{2\pi r L}(2\pi r L) = -I^2 R$$

The Poynting vector is the power per unit area carried by electric and magnetic fields. Thus the Poynting flux (SA_{wall}) is electromagnetic power directed into the resistor. It matches the power dissipated by the resistor $(I^2 R),$ which is what we expect from energy conservation.

35

AC CIRCUITS

35.15. **Model:** The current and voltage of a resistor are in phase, but the capacitor current leads the capacitor voltage by 90°.

Visualize: Please refer to Figure EX35.15.

Solve: From Equation 35.14, the peak voltages are $V_R = IR$ and $V_C = IX_C$, where

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi fC} = \frac{1}{2\pi(1.0\times10^4\text{ Hz})(80\times10^{-9}\text{ F})} = 199\,\Omega$$

The peak current is

$$I = \frac{\mathcal{E}_0}{\sqrt{X_C^2 + R^2}} = \frac{10\text{ V}}{\sqrt{(199\,\Omega)^2 + (150\,\Omega)^2}} = 0.0401\text{ A}$$

Thus, $V_R = (0.0401\text{ A})(150\,\Omega) = 6.0\text{ V}$ and $V_C = IX_C = (0.0401\text{ A})(199\,\Omega) = 8.0\text{ V}$.

35.37. **Visualize:** Please refer to Figure P35.37.

Solve:

(a) The voltage across the capacitor is

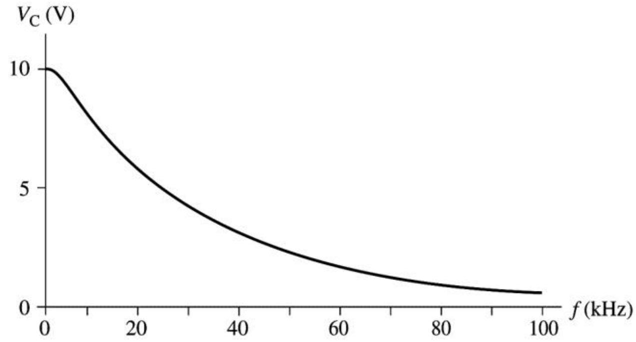
$$V_C = IX_C = \frac{\mathcal{E}_0}{\sqrt{R^2 + X_C^2}}X_C = \frac{\mathcal{E}_0/(\omega C)}{\sqrt{R^2 + (\omega C)^{-2}}} = \frac{\mathcal{E}_0}{\sqrt{(\omega RC)^2 + 1}}$$

$$= \frac{10\text{ V}}{\sqrt{4\pi^2 f^2(16\,\Omega)^2(1.0\times10^{-6}\text{ F})^2 + 1}} = \frac{10\text{ V}}{\sqrt{1 + (1.0106\times10^{-8}\text{ s}^2)f^2}}$$

The values of V_C at a few frequencies are in the following table.

f (kHz)	V_C (V)
1	9.95
3	9.57
10	7.05
30	3.15
100	0.990

(b)



Assess: For the voltage across the capacitor, the circuit is a low-pass filter.