UNIVERSITY EXAMINATIONS

UNIVERSITEITSEKSAMENS

UNISA university of south africa

MAT3705

May/June 2015

COMPLEX ANALYSIS

Duration 2 Hours

100 Marks

EXAMINERS .
FIRST DR L LINDEBOOM
EXTERNAL DR FES BULLOCK

Closed book examination.

This paper consists of 3 pages

ANSWER ALL THE QUESTIONS.

QUESTION 1

1 1 If $|z| = 1$ show that

$$\left|\frac{z+2}{2z+1}\right| = 1 \tag{6}$$

1 2 Determine and sketch the region

$$\{z \in \mathbb{C} \,|\mathrm{Re}(\,|z|^2 + 2z) \leq 3\}$$

State giving reasons ,whether the region is open, closed, or neither open nor closed (7)

[13]

QUESTION 2

2 1 Find all values of $z \in \mathbb{C}$ for which $\sinh z = \sqrt{2}$ (8)

2 2 Find all the points of $z \in \mathbb{C}$ for which $\mathrm{Re}\, e^{iz} = 0$ (6)

[14]

QUESTION 3

Let $z = x + iy$ ($x \in \mathbb{R}$, $y \in \mathbb{R}$)and define $f(z)$ as

$$f(z) = (xy^2 - 3) + i(-4y - 2xy + 3)$$

3 1 Determine where $f(z)$ is differentiable (8)

3 2 Is g analytic at any point of $\mathbb{C}$? Justify your answer (2)

[10]

QUESTION 4

4 1 Use Cauchy's intergral formula (or its extensions) to compute the following integrals (each contour is positively oriented)

4 1 1 $\int_{|z|=1} \frac{z}{4z+1} dz$ (4)

4 1 2 $\int_{|z|=3} \frac{e^{3z}}{(z+2)^4} dz$ (6)

4 2 Suppose that $f(z)$ is analytic on $\mathbb{C}$, $f(0) = 1 + i$ and $|f(z)| \leq \sqrt{2}$ if $|z| < 1$ What is the value of $f'(0)$? (6)

[16]

QUESTION 5

5 1 Compute the Laurent expansion of the following functions in the given regions Give at least four non-zero terms

5 1 1 $f(z) = \frac{1}{(z+2)(z-3)^3}$ for $0 < |z-3| < 5$ (7)

5 1 2 $g(z) = \frac{z-1}{z(z-3)}$ for $|z| > 3$ (7)

[14]

QUESTION 6

6 1 Find all the isolated singularities of the function

$$f(z) = \frac{z^2 - 1}{z^2 - 5iz - 4}$$

and compute the residues at these points (6)

6 2 Now use the results in part (6 1) to write down the value of

$$\int_C \frac{z^2 - 1}{z^2 - 5iz - 4} dz$$

for each of the following positively oriented contours

6 2 1 $C \quad |z| = 6$ (3)

6 2 2 $C \quad |z - i| = 1$ (4)

6 3 Give an example of a function with a pole of order 3 at i, and a removable singularity at $-i$ Justify all claims (5)

[18]

QUESTION 7

7 1 Show that

$$\int_{-\pi}^{\pi} \frac{\cos\theta + 1}{10 + 6\cos\theta} d\theta = \frac{1}{2i} \int_{|z|=1} \frac{z^2 + 2z + 1}{z(3z+1)(z+3)} dz$$

(6)

7 2 Now use the method of residues to evaluate

$$\int_{-\pi}^{\pi} \frac{\cos\theta + 1}{10 + 6\cos\theta} d\theta \quad (9)$$

[15]

TOTAL:[100]