UNIVERSITY EXAMINATIONS

UNIVERSITEITSEKSAMENS

MAT3700

October/November 2013

MATHEMATICS III (ENGINEERING)

Duration : 2 Hours

84 Marks

EXAMINATION PANEL AS APPOINTED BY THE DEPARTMENT.

Use of a non-programmable pocket calculator is permissible.

Closed book examination.

This examination question paper remains the property of the University of South Africa and may not be removed from the examination venue.

This examination question paper consists of 3 pages including this cover page plus

Formulae sheets (page (i) to (v), plus
A table of integrals (page (i) and (ii)) plus
A table of Laplace transforms (page iii).

Answer all the questions **in numerical order.**

Open Rubric

QUESTION 1

Solve the following differential equations:

1.1 $2x(y+1)dx - ydy = 0$, given that if $x = 0$ then $y = -2$. (6)

1.2 $x\frac{dy}{dx} - y = x\tan\left(\frac{y}{x}\right)$ [Hint: Put $y = vx$] (5)

1.3 $\frac{dy}{dx} - 2y\tan x = y^2\tan^2 x$ (6)

[17]

QUESTION 2

Find the general solutions of the following differential equations using **D-operator** methods:

2.1 $(D^2 - 9)y = 3e^x + x - \sin 4x$ (7)

2.2 $(D^2 - 4D + 4)y = e^{2x}(x^2 + 1)$ (7)

[14]

QUESTION 3

The conditions in a certain electrical circuit are represented by the following differential equation: $10\frac{d^2i}{dt^2} + 60\frac{di}{dt} + \frac{i}{0{,}004} = 240\cos 5t$. Determine the general and the steady state solutions for the current, *i* in terms of *t,* by using **D-operator methods.** (8)

[8]

QUESTION 4

Solve for **only** *x* in the following set of simultaneous differential equations by using **D-operator** methods:

$$(D^2 - 1)y + 5Dx = t$$
$$2Dy - (D^2 - 4)x = 2$$

(8)

[8]

QUESTION 5

Determine the following:

5.1 $L\left\{2t^4 + H(t-3) + 4\cos 5t\right\}$ (3)

5.2 $L^{-1}\left\{\frac{2}{(s+2)^3}\right\}$ (3)

[6]

[TURN OVER]

QUESTION 6

Determine the unique solution of the following differential equation by using

Laplace transforms: $y''(t)+6y'(t)+13y(t)=0$, if $y(0)=3$ and $y'(0)=7$. (7)

[7]

QUESTION 7

The motion of a mass-spring system, with no friction, is given by the equation

$$\frac{d^2y}{dt^2}+y=2\delta(t-2\pi)-2\delta(t-4\pi)$$

with an impulsive force at $t=2\pi$ and an equal and opposite force at $t=4\pi$.
If $y(0)=0$ and $y'(0)=0$, use Laplace transform methods to solve for *y*. (8)

[8]

QUESTION 8

If $A=\begin{bmatrix}2 & 2\\ -1 & 5\end{bmatrix}$, determine the eigenvalues of *A* and an eigenvector for *A.* (6)

[6]

QUESTION 9

Given the function defined by $f(x)=x, 0\le x\le 2$, find the half-range Fourier sine series for *f(x)*. Sketch the function within and outside of this range. (10)

[10]

Full marks = 84

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