UNIVERSITY EXAMINATIONS

UNIVERSITEITSEKSAMENS

MAT3700

May/June 2014

MATHEMATICS III (ENGINEERING)

Duration : 2 Hours

84 Marks

EXAMINATION PANEL AS APPOINTED BY THE DEPARTMENT.

Use of a non-programmable pocket calculator is permissible.

Closed book examination.

This examination question paper remains the property of the University of South Africa and may not be removed from the examination venue.

This examination question paper consists of 3 pages including this cover page plus

Formulae sheets (page (i) to (v), plus
A table of integrals (page (i) and (ii)) plus
A table of Laplace transforms (page iii).

Please answer all the questions **in numerical order** as far as possible.

QUESTION 1

Solve the following differential equations:

1.1 $\dfrac{dy}{dx}=\dfrac{y^2}{x^2+9}$ (4)

1.2 $\left(x^2-y^2\right)dy=xydx$ [Hint: Put $y=vx$] (7)

1.3 $\dfrac{dy}{dx}+xy=xe^{-x^2}y^{-3}$ (7)

[18]

QUESTION 2

Find the general solutions of the following differential equations using **D-operator** methods:

2.1 $D\left(D^2-1\right)y=7$ (6)

2.2 $\left(D^2-6D+9\right)y=x^3e^{3x}$ (6)

[12]

QUESTION 3

The conditions in a certain electrical circuit are represented by the following differential equation: $\dfrac{d^2i}{dt^2}+2\dfrac{di}{dt}+2i=85\sin 3t$. Determine the general solution for the current, *i* ,in terms of *t,* by using **D-operator methods**, given that for t = 0, *i* = 0 and $\dfrac{di}{dt}=-20$. (9)

[9]

QUESTION 4

Solve for ***y*** in the following set of simultaneous differential equations by using **D-operator** methods:

$$(4D+3)x-Dy=\sin t$$
$$Dx+y=\cos t$$
(8)

[8]

QUESTION 5

5.1 A function f(t) is defined by $f(t)=\begin{cases} t & 0\le t\le 2 \\ 2 & 2\le t\end{cases}$.

Write f(t) in terms of the Heaviside unit step function and determine the Laplace transform of f(t). (4)

5.2 Determine $L^{-1}\left\{\dfrac{6}{s^2-4s+13}\right\}$ (3)

[7]

QUESTION 6

Determine the unique solution of the following differential equation by using **Laplace transforms:** $y''(t)-4y'(t)+3y(t)=e^{2t}$, if $y(0)=0$ and $y'(0)=1$.

(7)

[7]

QUESTION 7

A vehicle rests on a spring-shock absorber system on each of four wheels. The system yields the model $y''(t)+2y'(t)+17y(t)=e^{-t}\cos 4t$, where $e^{-t}\cos 4t$ represents the force resulting from a bumpy road. If $y(0)=0$ and $y'(0)=0$, use **Laplace transforms** to solve the differential equation**.** (7)

[7]

QUESTION 8

If $A=\begin{bmatrix}3 & 2 & 2\\ 2 & 2 & 0\\ 2 & 0 & 4\end{bmatrix}$, find an eigenvector corresponding to the eigenvalue $\lambda=0$.

Also verify that 0 is an eigenvalue of *A.* (6)

[6]

QUESTION 9

Given the function defined by $f(t)=\pi-t,\ 0<t<\pi$, write down the Fourier half range cosine series expansion for *f(t).* (10)

[10]

Full marks = 84

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