MAT1510

PRECALCULUS MATHS 1

Oct-Nov 2016

QUESTION 1

Given, f(x) = x + √x + 1 g(x) = |2x + 1| h(x) = log

3

3 1.1 |−2x − 3| ≤ 1 D

f

x − 2log

x

and D

h D

f

, D

g : x + 1 ≥ 0 (in the realm of real numbers, only square roots of non-negative numbers exist)
⇒ x ≥ −1 D

g

: there are restrictions to the values of x, all values of x are valid

D

h

: using the change of base fomula we can write h(x) = log

3

2 log

3

3 log

3

⇒ h(x) = log

3

x −

2 log

3

x −

x

x

, so we have the following restrictions: $x > 0$ and log

3

$x \neq 0 \Rightarrow x \neq 30$ so $x \neq 1$

D

f

:$x \in R, x\ x \geq -1$ D

g

:$x \in R$, D

h

:$x \in R,\ x > 0,\ x \neq 1$

1.2 $f(x) = 5$, $x + \sqrt{x + 1} = 5$

$\sqrt{x + 1} = 5 - x$ Squaring both sides

$(\sqrt{x + 1})$

2

$= (5 - x)2$

$x + 1 = (5 - x)(5 - x)$

$x + 1 = 25 - 10x + x2$

$x2 - 11x + 24 = 0$ $(x - 8)(x - 3) = 0$ Solution set: either $x = 8$ or $x = 3$ [Checking: forx = 8, LHS $8 + \sqrt{9} = 11$ RHS = 5 LHS ≠ RHS. for x = 3, LHS = $3 + \sqrt{4} = 5$ RHS = 5 LHS = RHS]

Solution set:x = 3

1.3 $g(x) \geq 1$, $|2x + 1| \geq 1$

If $2x + 1$ is greater than zero, then $|(2x + 1)| = 2x + 1$, hence

$2x + 1 \geq 1$

$2x \geq 0$

$x \geq 0$

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If 2x + 1 is less than zero, then $|(2x + 1)| = -(2x + 1)$, hence

$-(2x + 1) \geq 1$

$-2x - 1 \geq 1$

$-2x \geq 2$

$x \leq -1$

Solution set: $x \in R, x \leq -1,\ x \geq 0$

1.4 h(x) = 1 applying the change of base formula on the original function we have

h(x) = log

3

x −

log

2

3

x We need to solve the equation log

3

x −

log 2

3

= 1

Set log

3

x x = χ , so we now have

χ −

χ 2

= 1

$\chi 2 - 2 = \chi$

$\chi 2 - \chi - 2 = 0$

$(\chi - 2)(\chi + 1) = 0$

Either $\chi = 2$ $\chi = -1$ If $\chi = 2$ then log

3

$x = 2 \Rightarrow 32 = x$

$x = 9$

If $\chi = -1$ then log

3

x = −1 ⇒ 3−1 = x

x =

1 3 Solution set: x = 9,x =

1 3

Question 2 Using a =

5 9

(b – 32) Making b the subject

a 9

=

5 9

(b – 32)

5 b a = = 9 5

b a − + 32 32

If a = 25, then b =

5 9

25 + 32 = 45 + 32 = 77

If a = 40, then b =

9 5

40 + 32 = 72 + 32 = 104 Therefore the temperatures range from 77 °F to 104 °F

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Question 3

3.1 Given y = f(x) = a(x − 1)2 + k and y = g(x) = bx + c From the description we establish that we know two points on each graph as follows

f(x) g(x) Point 1 -1,15 0,0 Point 2 2,3 2,3

Using point 1 on f(x)

15 = a(−1 − 1)2 + k 15 = 4a + k Using point 2 on f(x)

3 = a(2 − 1)2 + k 3 = a + k Solving these two equations simultaneously we have

15 = 4a + k 3 = a + k Subtracting equation 2 from equation 1 we get 12 = 3a Therefore a = 4 Substituting a we get 3 = 4 + k ⇒ k = −1 Therefore f(x) = 4(x − 1)2 − 1

3.2 A and B are the roots (the x-intercepts) determined by f(x) = 0

4(x − 1)2 − 1 = 0

4(x − 1)2 = 1

(x − 1)2 =

1 4

x − 1 = ±

1 2

x = ±

1 2

+ 1

x =

1 2

3 2 Coordinates A(

or

1 2

,0) ; B(

3 2

,0) 3.3 Using point 1 on g(x)

0 = b0 + c 0 = 1 + c c = −1 Using point 2 on g(x)

y = bx + c 3 = b2 − 1 b2 = 4

b = 2 y = 2x − 1 3.4 Paper not Clear

3.5 Given f(x) · g(x) ≥ 0 The product of two numbers is greater than or equal to zero, if either one of the numbers is zero, or both numbers are greater than zero (positive) or both numbers are less than zero (negative).

So on graph we look for a range where one of the following is satisfied:

(i) one of the graphs is zero (ii) both graphs are above the x-axis (iii) both graphs are below the x-axis Solution set x ∈ R,0 ≤ x ≤

1 2

3 2

3.6 (a)

y = 2x − 1 Swop x and y

x = 2y − 1 2y = x + 1 Taking log

2

,x ≥

both sides we have

g−1 = y = log

2

(x + 1)

(b) log0 is undefined so x + 1 > 0 ⇒ x > −1

D

g

:x ∈ R, x > −1

(c) g ∘ g−1 = 2log

2

−1

(x+1) − 1 = x + 1 − 1 = x

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