

MAT1510

October/November 2012

PRECALCULUS MATHEMATICS A

Duration 2 Hours

100 Marks

 EXAMINERS
 FIRST
 SECOND

 MRS NH PENZHORN
 MRS JC BEDEKER

Closed book examination

This examination question paper remains the property of the University of South Africa and may not be removed from the examination venue

This paper consists of 6 pages

THE USE OF A POCKET CALCULATOR IS NOT PERMITTED.**ANSWER ALL THE QUESTIONS.****QUESTION 1**Suppose the functions f and g are defined by

$$f(x) = \frac{1}{|x + 2|}$$

and

$$g(x) = x - \sqrt{\frac{3}{2}x + 10} + 2$$

1 1 Write down the sets D_f and D_g that represent the domains of f and g respectively (2)1 2 Solve the inequality $f(x) > 3$ for $x \in D_f$ (5)1 3 Solve the equation $g(x) = 0$ for $x \in D_g$ (8)**[15]****QUESTION 2**The function h models the height of a rocket in terms of time The equation of the function

$$h(t) = 250t - 5t^2$$

gives the height $h(t)$ of the rocket after t seconds, where $h(t)$ is in metres**[TURN OVER]**

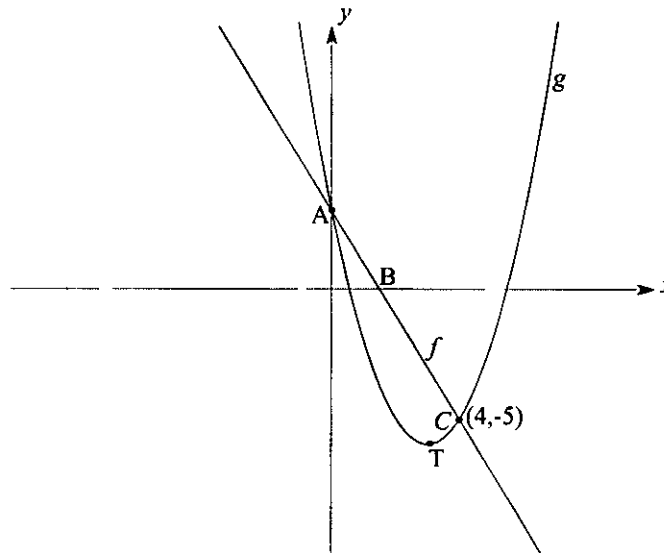
2.1 Use the method of completing the square to write the equation of h in the form

$$h(t) = a(t - h)^2 + k \quad (4)$$

2.2 Use the form of the equation in 2.1 to answer the following questions

- (a) After how many seconds will the rocket reach its maximum height?
 (b) What is the maximum height reached by the rocket?

(2)

[6]**QUESTION 3**

The sketch shows the graphs of functions f and g where f is defined by

$$y = f(x) = mx + k$$

and g is defined by

$$y = g(x) = x^2 - 6x + 3$$

The graphs of f and g intersect at points A and $C(4, -5)$. Point A lies on the y -axis. T is the turning point of the graph of g . B is a point on the x -axis.

- 3.1 Determine the coordinates of point T as well as point A (3)
 3.2 Determine the x -intercepts of the graph of g . Leave the answer in surd form (4)
 3.3 Find the values of m and k and then write down the equation of f (4)
 3.4 Suppose the function d models the vertical distance between the graphs of f and g on the interval $[0, 4]$. Find an equation that defines the function d in terms of x for $x \in [0, 4]$ (2)

[TURN OVER]

3.5 Suppose x_B is the value of the x -intercept of the graph of f . The graph of f cuts the x -axis at the point $B(x_B, 0)$.

Consider the interval $(x_B, 4)$ on the x -axis. Use the graphs to answer the following.

Is the inequality $f(x) > g(x)$ true for $x \in (x_B, 4)$?

Give a reason for your answer. (3)

[16]

QUESTION 4

Suppose the variable t represents time in years, n_0 is the number of people in a certain population when $t = 0$ ($n_0 > 0$), and the population increases according to the formula

$$n(t) = n_0 e^{rt},$$

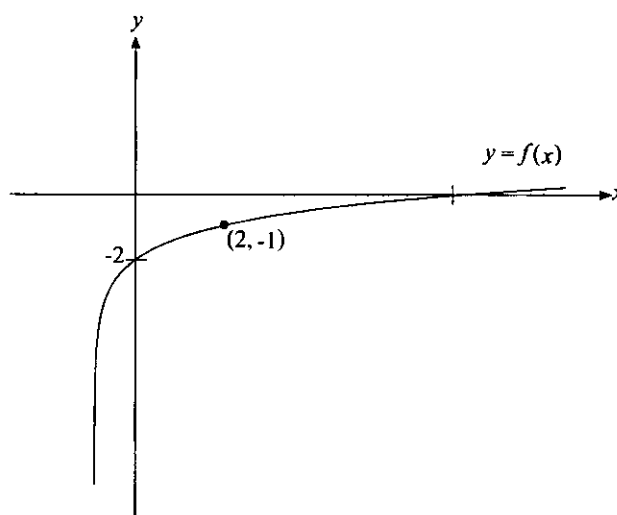
where $n(t)$ is the number of people at time t .

4.1 If it takes 40 years for the population to double, show that $r = \frac{1}{120} \ln 8$. (5)

4.2 Determine the number of people, that is, the size of the population, after 120 years. Give the answer in terms of n_0 . (3)

[8]

QUESTION 5



The sketch shows the graph of function f where f is defined by

$$y = f(x) = \log_a(x+1) + b; \quad a > 0 \text{ and } a \neq 1$$

5.1 Determine the values of a and b and then write down the equation of f . (7)

[TURN OVER]

- 5.2 Write down the equation of the vertical asymptote of the graph of f , as well as the domain and the range of f , that is, D_f and R_f , respectively (3)
- 5.3 Use the graph (not the algebraic expression of $f(x)$) to solve the inequality $f(x) < 0$ (5)
- 5.4 Determine the equation of the inverse function f^{-1} (3)
- 5.5 Show that $(f^{-1} \circ f)(x) = x$ for all $x \in D_f$. (3)

[21]

QUESTION 6

Solve each of the following

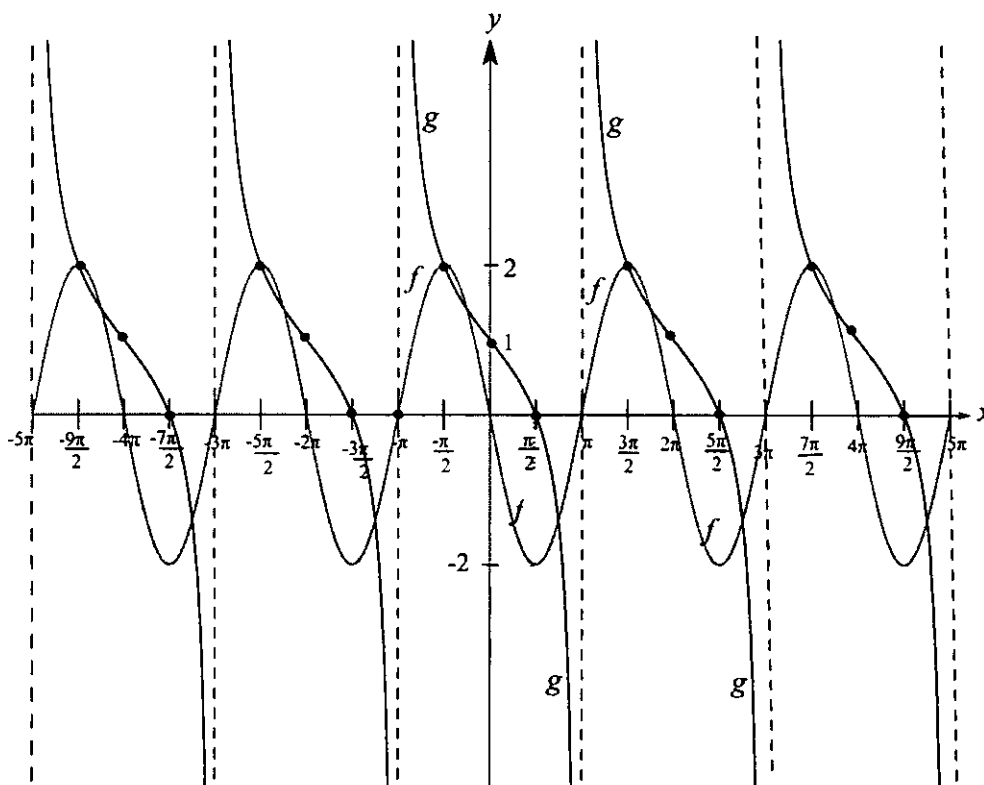
6.1 $\log_2 x^3 - \log_{\frac{1}{2}} x > 8$ (5)

6.2 $2 \cos^2 x + 4 \cos x = 1 + 2 \sin^2 x$ for $x \in [0, 2\pi)$

(Hint: Write $\sin^2 x$ in terms of $\cos x$) (7)

[12]

QUESTION 7



[TURN OVER]

The sketch shows the graphs of the functions f and g where f is defined by

$$y = f(x) = a \cos k(x - b), \quad k > 0,$$

and g is defined by

$$y = g(x) = \cot p(x - r) + s, \quad p > 0$$

Use the graphs of f and g to determine the following

7.1 For the function f , determine

- (a) the amplitude of the graph,
- (b) the period of f ,
- (c) the value of k ,
- (d) the value of b , and then
- (e) write down the equation of f (5)

7.2 For the function g , determine

- (a) the period of g ,
- (b) the value of p ,
- (c) the value of r ,
- (d) the value of s , and then
- (e) write down the equation of g (5)

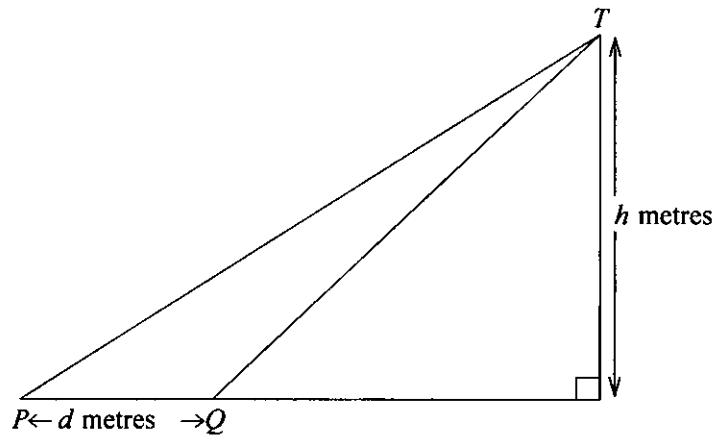
7.3 Use the Addition Formula for Sine to show that $4 \sin 2 \left(x + \frac{\pi}{6} \right) = 2 \left(\sin 2x + \sqrt{3} \cos 2x \right)$ (3)

[13]

QUESTION 8

Consider the following diagram. P and Q represent the positions of two observers on the same horizontal level as the foot of a mountain. T represents the highest peak of the mountain. The observers are in the same vertical plane as T . The altitude of T is h metres and the distance from P to Q is d metres. The angle of depression of P from position T is β , and the angle of elevation of T from position Q is α .

[TURN OVER]



8.1 Copy the diagram in your exam script. Show the angles α and β on your diagram. (2)

8.2 Use the **Law of Sines** to show that

$$d = \frac{h \sin(\alpha - \beta)}{\sin \alpha \sin \beta} \quad (7)$$

[9]

TOTAL: [100]