UNIVERSITY EXAMINATIONS

UNIVERSITEITSEKSAMENS

UNISA university of south africa

MAT1510

October/November 2011

PRECALCULUS MATHEMATICS A

Duration . 2 Hours

100 Marks

EXAMINERS :
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SECOND MRS JC BEDEKER

This paper consists of 5 pages

THE USE OF A POCKET CALCULATOR IS NOT PERMITTED.
Answer ALL the questions.

QUESTION 1

The functions h and l are defined by

$$h(x) = \frac{3}{|3-x|}$$

and

$$l(x) = x - \sqrt{x} - 2.$$

1 1 Write down D_h (the domain of h) and solve the inequality $h(x) \geq 1$ (7)

1 2 Write down D_l (the domain of l) and solve the equation $l(x) = 0$. (7)

[14]

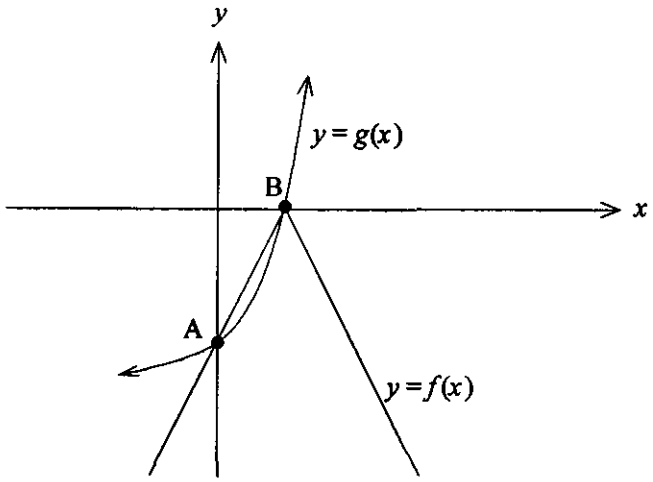
QUESTION 2

If they work together, Mandla and George can paint a room in three–fifths of the time that it takes Mandla to paint it by himself George takes 5 hours to paint the room by himself Determine how long it takes Mandla to paint the room by himself

[8]

[TURN OVER]

QUESTION 3



The sketch shows the graphs of the functions f and g defined by

$$y = f(x) = -3|x - 1|$$

and

$$y = g(x) = b^x + k, \text{ where } b > 0 \text{ and } b \neq 1$$

3.1 Determine the coordinates of A and B. (4)

3 2 Use the coordinates of A and B to determine the values of b and k, and then write down the equation of g. (5)

3 3 (a) Give the equation of the horizontal asymptote of the graph of g as well as the sets D_g and R_g (the domain and the range of g, respectively) (Answer this question in terms of b or k if you could not solve for b or k in 3.2.)

(b) Suppose the graph of the function l is obtained by

- shifting the graph of f horizontally 2 units to the left
- then shifting the resulting graph vertically 3 units upwards
- and then shrinking the resulting graph vertically by a factor $\frac{1}{3}$.

Give the equation of l in the form

$$y = l(x) \ldots .$$ (6)

3 4 Answer the following (leave your answers in terms of b and/or k if you were unable to find the numerical values of b and/or k in 3.2):

(a) Why is the function g one-to-one?

(b) Determine the equation that defines the function g^{-1}.

(c) Show that $\left(g \circ g^{-1}\right)(x) = x$ for all $x \in D_{g^{-1}}$. (8)

3.5 (a) Suppose the graph of the funtion h is a parabola that has vertex A and that passes through B. Give the equation of h in the form

$$y = h(x) = a(x - p)^2 + q. \quad (3)$$

(b) For a specific value of x the quadratic function h has a maximum or a minimum value. Find this specific value of x, as well as the corresponding value of the function h. State whether, in this case, the value of h that you determined is a maximum or a minimum, and give a reason for your answer (2)

[28]

QUESTION 4

Suppose the variable t represents time in years, n_0 is the number of people in a certain population when $t = 0$, and the population increases according to the formula

$$n(t) = n_0 e^{rt},$$

where $n(t)$ is the number of people at time t and $r = \frac{1}{50} \ln 2$.
Determine the doubling time of this population, that is, the time it takes for the population to double. **[6]**

QUESTION 5

Solve each of the following equations

5 1 $\log_{\frac{1}{3}}(x - 1) + \log_{\frac{1}{3}} x = \log_{\frac{1}{3}} 2$

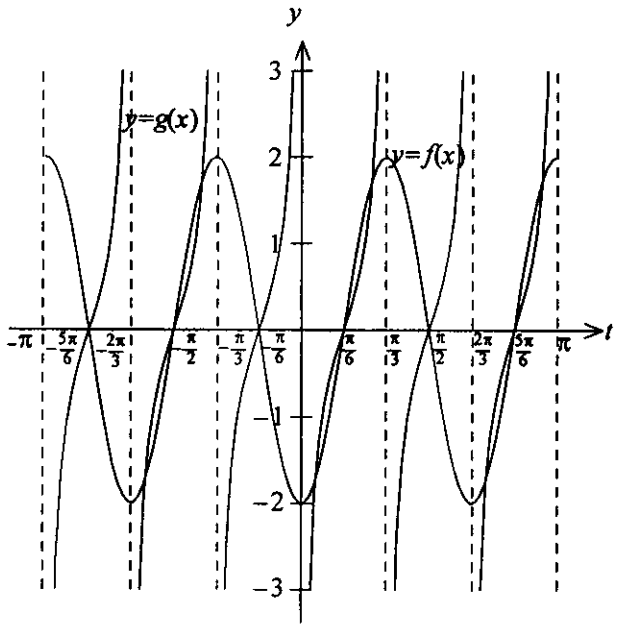
[Hint First make sure that you know for which values of x this equation is defined.] (6)

5 2 $\sin^2 t - \cos^2 t + \sin t = 0$ for $t \in [-2\pi, 2\pi)$ (8)

[14]

QUESTION 6

The graphs of f and g are sketched below, on the interval $[-\pi, \pi]$.



The function f is defined by

$$y = a\cos k(t-b) \quad \text{or} \quad y = a\sin k(t-b), \text{ where } k > 0 \text{ and } b \geq 0.$$

The function g is defined by

$$y = \tan c(t-d), \text{ where } c > 0 \text{ and } d > 0$$

Use the graphs of f and g to answer the following questions

6.1 (a) What is the amplitude of the graph of f?

(b) What is the period of the function f?

(c) Determine the values of a, k and b, and hence write down the equation of f (Remember the conditions given above, $k > 0$ and $b \geq 0$.) (6)

6.2 (a) What is the period of the function g?

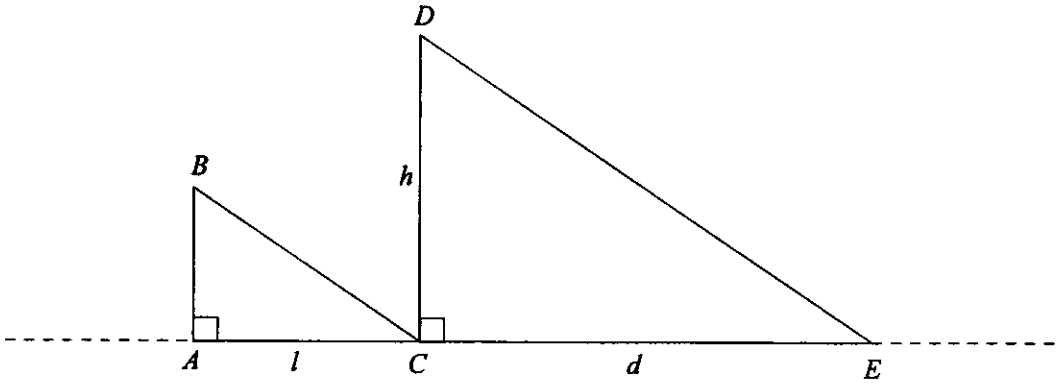
(b) Determine the values of c and d, and hence write down the equation of g (Remember the conditions given above, $c > 0$ and $d > 0$) (4)

6.3 **Use the graphs of f and g (not** the trigonometric expressions representing $f(t)$ and $g(t)$) to solve the inequality $f(t)\,g(t) < 0$ for $t \in (-\pi, 0)$ (4)

[14]

QUESTION 7

In the following diagram AB represents a tree whose shadow at a given time of day reaches the foot of a building CD At the same time of day the shadow of the building reaches the point E.



The angle of elevation of the top of the building from the top of the tree is α. The angle of depression of the bottom of the building from the top of the tree is β Suppose the shadow lengths (in metres) of the tree and the building are l and d respectively, and the height (in metres) of the building is h, as shown in the sketch

7 1 Copy the diagram in your script Complete the triangle ΔBCD. Draw a short horizontal line through B and fill in the angles α and β on your diagram (3)

7 2 **Use the Law of Sines at least once** to show that the length of the building's shadow can be expressed in terms of the length of the tree's shadow by

$$d = \frac{l \sin(\alpha + \beta) \cot \beta}{\cos \alpha \cos \beta}.$$

Hint: Since the sun creates the shadow cast by both the tree and the building, BC is parallel to DE (7)

7 3 Prove the identity

$$\frac{\sin(\alpha + \beta) \cot \beta}{\cos \alpha \cos \beta} = \tan \alpha \cot \beta + 1 \quad (3)$$

7.4 Use the result in 7 2 or 7.3 to show that $d = 2l$ if $\alpha = \beta = 45°$ (3)

[16]

TOTAL: [100]