UNISA university of south africa

MAT1510

October/November 2010

PRECALCULUS MATHEMATICS A

Duration 2 Hours

100 Marks

EXAMINERS
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This paper consists of 6 pages

THE USE OF A POCKET CALCULATOR IS NOT PERMITTED.

ANSWER ALL THE QUESTIONS.

QUESTION 1

The functions f, g and h are defined by

$$f(x) = x - \sqrt{\frac{3x-1}{2}} - 1.$$
$$g(x) = \frac{1}{x-4}$$

and

$$h(x) = \frac{3}{x}$$

respectively

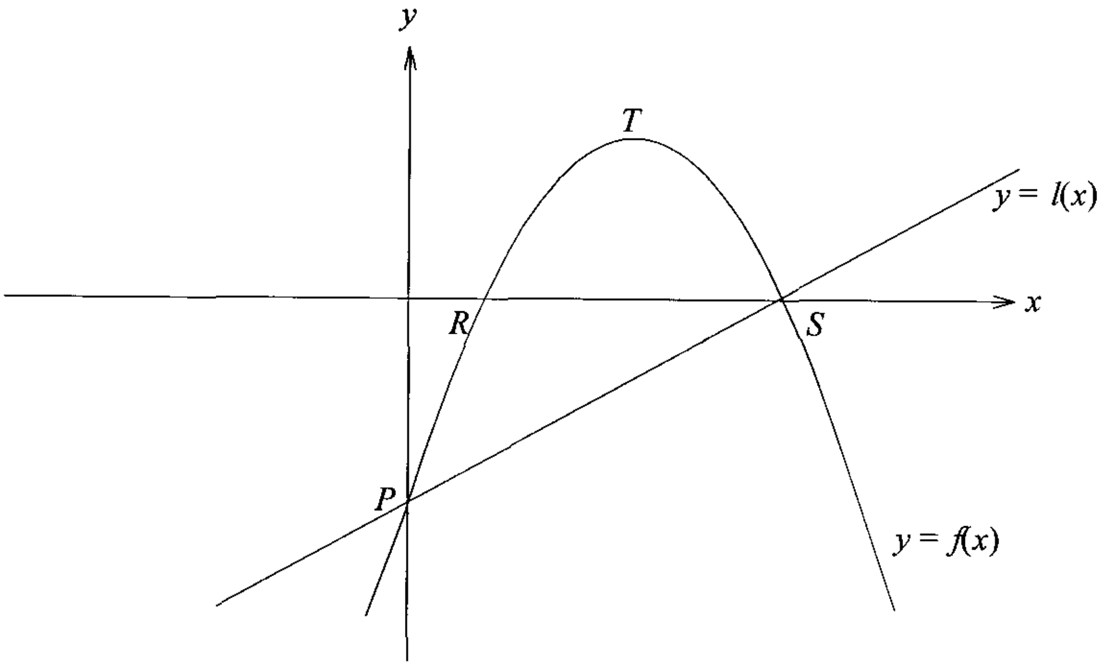
1 1 Write down the sets D_f, D_g, D_h and D_{g-h} that represent the domains of f, g, h and $g - h$ respectively (4)

1 2 Solve the equation $f(x) = 0$ for $x \in D_f$ (7)

1 3 Solve the inequality $(g - h)(x) \leq 0$ (that is $g(x) - h(x) \leq 0$) for $x \in D_{g-h}$ (7)

[18]

QUESTION 2



The sketch shows the graphs of the functions f and l where f is defined by

$$y = f(x) = -(x-3)^2 + 4$$

The graph of f is a parabola with turning point (vertex) T, x-intercepts at the points R and S, and y-intercept at P. The graph of l is a straight line which passes through the points P and S.

2.1 Write down the coordinates of the points T, P, R and S (5)

2.2 (a) Restrict the domain D_f of the function f to D_{f_r}, such that f_r defined by

$$f_r(x) = f(x) \qquad \text{for } x \in D_{f_r},$$

is a one-to-one function. Write down D_{f_r} and R_{f_r}

(b) Determine the equation of the inverse function f_r^{-1} (6)

2.3 Determine the equation of the straight line l (3)

2.4 Suppose the point S has coordinates (a, b) and the function d models the vertical distance between the graphs of f and l. Find an equation that defines the function d in terms of x for $0 \leq x \leq a$, and then determine the maximum vertical distance between the graphs of f and l on the interval $[0, a]$ (5)

[19]

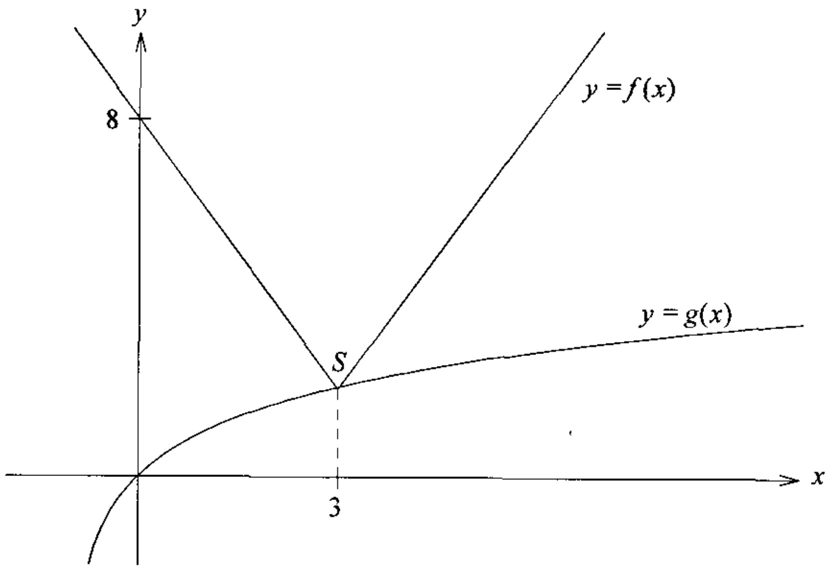
QUESTION 3

3 1 A certain radioactive substance has a half-life of 140 days. Suppose a sample of this substance has a mass of 320 mg Use the equation $m(t) = m_0 e^{-rt}$ to find the equation of the function that models the amount of the sample remaining at time t, in this specific case (4)

3 2 Use the equation of the function determined in 3 1 to find the mass remaining after one year and 55 days (5)

[9]

QUESTION 4



The sketch shows the graphs of the functions f and g The function f is defined by

$$f(x) = a|x - b| + 2$$

and the function g is defined by

$$g(x) = \log_p (x - r)$$

where $p > 0,\ p \neq 1$
S is the salient point of the graph of f, and the graph of g passes through the origin and S

4 1 Determine the values of a and b, and hence write down the equation of f (4)

4 2 Find the values of p and r, and hence write down the equation of g (6)

4 3 Write down the set D_g (the domain of the function g) Then, **use the graphs** (**not** the algebraic expressions for $f(x)$ and $g(x)$) to solve the inequality $g(x) < f(x)$ (3)

[TURN OVER]

4 4 Determine the equation of the inverse function g^{-1}

[If you were not able to find the values of p and r in 4 2, you may answer the question in terms of p and r] (3)

4 5 Show that $\left(g^{-1} \circ g\right)(x) = x$. Include all the steps of your reasoning (3)

[19]

QUESTION 5

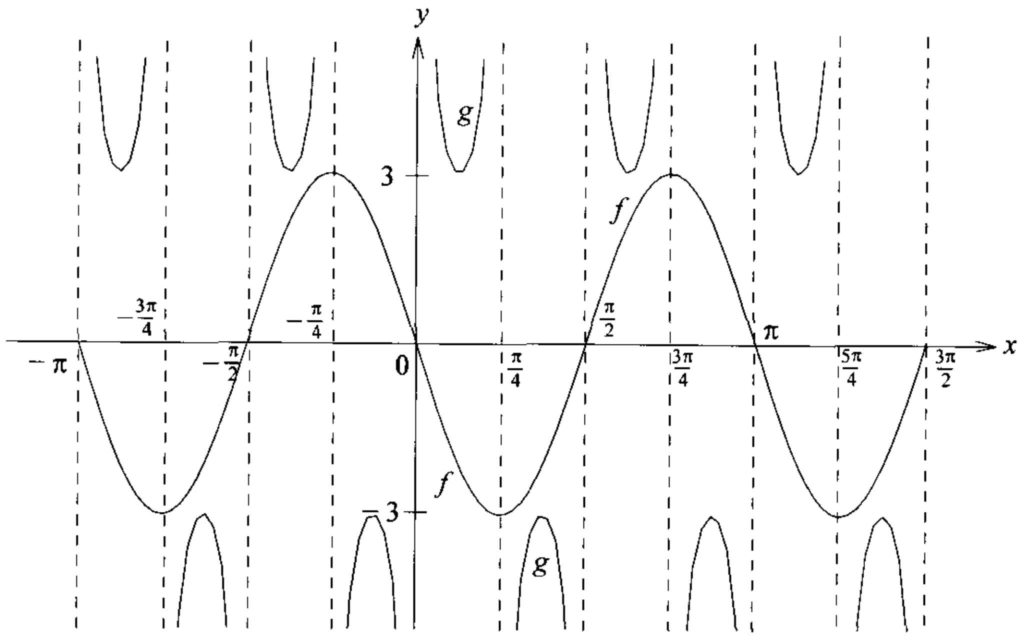
5 1 Solve the equation $\tan 2x \cos x - \cos x = \frac{1}{2} - \frac{1}{2} \tan 2x$ for $x \in [0, \pi)$

[**Hint Do not** write $\tan 2x$ in terms of $\sin 2x$, $\cos 2x$ or $\tan x$] (7)

5 2 Solve the equation $e^{x} - 15e^{-x} + 2 = 0$ Leave the answer in logarithmic form (5)

[12]

QUESTION 6



The sketch shows the graphs of the functions f and g The function f is defined by

$$y = a \cos k(x - b), \quad k > 0$$

and the function g is defined by

$$y = m \csc p(x - n), \quad p > 0$$

Use the graphs of f and g to answer the following

6 1 For the function f determine

(a) the amplitude $|a|$,

(b) the period,

(c) the value of k,

(d) the phase shift b, and then

(e) write down the equation of f (5)

6 2 For the function g determine

(a) the value of m,

(b) the period,

(c) the value of p,

(d) the phase shift n, and then

(e) write down the equation that defines g (5)

6 3 The graph of the function h can be obtained by

- shifting the graph of g horizontally $\frac{\pi}{8}$ units to the left, and then
- reflecting the resulting graph in the y-axis

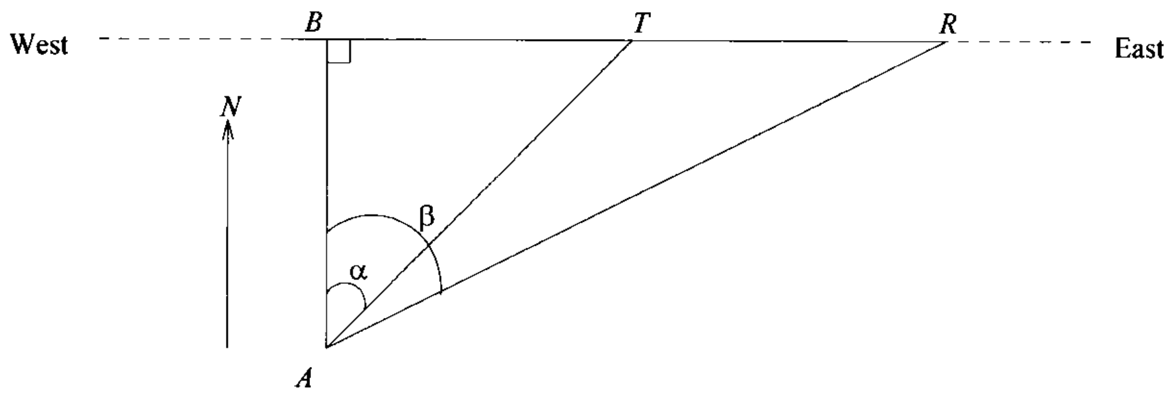
Write down the equation of h as

$$y = h(x) =$$ (2)

[12]

QUESTION 7

Thabo and Randall find themselves lost on a hiking expedition They know, however, that the main road running west–east cannot be too far away They start walking in different directions to get to the main road The following diagram illustrates the situation



Thabo and Randall start to walk from point A Thabo walks in the direction α degrees east of north and Randall in the direction β degrees east of north Thabo gets to the road at point T and Randall arrives at R AB represents the shortest distance from A to the main road

7 1 Suppose the distance AB is a kilometres and the distance TR is d kilometres

Use the **Law of Sines** to show that

$$a = \frac{d \cos \beta}{\sec \alpha \sin (\beta - \alpha)}$$

(6)

7 2 Show that the equation in 7 1 can also be written in the form

$$a = \frac{d}{\tan \beta - \tan \alpha}$$

(2)

7 3 Suppose Thabo walks in the direction 45° east of north, and Randall in the direction 60° east of north, and they arrive at the road $2\left(\sqrt{3} - 1\right)$ kilometres apart Use the equation in 7 2 to calculate the shortest distance from A to the road **Simplify** the answer (3)

[11]

TOTAL: [100]