

MAT1510

January/February 2011

PRECALCULUS MATHEMATICS A

Duration 2 Hours

100 Marks

EXAMINERS
FIRST
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This paper consists of 6 pages

THE USE OF A POCKET CALCULATOR IS NOT PERMITTED.**ANSWER ALL THE QUESTIONS.****QUESTION 1**The functions f , g and h are defined by

$$f(x) = 2x,$$
$$g(x) = 1 - \sqrt{4 - 3x}$$

and

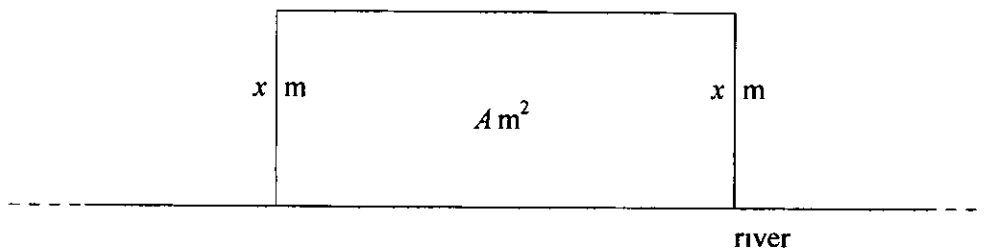
$$h(x) = \frac{4}{|3x + 2|}$$

respectively

1 1 Write down the sets D_f (domain of f), D_g (domain of g), as well as $D_f \cap D_g$ (3)1 2 Solve the equation $f(x) = g(x)$ for $x \in D_f \cap D_g$ (7)1 3 Write down the set D_h , and then solve the inequality $h(x) \leq 1$ for $x \in D_h$ (5)**[15]****[TURN OVER]**

QUESTION 2

A farmer has 800 metres of fencing and wants to fence off a rectangular field that borders a straight river. He does not need a fence along the river. Suppose the width of the field is x metres and the area is A square metres. The following diagram illustrates the situation.



- 2.1 Find the equation of the function that models the area A in terms of x if 800 m of fencing are used (4)
- 2.2 Using 800 m of fencing the farmer can fence off rectangles with different dimensions. What are the dimensions of the field with maximum area? (3)
- 2.3 If 1 hectare = 10^4 square metres, what is the maximum area of the field in terms of hectares? (2)
- [9]

QUESTION 3

Suppose the function g is defined by

$$y = g(x) = -2x^2 + 8x + 7$$

- 3.1 Restrict the domain of g such that the function g_r , defined by

$$g_r(x) = g(x) \text{ for all } x \in D_{g_r},$$

is a one-to-one function, and such that the domain D_{g_r} contains only positive numbers

Write down R_{g_r} as well (4)

- 3.2 Determine the equation that defines the inverse function g_r^{-1} (5)

- 3.3 Write down the sets which represent $D_{g_r^{-1}}$ and $R_{g_r^{-1}}$ (2)

- 3.4 Show that

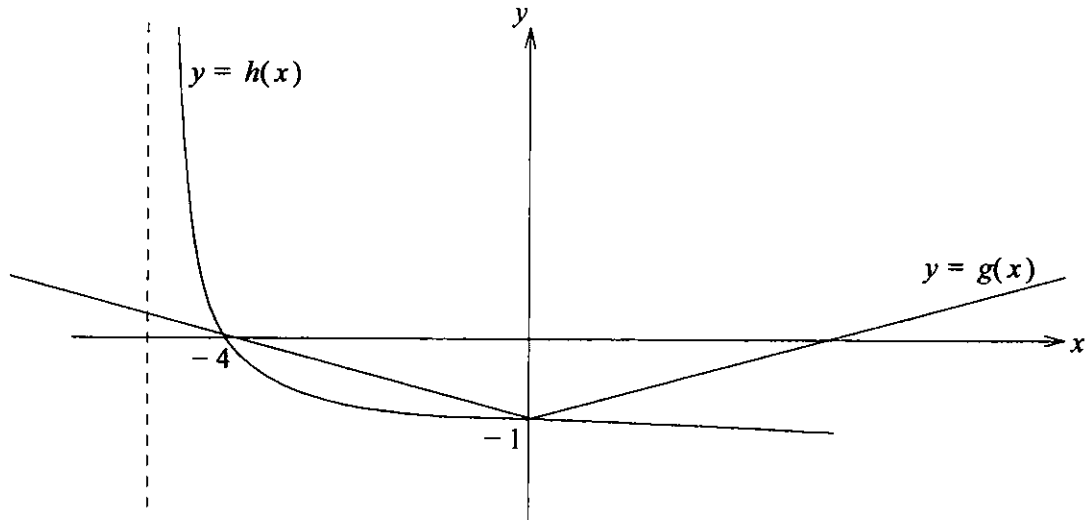
$$(g_r^{-1} \circ g_r)(x) = x \text{ for } x \in D_{g_r}$$

Include all the steps of your reasoning (3)

[14]

[TURN OVER]

QUESTION 4



The sketch shows the graphs of the functions g and h defined by

$$y = g(x) = m|x - k| + l$$

and

$$y = h(x) = -\log_a(x - b) \text{ where } a > 1$$

- 4.1 Use the graph of h to find a and b , and hence write down the equation of h (5)
- 4.2 Write down the sets which represent the domain and the range of the function h , and the equation of the asymptote of the graph of h (3)
- 4.3 Suppose the function r is defined by $y = r(x) = \log_{\frac{1}{a}}(x - b)$, where a and b are the same as in 4.1. Explain why the graph of r is the same as the graph of h . You do not need the values of a and b to answer this question (2)
- 4.4 Find the equation of the inverse function h^{-1} (3)
- 4.5 Show that $(h \circ h^{-1})(x) = x$. Include all the steps of your reasoning (3)
- 4.6 Use the graph of g to find m , k and l , and hence write down the equation of g (4)

[20]

[TURN OVER]

QUESTION 5

Solve each of the following for x

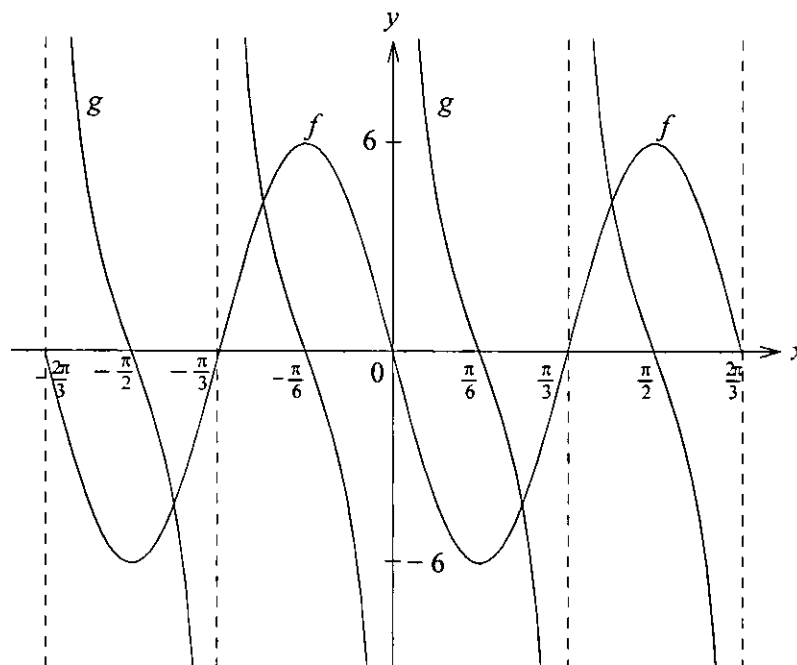
$$5.1 \quad 3^{2x+3} - 4 \cdot 3^{x+1} + 1 = 0 \quad (6)$$

$$5.2 \quad 2 \sin x \tan \frac{x}{2} - \tan \frac{x}{2} = 1 - 2 \sin x \text{ for } x \in (-\pi, \pi)$$

(Hint Do not write $\tan \frac{x}{2}$ in terms of $\sin \frac{x}{2}$, $\cos \frac{x}{2}$ or $\tan x$) (8)

[14]

QUESTION 6



The sketch shows the graphs of the functions f and g . The function f is defined by

$$y = f(x) = a \sin k(x - b)$$

where k is a **positive** constant. The function g is defined by

$$y = g(x) = n \cot mx$$

where m is a **positive** constant and $(\frac{\pi}{4}, -4)$ is a point on the graph of g .

6.1 For the function f , determine

(a) the amplitude $|a|$,

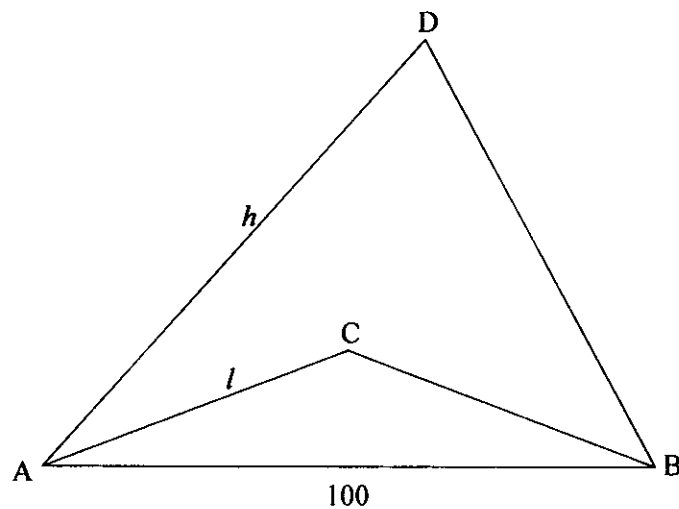
[TURN OVER]

- (b) the period,
 (c) the value of k ,
 (d) the phase shift, and hence b , and then
 (e) write down the equation that defines f (5)

6.2 For the function g , determine

- (a) the period,
 (b) the value of m ;
 (c) the value of n , and then
 (d) write down the equation that defines g (6)
 [11]

QUESTION 7



In the sketch C and D represent the tops of two trees. The trees are planted on the straight line connecting A and B . A , B , C and D are points in the same vertical plane. From both A and B the angle of elevation of C is α . The angle of elevation of D from A is β and the angle of elevation of D from B is 2α . Suppose the distance from A to B is 100 metres, and h and l represent the distances (in metres) from A to D , and from A to C , respectively.

7.1 Copy the diagram in your exam script. Fill in the angles α and β on your diagram (3)

7.2 Use the Law of Sines to show that

$$l = 50 \sec \alpha$$

(5)

[TURN OVER]

7.3 Use the Law of Sines to show that

$$h = \frac{100}{\cos \beta + \cot 2\alpha \sin \beta} \quad (5)$$

7.4 Use the formula in 7.3 to calculate the distance from A to D (in metres) if $\alpha = 30^\circ$ and $\beta = 45^\circ$. Simplify the answer and leave it in surd form (in terms of square roots) but rationalise the denominator. (4)

[17]

TOTAL: [100]