UNIVERSITY EXAMINATIONS

UNIVERSITEITSEKSAMENS

UNISA university of south africa

MAT1510

October/November 2017

PRECALCULUS MATHEMATICS A

Duration 2 Hours

100 Marks

EXAMINERS

FIRST DR B HUNTLEY
SECOND DR O IGHEDO

Closed book examination

This paper consists of 7 pages

Answer All Questions

[TURN OVER]

QUESTION 1

The functions f and g are defined by

$$f(x) = \frac{1}{|-3x+5|}$$

and

$$g(x) = \log_3 x + \log_3 (x-8)$$

1.1 Write down the domain D_f of the function f and the domain D_g of the function g, using interval notation (2)

1.2 Solve each of the following for x

(a) $f(x) < 1$ (6)

(b) $g(x) > 2$ (7)

(c) $\sin 2x = 2\sin 2x \cos 3x$ where $x \in [0, \pi]$ (8)

[23]

QUESTION 2

A farmer can use two different tractors to plough a field
If tractor A works alone, the field can be ploughed in x hours
Working alone, it takes tractor B 1 hour longer to plough the field than it takes tractor A to plough the field
If both tractors work together, it takes them 1 hour and 12 minutes to plough the field

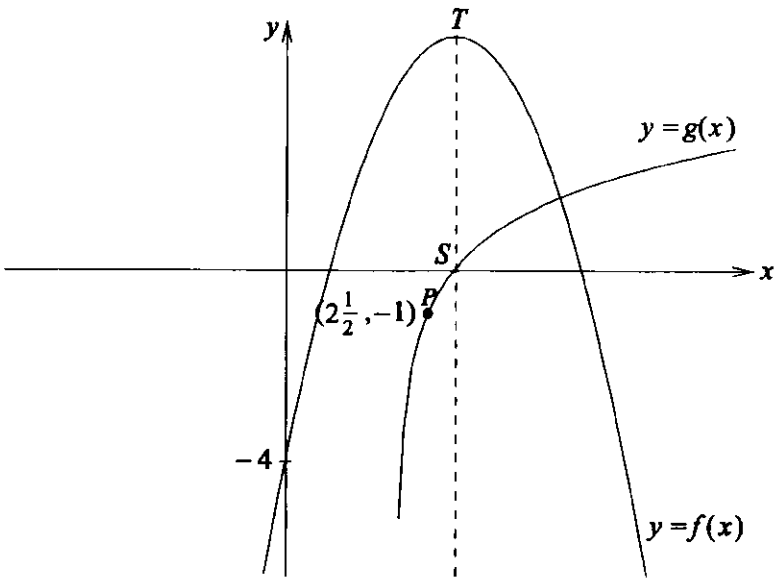
2.1 Express 1 hour and 12 minutes as hours

2.2 How long does it take to plough the field when

(a) tractor A works alone?

(b) tractor B works alone? **[10]**

QUESTION 3



The sketch shows the graphs of the functions f and g The equations that define f and g are

$$y = f(x) = a(x-3)^2 + 5$$

and

$$y = g(x) = \log_b(x-k), \quad \text{where} \quad b > 0 \quad \text{and} \quad b \neq 1,$$

respectively The graph of f is a parabola with vertex T and y intercept -4 The graph of g passes through the points $P\left(2\frac{1}{2}, -1\right)$ and S S is the point where the graph of g cuts the x–axis T and S lie on the same vertical line

3 1 Write down the coordinates of the points T and S (2)

3 2 Show that $a = -1$, and hence write down the equation that defines f in the form

$f(x) = ax^2 + bx + c$ (3)

3 3 Determine the values of b and k, and hence write down the equation of g (5)

3 4 Leave your answers to (a), (b) and (c) below in terms of b and/or k if you were unable to find the values of b and/or k in 3 3

(a) If the graph of the function h is the reflection of the graph of g in the y–axis, give the equation of h in the form

$$y = h(x) =$$

(b) If the graph of the function ℓ is the reflection of the graph of g in the x–axis, give the equation of ℓ in the form

$$y = \ell(x) =$$

(c) Give the equation of the vertical asymptote of the graph of g (3)

3 5 (a) Why is f not a one–to–one function? (1)

(b) Restrict the domain of f such that the function f_r defined by

$$f_r(x) = f(x) \qquad \text{for all } x \in D_{f_r}$$

is a one–to–one function Give the restricted domain D_{f_r} (1)

(c) Determine an equation for the inverse function f_r^{-1}. (5)

(d) Write down the domain $D_{f_r^{-1}}$ of the inverse function f_r^{-1}, and show that

$$\left(f_r \circ f_r^{-1}\right)(x) = x \qquad \text{for all } x \in D_{f_r^{-1}}$$ (5)

[25]

QUESTION 4

The area covered by water weed on a dam increases exponentially according to the formula

$$A(t) = A(0)\, e^{kt}$$

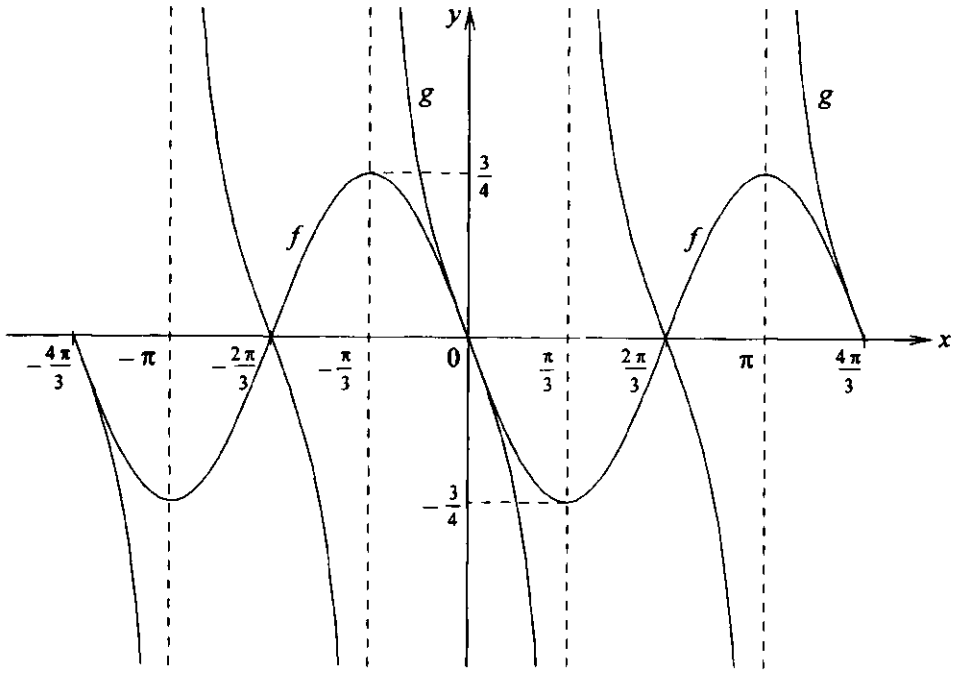
where $A(t)$ is the area of the water weed in square metres after t days The initial area of water weed was 100 m^2 After 10 days the area was 150 m^2

4 1 Find the value of k in the above formula Leave your answer in terms of ln (5)

4 2 Use the formula to determine the area of water weed after 20 days (5)

[10]

QUESTION 5



The sketch shows the graphs of the functions f and g The functions f and g are defined by

$$y = f(x) = a\cos k(x - b)$$

and

$$y = g(x) = c\tan dx$$

respectively The constants k and d are positive, and the graph of g passes through the point $\left(-\frac{\pi}{6}, \frac{3}{4}\right)$ Use the sketch to answer the following

5 1 For the function f, determine

(a) the amplitude

(b) whether a is positive or negative, give the value of a

(c) the value of k

(d) the phase shift b

(e) and thus the equation that defines f (5)

5 2 For the function g, determine

(a) the period

(b) the value of d

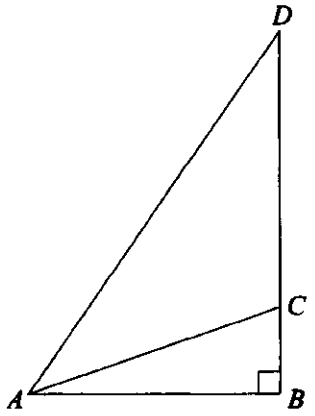
(c) the value of c (show all working out)

(d) and thus the equation that defines g (6)

[11]

QUESTION 6

In the following sketch $C\hat{A}B = 15°$, $C\hat{A}D = 45°$ and AC represents a distance of 100 metres



6 1 What is the measure of the angle of elevation from A to C? (1)

6 2 Determine the angle of depression from D to A (1)

6.3 **Use the Law of Sines** to determine the distance represented by DC Leave your answer in surd form (6)

[8]

QUESTION 7

In a certain triangle PQR, where the lengths of the sides opposite the angles $\hat{P}$, $\hat{Q}$ and $\hat{R}$ are given by p, q and r respectively, the following formula holds

$$q^2 = p^2 + r^2 - 2pr\cos\hat{Q}$$

Suppose we have a triangle ABC where $\hat{B} = \hat{Q}$ and the lengths of the sides opposite $\hat{A}$, $\hat{B}$ and $\hat{C}$ are $2p$, b and $2r$ respectively Use the given formula to show that $b = 2q$

Hint: Draw a sketch of triangles PQR and ABC to make it easier to apply the formula correctly [5]

QUESTION 8

Two cars leave from the same place and travel along two straight roads The two roads make an angle of 60° with each other at the point from which the cars begin their journey The one car travels at 60 km/h and the other at x km/h If the cars are $10\sqrt{13}$ km apart after half an hour, what is the speed of the other car?

Apply the Law of Cosines to solve for x As part of your answer, sketch a diagram to illustrate the situation **[8]**

TOTAL: [100]