UNIVERSITY EXAMINATIONS

UNIVERSITEITSEKSAMENS

UNISA university of south africa

MAT1510

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PRECALCULUS MATHEMATICS A

Duration 2 Hours

100 Marks

EXAMINERS

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Closed book examination.

This examination question paper remains the property of the University of South Africa and may not be removed from the examination venue.

This paper consists of 5 pages

The use of a pocket calculator is not permitted
Answer all the questions
All calculations must be shown

[TURN OVER]

QUESTION 1

The functions f and g are defined by

$$f(x) = \frac{1}{|1-3x|} \quad \text{and} \quad g(x) = \log_{\frac{1}{3}}\left(\frac{1}{3x-2}\right) - \log_3(x)$$

respectively

(1.1) *Write down the sets* D_f (the domain of f) and D_g (the domain of g) (3)

(1.2) Solve the inequality $f(x) > 2$ for $x \in D_f$ (5)

(1.3) Solve the inequality $g(x) \geq 0$ for $x \in D_g$ (10)
Hint: *Use the Change of Base Formula*

[18]

QUESTION 2

Suppose we have two numbers x and y whose difference is 8 and $x < y$

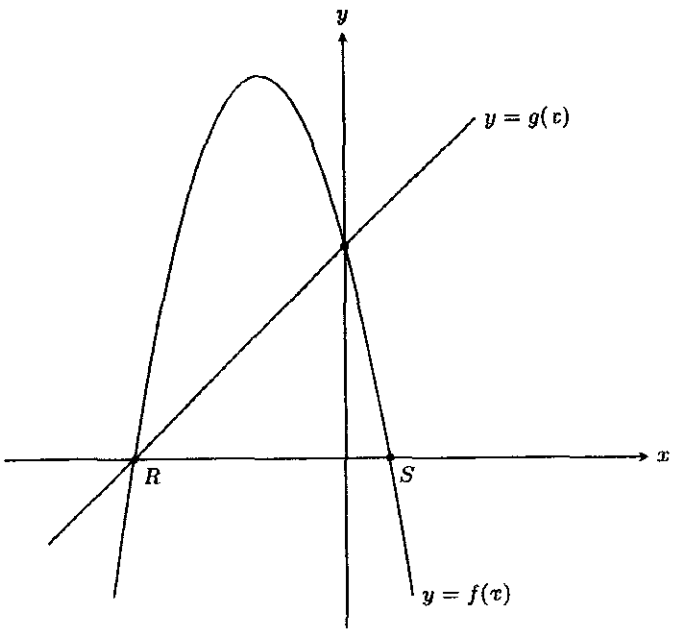
(2.1) Find a function that models the number y in terms of the number x (2)

(2.2) Find a function that models the sum S of the squares of the two numbers in terms of x (3)

(2.3) Determine the values of the two numbers such that S is a minimum Include the steps of your reasoning (3)

[8]

QUESTION 3



The sketch shows the graphs of the functions f and g where

$$f(x) = a(x-h)^2 + k, \quad \text{and} \quad g(x) = mx + c$$

The graph of f is a parabola which cuts the $x-$axis at the points R and S The turning point is $(-2, 9)$ The graph of g is a straight line The graphs of f and g cut each other at R on the $x-$axis and at 5 on the $y-$axis

(3.1) Determine the values of a, h and k, and thus write down the equation of f (4)

(3.2) Determine the coordinates of R and S (4)

(3.3) Find the values of m and c, and thus write down the equation of g (3)

(3.4) Suppose the $x-$coordinate of R is x_R and the vertical distance between corresponding points on the graphs of f and g is given by $d(x)$ where $x_R \leq x \leq 0$ What is the maximum value of $d(x)$? (5)

(3.5)

(a) Shift the graph of f vertically 4 units downwards Suppose we obtain the graph of l Write down the equation (1)

$$y = l(x) =$$

(b) Shift the graph of l horizontally 3 units to the right Suppose we obtain the graph of n Write down the equation (1)

$$y = n(x) =$$

[18]

QUESTION 4

(4.1) Prove the identity (5)

$$\frac{1}{1-\sin\theta} = \sec^2\theta + \tan\theta\sec\theta$$

(4.2) For which values of θ is the identity in (4.1) undefined? (3)

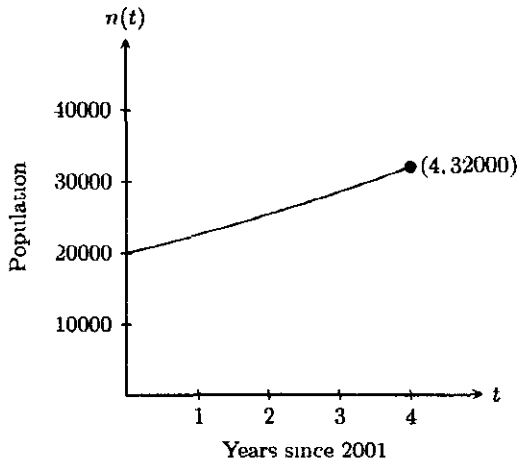
(4.3) Solve the equation (8)

$$\cos 2x \csc^2 x = 2\cos 2x \quad \text{for} \quad x \in (-\pi, \pi)$$

[16]

QUESTION 5

The graph below shows the population in a town from 2001 to 2005 Assume that the population grows exponentially at the same rate



(5.1) What was the town's population in 2001? (1)

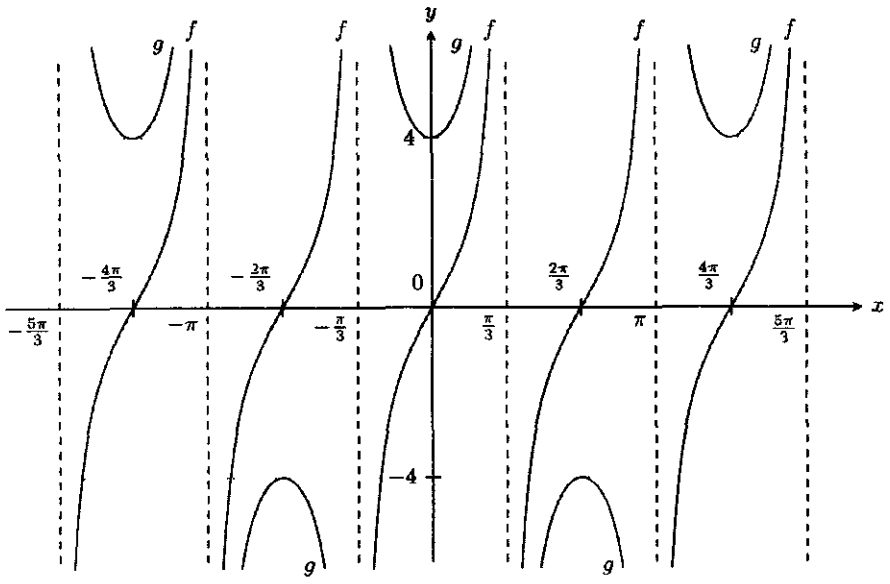
(5.2) Find a formula for the population t years after 2001 (5)
Hint: Use $n(t) = n_0 e^{rt}$

(5.3) Use the formula in (5.2) to determine how much time it takes for the 2001 population to double. Leave the answer in terms of ln (4)

(5.4) What is the projected population for 2009? (6)

(5.5) Will the 2001 population double before or after 2009? Give a reason for your answer (2)
[18]

QUESTION 6



The sketch shows the graphs of the functions f and g The functions f and g are defined by

$$y = p\tan cx$$

and

$$y = a\csc k\,(x - b)$$

respectively The constants c and k are positive The point $(\frac{\pi}{6}\ 2)$ lies on the graph of f Use the sketch to answer the following

(6.1) For the function f determine

(a) The period (1)

(b) The value of c (1)

(c) The value of p and thus (2)

(d) Write down the equation that defines f (1)

(6.2) For the function g determine

(a) The period (1)

(b) The value of k (1)

(c) The phase shift b (1)

(d) The value of a and thus (1)

(e) Write down the equation that defines g (1)

[10]

QUESTION 7

(7.1) Use the special triangles and the addition formula for sine to determine the value of $\sin 75°$ Leave the answer in surd form if necessary (5)

(7.2) Suppose we have a triangle ABC where $\angle ABC = 75°$ $\angle BAC = 60°$ and the length of AC is 10 cm

(a) Briefly sketch the triangle, displaying all the given information (2)

(b) Use the Law of Sines, and your answer to (7 1) to determine the length of AB Leave the answer in surd form if necessary (5)

[12]

TOTAL MARKS: [100]