UNIVERSITY EXAMINATIONS

UNIVERSITEITSEKSAMENS

UNISA

MAT1510

October/November 2016

PRECALCULUS MATHEMATICS A

Duration . 2 Hours

100 Marks

EXAMINERS :

FIRST .	DR MM KAMGA PENE	DR JM MANALE
SECOND	DR B HUNTLEY	

Closed book examination.

This paper consists of 5 pages.

The use of a pocket calculator is not permitted
Answer all the questions.
All calculations must be shown

[TURN OVER]

QUESTION 1

The functions f, g and h are defined by

$$f(x) = x + \sqrt{x+1} \qquad g(x) = |2x+1| \qquad \text{and} \qquad h(x) = \log_3 x - 2\log_x 3$$

respectively

(1.1) Write down the sets D_f, D_g and D_h that represent the domains of f, g and h respectively (3)

(1.2) Solve the equation $f(x) = 5$ (8)

(1.3) Solve the inequality $g(x) \geq 1$ (4)

(1.4) Solve the equation $h(x) = 1$ (9)
Hint: Use the Change of Base Formula

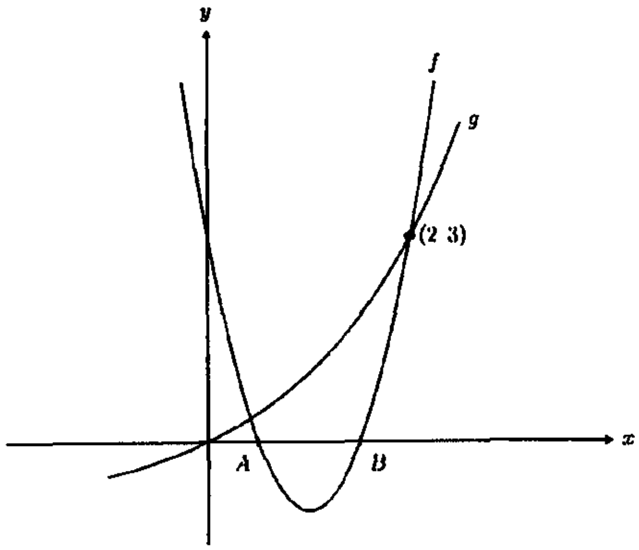
[24]

QUESTION 2

In summer the day temperatures of a certain town in South Africa range between 25 degrees Celsius and 40 degrees Celsius What range of temperatures does this correspond to on the Fahrenheit scale? The relationship between a degrees Celsius ($a°C$) and b degrees Fahrenheit ($b°F$) is given by the equation $a = \frac{5}{9}(b-32)$

[4]

QUESTION 3



The sketch shows the graphs of the functions f and g defined by

$$y = f(x) = a(x-1)^2 + k$$

and

$$y = g(x) = b^x + c, \quad \text{where} \quad b > 0 \quad b \neq 1$$

The graph of f passes through the point $(-1, 15)$ and the graph of g passes through the origin. The two graphs cut each other (intersect) at the point $(2, 3)$.

(3.1) Determine the values of a and k and thus the equation of f (6)

(3.2) Determine the coordinates of the points A and B (3)

(3.3) Determine the values of b and c and thus the equation of g (6)

(3.4) Describe the transformation that you need to apply to the graph of f to obtain the graph of the function h where $h(x) = a x^2 + k$. Explain why the graph of h is symmetric with respect to the y-axis. (2)

(3.5) **Use the graphs** (not the algebraic expressions) of f and g to solve the inequality $f(x) \cdot g(x) \geq 0$ (4)

(3.6)

(a) Determine an equation for the inverse function g^{-1} (2)

(b) Write down the set $D_{g^{-1}}$ (the domain of g^{-1}) (2)

(c) Show that $(g \circ g^{-1})(x) = x$ for $x \in D_{g^{-1}}$ (3)

[28]

QUESTION 4

(4.1) The function l is defined by (3)

$$y = l(x) = 2x^2 + 4x + 3$$

Write this equation in the form

$$y = l(x) = a(x - h)^2 + k$$

(4.2) Restrict the domain of l such that the function l_r defined by (1)

$$l_r(x) = l(x) \quad \text{for all} \quad x \in D_{l_r}$$

is a one-to-one function. Write down the set D_{l_r}

(4.3) Determine the equation that defines the inverse function l_r^{-1} and write down the set $D_{l_r^{-1}}$ (5)

(4.4) Show that $(l_r^{-1} \circ l_r)(x) = x$ for all $x \in D_{l_r}$ (3)

[12]

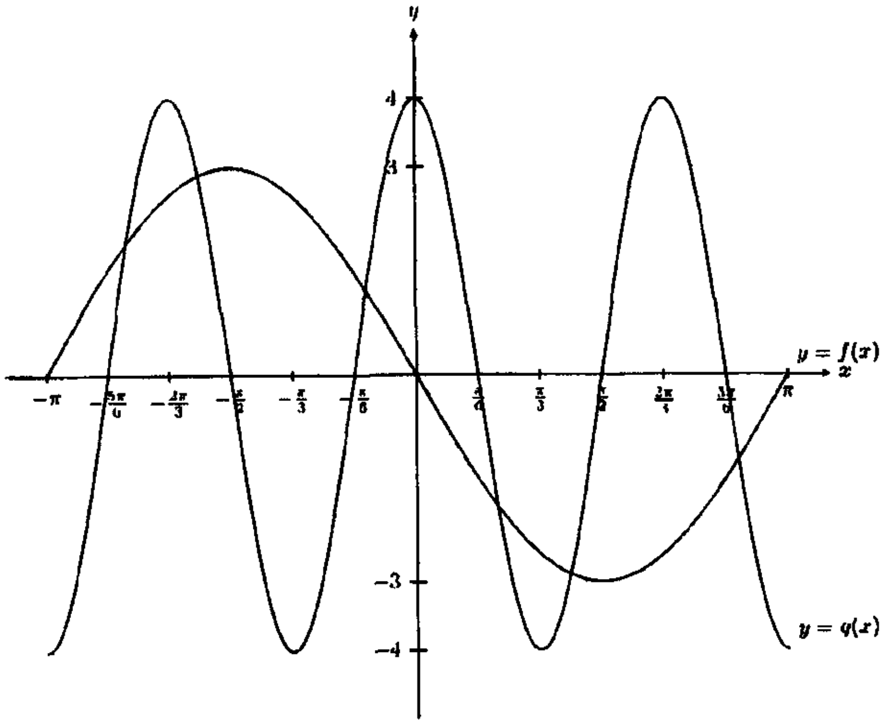
QUESTION 5

Solve the equation

$$2\sin\frac{t}{2} = -\sin\frac{t}{2}\sec t \quad \text{for} \quad t \in [-2\pi, 2\pi]$$

[10]

QUESTION 6



The sketch shows the graphs of the functions f and g. The function f is defined by

$$y = a\sin k(x - b), \quad k > 0$$

and the function g is defined by

$$y = p\cos c(x - q) \quad c > 0$$

Use the graphs of f and g to answer the following

(6.1) For the function f determine

- **(a)** The amplitude $|a|$ and hence a (1)
- **(b)** The period of the function (1)
- **(c)** The value of k (1)
- **(d)** The phase shift b, and then (1)
- **(e)** Write down the equation of f (1)

(6.2) For the function g determine

- **(a)** The amplitude $|p|$, and hence p (1)
- **(b)** The period of the function (1)
- **(c)** The value of c (1)
- **(d)** The phase shift q and then (1)
- **(e)** Write down the equation of g (1)

(6.3) The graph of the function h can be determined by (2)

- reflecting the graph of f around the x-axis and then
- shifting the resulting graph horizontally $\frac{\pi}{6}$ units to the right

Write down the equation of h as

$$y = h(x) =$$

[12]

QUESTION 7

A jogger runs north for $1\frac{1}{2}$ hour along a straight road. She then turns 60° to the right to run in a north-easterly direction, along another straight road, for two hours. The jogger runs the complete distance at a constant speed of 10 km/h.

(7.1) Give a diagram to represent this information. (3)

(7.2) How far is the jogger from her starting position after $3\frac{1}{2}$ hours? Use the Law of Cosines to calculate the distance. Leave your answer in surd form if necessary. (7)

[10]

TOTAL MARKS: [100]