

MAT1510
PRECALCULUS MATHS 1
Jan-Feb 2016

QUESTION 1

Solve each of the following for x

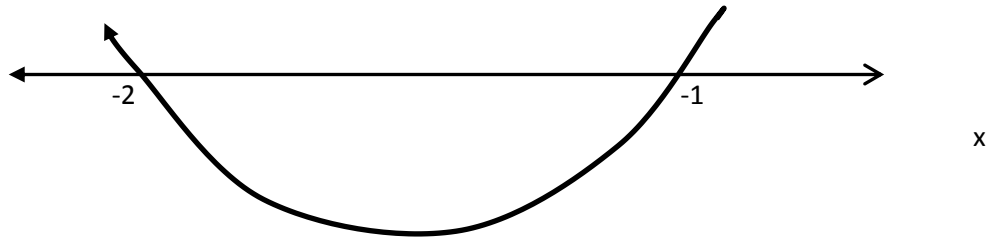
1.1 $|-2x - 3| \leq 1$

There are several ways of solving this problem; we will highlight here only two methods.

Method 1

The equation involved the $| |$ modulus sign. To remove the modulus sign, we square both sides of the equation to get,

$$\begin{aligned} |-2x - 3|^2 &\leq 1^2 \\ (-2x - 3)(-2x - 3) &\leq 1 \\ 4x^2 + 6x + 6x + 9 &\leq 1 \\ 4x^2 + 12x + 8 &\leq 0 \\ x^2 + 3x + 2 &\leq 0 \\ (x + 2)(x + 1) &\leq 0 \end{aligned}$$



So the solution set for the above equation is

$$-2 \leq x \leq -1$$

Method 2

Since the term $-2x - 3$ is inside the modulus sign, it can be negative or positive.

Taking it to be positive we have

$$\begin{aligned} -2x - 3 &\leq 1 \\ -2x &\leq 4 \end{aligned}$$

Dividing both sides by -2 (and note the sign change due to division by negative), we have

$$x \geq -2$$

Taking it to be negative we have

$$\begin{aligned} -(-2x - 3) &\leq 1 \\ 2x + 3 &\leq 1 \\ 2x &\leq -2 \\ x &\leq -1 \end{aligned}$$

Combining the two solutions, therefore the full solution set is. $-2 \leq x \leq -1$

1.2 $2^{x+1} + 2^{3-x} = 17$

Applying laws of indices we have

$$2^x \cdot 2^1 + 2^3 \cdot 2^{-x} = 17$$

$$2 \cdot 2^x + \frac{8}{2^x} = 17$$

Rearranging, we have

$$2 \cdot (2^x)^2 + 8 = 17 \cdot 2^x$$

Set 2^x into another variable say χ , in other words set $2^x = \chi$ so we have

$$2\chi^2 + 8 = 17\chi$$

$$2\chi^2 - 17\chi + 8 = 0$$

Solving the quadratic equation we have

$$(2\chi - 1)(\chi - 8) = 0$$

Either $\chi = \frac{1}{2}$ or $\chi = 8$

If $\chi = \frac{1}{2}$, then $2^x = \frac{1}{2}$ and $2^x = 2^{-1}$ therefore $x = -1$

If $\chi = 8$, then $2^x = 2^3$ therefore $x = 3$

So $x = -1$ or $x = 3$

1.3 $2 \sin x \tan 2x + \tan 2x = 0$

$$2 \sin x \frac{\sin 2x}{\cos 2x} + \frac{\sin 2x}{\cos 2x} = 0$$

$$2 \sin x \frac{2 \sin x \cos x}{2 \cos^2 x - 1} + \frac{2 \sin x \cos x}{2 \cos^2 x - 1} = 0$$

$$\frac{4 \sin^2 x \cos x}{2 \cos^2 x - 1} + \frac{2 \sin x \cos x}{2 \cos^2 x - 1} = 0$$

$$\frac{4 \sin^2 x \cos x + 2 \sin x \cos x}{2 \cos^2 x - 1} = 0$$

$$\frac{2 \sin x \cos x (2 \sin x + 1)}{2 \cos^2 x - 1} = 0$$

$$2 \sin x \cos x (2 \sin x + 1) = 0$$

$$\sin x \cos x (2 \sin x + 1) = 0$$

From the above equation either $\sin x = 0$ or $\cos x = 0$ or $2 \sin x + 1 = 0$

If $\sin x = 0$, then the general solution;

$$x = 0 + 2\pi k$$

$$x = \pi + 2\pi k$$

If $\cos x = 0$, then the general solution;

$$x = \frac{\pi}{2} + 2\pi k$$
$$x = -\frac{\pi}{2} + 2\pi k$$

If $2\sin x + 1 = 0$, $\sin x = \frac{-1}{2}$ then the general solution;

$$x = -\frac{\pi}{6} + 2\pi k$$
$$x = \frac{7\pi}{6} + 2\pi k$$

Where, $k = 0, \pm 1, \pm 2, \dots$

Particular solutions in the range $[0, 2\pi)$

$$x = 0, \frac{\pi}{2}, \pi, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$$

QUESTION 2

Given $h(t) = 32t - 4t^2$

2.1 If $h(t) = 48 \Rightarrow 32t - 4t^2 = 48$

Re- writing the quadratic equation in standard form we have

$$4t^2 - 32t + 48 = 0$$

And dividing the equation by 4 we no have

$$t^2 - 8t + 12 = 0$$

$$(t - 6)(t - 2) = 0$$

Solving we get

Either $t = 2$ or $t = 6$

So the ball will be 48 m above the ground when $t = 2 s$ and when $t = 6 s$

2.2 $h(t)$ is a quadratic (parabolic) function. Parabolic function fail the horizontal line test and there if is not a one-to-one function.

2.3 Starting from $h(t) = -4t^2 + 32t$, factoring out -4

then completing the square

$$h(t) = -4[t^2 - 8t]$$

$$h(t) = -4[(t-4)^2 - 16]$$

Annotations:

- $\frac{1}{2}$ of coefficient of t
- Take down t
- Subtract the square of 4

Re-writing the expression more neatly now we have

$$h(t) = -4[(t-4)^2 - 16]$$

Simplifying we have

$$h(t) = -4(t-4)^2 + 64$$

2.3 From the above equation;

- (a) the maximum height of the ball is **64 m**
- (b) it takes **4 s** to reach the maximum height.

QUESTION 3

Given $y = h(t) = \log_3(x+2)$

3.1 $D_h: x \in \mathbb{R}, x > -2$ $R_h: y \in \mathbb{R}$

3.2

(a.) $y = \log_3(x+2)$

Applying laws of logarithms and indices

$$\begin{aligned} \Rightarrow 3^y &= (x+2) \\ x &= 3^y - 2 \end{aligned}$$

Swopping x and y we get

$$h^{-1} = y = 3^x - 2$$

(b.) $D_{h^{-1}}: x \in \mathbb{R}$,

[NB: $D_{h^{-1}} = R_h$ and $R_{h^{-1}} = D_h$]

3.3

(a.) $(h^{-1} \circ h)(x) = 3^h - 2 = 3^{\log_3(x+2)} - 2$

From laws of logarithms $a^{\log_a x} = a$, hence

$$(h^{-1} \circ h)(x) = x + 2 - 2 = x$$

QED.

(b.) $(h^{-1} \circ h)(x) = x$ therefore $D_{h^{-1} \circ h}: x \in \mathbb{R}$