CHAPTER

5 SWAP MARKETS AND CONTRACTS

LEARNING OUTCOMES

After completing this chapter, you will be able to do the following:

- Describe the characteristics of swap contracts.
- Explain how swaps are terminated.
- Define and give examples of the types of currency swaps.
- Calculate the payments on a currency swap.
- Define and give an example of a plain vanilla interest rate swap.
- Calculate the payments on an interest rate swap.
- Define and give examples of the types of equity swaps.
- Calculate the payments on an equity swap.
- Distinguish between the pricing and valuation of swaps.
- Explain the equivalence of swaps to combinations of other instruments.
- Explain how interest rate swaps are equivalent to a series of off-market forward rate agreements (FRAs).
- Explain how a plain vanilla swap is equivalent to a combination of an interest rate call and an interest rate put.
- Determine the fixed rate on a plain vanilla interest rate swap and the market value of the swap during its life.
- Determine the fixed rate, if applicable, and the foreign notional principal for a given domestic notional principal on a currency swap, and determine the market values of each of the different types of currency swaps during their lives.
- Determine the fixed rate, if applicable, on an equity swap and the market values of the different types of equity swaps during their lives.
- Define, explain, and interpret the characteristics of swaptions, including the difference between payer and receiver swaptions.
- Explain why swaptions exist and give examples of how they are used.
- Show how the payoffs of an interest rate swaption are like those of an option on a coupon-bearing bond.
- Calculate the value of an interest rate swaption on the expiration day.
- Explain how the market value of a swaption at expiration can be received in different ways.

$$N(-0.31) = 1 - N(0.31) = 1 - 0.6217 = 0.3783 = N(d_2)$$

The value of the call option is

$$c = 144.03(0.4325) - 150e^{-0.0375(0.1781)}(0.3783) = 5.926$$

Consistent with the equivalence of options on the underlying and options on the forward contract, this value is the same as the value of the call option on the forward contract in Part A (the slight difference comes from rounding).

C. The values for $N(d_1)$ and $N(d_2)$ are the same as in Part A. Putting these values into the formula for the put option in the Black model produces

$$p = e^{-0.0375(0.1781)}[150(1 - 0.3783) - 145(1 - 0.4325)] = 10.894$$

D. The values for $N(d_1)$ and $N(d_2)$ are the same as in Part B. Putting these values into the formula for the put option in the Black–Scholes–Merton model produces

$$p = 150e^{-0.0375(0.1781)}(1 - 0.3783) - 144.03(1 - 0.4325) = 10.897$$

Consistent with the equivalence of options on the underlying and options on the forward contract, this value is the same as the value of the put option on the forward contract in Part C (the slight difference comes from rounding).

20. A. Calculate the values of d_1 and d_2. The time to expiration is $T = 180/365 = 0.4932$.

$$d_1 = \frac{\ln(0.0725/0.075) + [(0.04)^2/2](0.4932)}{0.04\sqrt{0.4932}} = -1.1928$$

$$d_2 = -1.1928 - 0.04\sqrt{0.4932} = -1.2209$$

Using the normal probability table,

$$N(-1.19) = 1 - N(1.19) = 1 - 0.8830 = 0.1170 = N(d_1)$$

$$N(-1.22) = 1 - N(1.22) = 1 - 0.8888 = 0.1112 = N(d_2)$$

The value of the call option is

$$c = e^{-0.05(0.4932)}[0.0725(0.1170) - 0.075(0.1112)] = 0.00013903$$

This value is computed based on the assumption that the option payoff is made at expiration. For the 90-day interest rate option, however, the payment is made 90 days after expiration. So, discount back 90 days:

$$0.00013903e^{-0.0725(90/365)} = 0.00013657$$

Convert to periodic rate based on a 90-day rate and using the customary 360-day year.

$$0.00013657(90/360) = 0.00003414$$

B. The values of $N(d_1)$ and $N(d_2)$ are the same as in Part A. Plug these values into the Black model for the put option.

$$p = e^{-0.05(0.4932)}[0.075(1 - 0.1112) - 0.0725(1 - 0.1170)] = 0.00257813$$

Now discount back 90 days:

$$0.00257813e^{-0.0725(90/365)} = 0.00253245$$

Convert to periodic rate based on 90-day rate:

$$0.00253245(90/360) = 0.00063311$$

(overpriced) will yield a risk-free profit of \$9 − \$8.41 = \$0.59. The initial up-front cash is generated as follows:

Transaction	
Sell put	9
Buy synthetic put	
Long call	−3.5
Short forward	0
Long bond	−4.91
Total	0.59

As shown below, at expiration no cash will be received or paid out.

	Value at Expiration	
Transaction	$S_T < 50$	$S_T > 50$
Sell put	$-(50 - S_T)$	0
Buy synthetic put		
Long call	0	$S_T - 50$
Short forward	$-(S_T - 45)$	$-(S_T - 45)$
Long bond	50 − 45	50 − 45
Total	0	0

19. A. First calculate the values of d_1 and d_2. The time to expiration, T = 65/365 = 0.1781.

$$d_1 = \frac{\ln(145/150) + [(0.03)^2/2](0.1781)}{0.33\sqrt{0.1781}} = -0.1738$$

$$d_2 = -0.1738 - 0.33\sqrt{0.1781} = -0.3131$$

Using the normal probability table,

$$N(-0.17) = 1 - N(0.17) = 1 - 0.5675 = 0.4325 = N(d_1)$$

$$N(-0.31) = 1 - N(0.31) = 1 - 0.6217 = 0.3783 = N(d_2)$$

The value of the call option is

$$c = e^{-0.0375(0.1781)}[145(0.4325) - 150(0.3783)] = 5.928$$

B. With no cash flows on the underlying and continuously compounded interest, the price of the underlying asset can be calculated as

$$S_0 = 145e^{-0.0375(0.1781)} = 144.03$$

Now use the Black–Scholes–Merton model:

$$d_1 = \frac{\ln(144.03/150) + [0.0375 + (0.33)^2/2](0.1781)}{0.33\sqrt{0.1781}} = -0.1740$$

$$d_2 = -0.174 - 0.33\sqrt{0.1781} = -0.3133$$

Using the normal probability table,

$$N(-0.17) = 1 - N(0.17) = 1 - 0.5675 = 0.4325 = N(d_1)$$

18. Call price, c_0 = \$3.50
Put price, p_0 = \$9
Exercise price, X = \$50
Forward price, F(0,T) = \$45
Days to option expiration = 175
Risk-free rate, r = 4 percent
Time to expiration = 175/365 = 0.4795
Bond price = $[X - F(0,T)]/(1 + r)^T = (50 - 45)/(1+.04)^{0.4795}$ = \$4.91

A. Synthetic call = Long forward + $p_0 - [X - F(0,T]/(1 + r)^T = 0 + 9 - 4.91 =$ 4.09
Synthetic put = c_0 + Short forward + $[X - F(0,T)]/(1 + r)^T = 3.5 + 0 + 4.91 = 8.41$
Synthetic forward = $c_0 - p_0 + [X - F(0,T)]/(1 + r)^T = 3.5 - 9 + 4.91 = -0.59$

B.

Instrument	Actual Price	Synthetic Price	Mispricing/Profit
Call	3.50	4.09	0.59
Put	9	8.41	0.59
Forward	0	−0.59	0.59

C. The actual call is cheaper than the synthetic call. Therefore, an arbitrage transaction in which you buy the actual call (underpriced) and sell the synthetic call (overpriced) will yield a risk-free profit up front of \$4.09 − \$3.50 = \$0.59. The initial up-front cash is generated as follows:

Transaction	
Buy call	−3.5
Sell synthetic call	
Short forward	0
Short put	9
Long bond	−4.91
Total	0.59

As shown below, at expiration no cash will be received or paid out.

	Value at Expiration	
Transaction	$S_T < 50$	$S_T > 50$
Buy call	0	$S_T - 50$
Sell synthetic call		
Short forward	$-(S_T - 45)$	$-(S_T - 45)$
Short put	$-(50 - S_T)$	0
Long bond	50 − 45	50 − 45
Total	0	0

D. The actual put is more expensive than the synthetic put. Therefore, an arbitrage transaction in which you buy the synthetic put (underpriced) and sell the actual put

ii. For a \$5 change in the price of the underlying:
Change in call price = 0.4013(5) = \$2.0065
Change in put price = −0.5987(5) = −\$2.9935
Approximate new call price = 1.3144 + 2.0065 = \$3.3209
Approximate new put price = 3.0994 − 2.9935 = \$0.1059

C. For a \$1 change in the price of the underlying, the approximate new call price of \$1.7157 is different from but not too far off the actual call price of \$1.7538. Similarly, the approximate new put price of \$2.5007 is different from but not too far off the actual put price of \$2.5388. Therefore, the delta approximation is good but not perfect.

For a \$5 change in the price of the underlying, the approximate new call price of \$3.3209 is fairly far off the actual call price of \$4.2270. Similarly, the approximate new put price of \$0.1059 is quite far off the actual put price of \$1.0120. Therefore, the delta approximation is not particularly good.

The above comparisons indicate that the approximations based on delta are worse for larger moves in the underlying price.

17. A. First calculate the values of d_1 and d_2. The time to expiration is T = 275/365 = 0.7534. Adjust the price of the asset S_0 = \$100 − \$4.25 = \$95.75

$$d_1 = \frac{\ln(95.75/100) + [0.03 + (0.45)^2/2](0.7534)}{0.45\sqrt{0.7534}} = 0.1420$$

$$d_2 = 0.1420 - 0.45\sqrt{0.7534} = -0.2486$$

Using the normal probability table,

$$N(0.14) = 0.5557 = N(d_1)$$

$$N(-0.25) = 1 - N(0.25) = 1 - 0.5987 = 0.4013 = N(d_2)$$

The value of the call option is

$$c = 95.75(0.5557) - 100e^{-0.03(0.7534)}(0.4013) = 13.975$$

The value of the put option is

$$p = 100e^{-0.03(0.7534)}(1 - 0.4013) - 95.75(1 - 0.5557) = 15.990$$

B. First calculate the values of d_1 and d_2. The time to expiration is T = 275/365 = 0.7534. Adjust the price of the asset $S_0 = 100e^{-0.015(0.7534)} = 98.87626$

$$d_1 = \frac{\ln(98.876/100) + [0.03 + (0.45)^2/2](0.7534)}{0.45\sqrt{0.7534}} = 0.2242$$

$$d_2 = 0.22423 - 0.45\sqrt{0.7534} = -0.1664$$

Using the normal probability table,

$$N(0.22) = 0.5871 = N(d_1)$$

$$N(-0.17) = 1 - N(0.17) = 1 - 0.5675 = 0.4325 = N(d_2)$$

The value of the call option is

$$c = 98.876(0.5871) - 100e^{-0.03(0.7534)}(0.4325) = \$15.767$$

The value of the put option is

$$p = 100e^{-0.03(0.7534)}(1 - 0.4325) - 98.876(1 - 0.5871) = \$14.656$$

Pricing the two-period floorlet
The option values at expiration at t = 2 are

$p^{++} = \text{Max}(0, 0.09 - 0.114)/1.114 = 0$

$p^{+-} = \text{Max}(0, 0.09 - 0.0816)/1.0816 = 0.0078$

$p^{--} = \text{Max}(0, 0.09 - 0.0501)/1.0501 = 0.0380$

The risk-neutral probability is

$\pi = 0.5$ and $1 - \pi = 0.5$

Now find the option prices at time 1:

$$p^{+} = \frac{0.5(0) + 0.5(0.0078)}{1.0982} = 0.0036$$

$$p^{-} = \frac{0.5(0.0078) + 0.5(0.0380)}{1.0662} = 0.0215$$

The price of the two-period floorlet today is

$$p = \frac{0.5(0.0036) + 0.5(0.0215)}{1.0625} = 0.0118$$

Pricing the one-period floorlet
The option values at expiration at t = 1 are

$p^{+} = \text{Max}(0, 0.09 - 0.0982)/1.0982 = 0$

$p^{-} = \text{Max}(0, 0.09 - 0.0662)/1.0662 = 0.0223$

The price of the one-period floorlet today is

$$p = \frac{0.5(0) + 0.5(0.0223)}{1.0625} = 0.0105$$

The price of the two-period floor = 0.0118 + 0.0105 = 0.0223

16. A. The value of $N(d_1)$ is the approximate value of delta for calls and $N(d_1) - 1$ is the approximate value of delta for puts. So, first calculate the value of d_1. The time to expiration is T = 75/365 = 0.2055.

$$d_1 = \frac{\ln(28/30) + [0.035 + (0.40)^2/2](0.2055)}{0.40\sqrt{0.2055}} = -0.25$$

Using the normal probability table,

$N(-0.25) = 1 - N(0.25) = 1 - 0.5987 = 0.4013 = N(d_1)$

Therefore, the approximate value of delta for calls is 0.4013 and the approximate value of delta for puts is 0.4013 − 1 = − 0.5987.

B. Change in option price = Delta × Change in underlying asset price
Call delta = 0.4013
Put delta = −0.5987

i. For a $1 change in the price of the underlying:
Change in call price = 0.4013(1) = $0.4013
Change in put price = −0.5987(1) = −$0.5987
Approximate new call price = 1.3144 + 0.4013 = $1.7157
Approximate new put price = 3.0994 − 0.5987 = $2.5007

Now find the option prices at time 1:

$$p^{+} = \frac{0.5(0.0574) + 0.5(0.0023)}{1.0982} = 0.0272$$

$$p^{-} = \frac{0.5(0.0023) + 0.5(0)}{1.0662} = 0.0011$$

The put price today is

$$p = \frac{0.5(0.0272) + 0.5(0.0011)}{1.0625} = 0.0133$$

15. A. The two-period cap with an exercise rate of 8 percent consists of two caplets, a one-period call on the one-period interest rate with an exercise rate of 8 percent, and a two-period call on the one-period interest rate with an exercise rate of 8 percent. Price these caplets separately and sum them to obtain the price of the cap.

Pricing the two-period caplet

The option values at expiration at t = 2 are

$$c^{++} = \text{Max}(0,0.114 - 0.08)/1.114 = 0.0305$$

$$c^{+-} = \text{Max}(0,0.0816 - 0.08)/1.0816 = 0.0015$$

$$c^{--} = \text{Max}(0,0.0501 - 0.08)/1.0501 = 0$$

The risk-neutral probability is

$$\pi = 0.5 \text{ and } 1 - \pi = 0.5$$

Now find the option prices at time 1

$$c^{+} = \frac{0.5(0.0305) + 0.5(0.0015)}{1.0982} = 0.0146$$

$$c^{-} = \frac{0.5(0.0015) + 0.5(0)}{1.0662} = 0.0007$$

The price of the two-period caplet today is

$$c = \frac{0.5(0.0146) + 0.5(0.0007)}{1.0625} = 0.0072$$

Pricing the one-period caplet

The option values at expiration at t = 1 are

$$c^{+} = \text{Max}(0,0.0982 - 0.08)/1.0982 = 0.0166$$

$$c^{-} = \text{Max}(0,0.0662 - 0.08)/1.0662 = 0$$

The price of the one-period caplet today is

$$c = \frac{0.5(0.0166) + 0.5(0)}{1.0625} = 0.0078$$

The price of the two-period cap = 0.0072 + 0.0078 = 0.0150

B. The two-period floor with an exercise rate of 9 percent consists of two floorlets, a one-period put on the one-period interest rate with an exercise rate of 9 percent, and a two-period put on the one-period interest rate with an exercise rate of 9 percent. Price these floorlets separately and sum them to obtain the price of the floor.

$$1.0562 = 0.08(0.9523) + 1.08(0.9074)$$

Now compute the option prices. At time 2, the prices are

$$c^{++} = \text{Max}(0,0.9426 - 0.95) = 0$$

$$c^{+-} = \text{Max}(0,0.9977 - 0.95) = 0.0477$$

$$c^{--} = \text{Max}(0,1.0562 - 0.95) = 0.1062$$

The risk-neutral probability is

$$\pi = 0.5 \text{ and } 1 - \pi = 0.5$$

Now find the option prices at time 1:

$$c^{+} = \frac{0.5(0) + 0.5(0.0477)}{1.0982} = 0.0217$$

$$c^{-} = \frac{0.5(0.0477) + 0.5(0.1062)}{1.0662} = 0.0722$$

The call price today is

$$c = \frac{0.5(0.0217) + 0.5(0.0722)}{1.0625} = 0.0442$$

C. First calculate the payoff values of the put at expiration in two periods.

$$p^{++} = \text{Max}(0,0.85 - 0.8063) = 0.0437$$

$$p^{+-} = \text{Max}(0,0.85 - 0.8553) = 0$$

$$p^{--} = \text{Max}(0,0.85 - 0.9074) = 0$$

The risk-neutral probability is

$$\pi = 0.5 \text{ and } 1 - \pi = 0.5$$

Now find the option prices at time 1:

$$p^{+} = \frac{0.5(0.0437) + 0.5(0)}{1.0982} = 0.0199$$

$$p^{-} = \frac{0.5(0) + 0.5(0)}{1.0662} = 0$$

The put price today is

$$p = \frac{0.5(0.0199) + 0.5(0)}{1.0625} = 0.0094$$

D. The prices at time 2 of the 8 percent coupon bond with \$1 face value are the same as in Part B.

The payoff values of the put at expiration in two periods at $t = 2$ are

$$p^{++} = \text{Max}(0,1 - 0.9426) = 0.0574$$

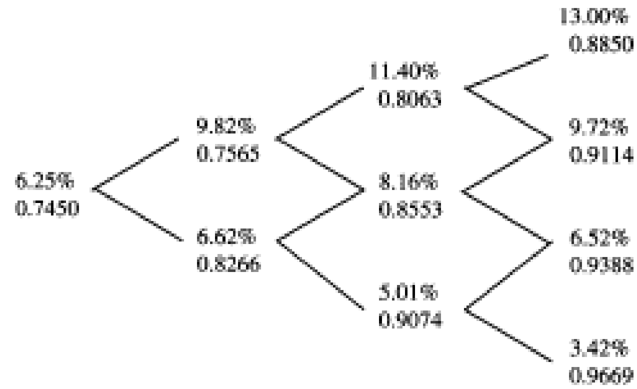
$$p^{+-} = \text{Max}(0,1 - 0.9977) = 0.0023$$

$$p^{--} = \text{Max}(0,1 - 1.0562) = 0$$

The risk-neutral probability is

$$\pi = 0.5 \text{ and } 1 - \pi = 0.5$$

14. **A.** Because the underlying is a four-period bond, redraw the interest rate tree to include only the four-year maturity bond prices.



Because the call expires in two periods, first we must calculate the payoff values of the call at expiration in two periods using the bond prices at time 2 in the above tree.

$$c^{++} = \text{Max}(0{,}0.8063 - 0.85) = 0$$

$$c^{+-} = \text{Max}(0{,}0.8553 - 0.85) = 0.0053$$

$$c^{--} = \text{Max}(0{,}0.9074 - 0.85) = 0.0574$$

The risk-neutral probability is

$$\pi = 0.5 \text{ and } 1 - \pi = 0.5$$

Now find the option prices at time 1:

$$c^{+} = \frac{0.5(0) + 0.5(0.0053)}{1.0982} = 0.0024$$

$$c^{-} = \frac{0.5(0.0053) + 0.5(0.0574)}{1.0662} = 0.0294$$

The call price today is

$$c = \frac{0.5(0.0024) + 0.5(0.0294)}{1.0625} = 0.0150$$

B. Because the bond expires in two periods, first we have to calculate the prices of the 8 percent coupon bond with \$1 face value at t = 2. At t = 2, the bond is a two-period bond. Consider the top node at t = 2. The one-and two-period zero-coupon bond prices are 0.8977 and 0.8063, respectively. Therefore, the coupon bond price at the top node at t = 2 is

$$0.9426 = 0.08(0.8977) + 1.08(0.8063)$$

Similarly, the coupon bond prices at the middle and bottom nodes at t = 2 are

$$0.9977 = 0.08(0.9246) + 1.08(0.8553)$$

C. Exercise price, X = \$70

Stock prices in the binomial tree, one and two periods from now are

$$S^+ = Su = 65(1.20) = \$78$$

$$S^- = Sd = 65(0.83) = \$53.95$$

$$S^{++} = Su^2 = 65(1.20)(1.20) = \$93.6$$

$$S^{+-} = Sud = 65(1.20)(0.83) = \$64.74$$

$$S^{--} = Sd^2 = 65(0.83)(0.83) = \$44.78$$

Put option values at expiration two periods from now are

$$p^{++} = \text{Max}(0, 70 - 93.6) = \$0$$

$$p^{+-} = \text{Max}(0, 70 - 64.74) = \$5.26$$

$$p^{--} = \text{Max}(0, 70 - 44.78) = \$25.22$$

The risk-neutral probability is

$$\pi = \frac{1.05 - 0.83}{1.20 - 0.83} = 0.5946, \text{ and } 1 - \pi = 0.4054$$

Now find the option prices at time 1:

$$p^+ = \frac{0.5946(0) + 0.4054(5.26)}{1.05} = \$2.03$$

$$p^- = \frac{0.5946(5.26) + 0.4054(25.22)}{1.05} = \$12.72$$

The put price today is

$$p = \frac{0.5946(2.03) + 0.4054(12.72)}{1.05} = \$6.06$$

D. The hedge ratios at each point in the binomial tree are calculated as follows:

At the current stock price of \$65,

$$n = \frac{p^- - p^+}{S^+ - S^-} = \frac{12.72 - 2.03}{78 - 53.95} = 0.4445$$

Therefore, today at time 0, the risk-free hedge would consist of a long position in 10,000 puts and a long position in 4,445 shares of the underlying stock.

At stock price \$78,

$$n^+ = \frac{p^{+-} - p^{++}}{S^{++} - S^{+-}} = \frac{5.26 - 0}{93.6 - 64.74} = 0.1823$$

Therefore, today at time 0, the risk-free hedge would consist of a long position in 10,000 puts and a long position in 1,823 shares of the underlying stock.

At stock price \$53.95,

$$n^- = \frac{p^{--} - p^{+-}}{S^{+-} - S^{--}} = \frac{25.22 - 5.26}{64.74 - 44.78} = 1$$

Now, the risk-free hedge would consist of a long position in 10,000 puts and a long position in 10,000 shares of the underlying stock.

Now find the option prices at time 1:

$$p^+ = \frac{0.5946(0) + 0.4054(0)}{1.05} = \$0$$

$$p^- = \frac{0.5946(0) + 0.4054(15.22)}{1.05} = \$5.88$$

The put price today is

$$p = \frac{0.5946(0) + 0.4054(5.88)}{1.05} = \$2.27$$

B. Unlike the hedge portfolio for calls, which has the opposite positions in the two instruments (calls and underlying stock), the hedge portfolio for puts has the same positions in the two instruments. Therefore, the current value of the hedge portfolio for puts is

$$H = nS + p$$

The possible values of the hedge portfolio one period later are

$$H^+ = nS^+ + p^+$$

$$H^- = nS^- + p^-$$

Setting H^+ equal to H^- and solving for n,

$$n = \frac{p^- - p^+}{S^+ - S^-}$$

Note that the above formula is the same as that for the hedge portfolio for calls, except that the p^+ and p^- have switched positions in the numerator. Similarly, the hedge ratios for the next time point are

$$n^+ = \frac{p^{+-} - p^{++}}{S^{++} - S^{+-}}$$

$$n^- = \frac{p^{--} - p^{+-}}{S^{+-} - S^{--}}$$

At the current stock price of \$65,

$$n = \frac{p^- - p^+}{S^+ - S^-} = \frac{5.88 - 0}{78 - 53.95} = 0.2445$$

Therefore, today at time 0, the risk-free hedge would consist of a long position in 10,000 puts and a long position in 2,445 shares of the underlying stock.

At a stock price of \$78,

$$n^+ = \frac{p^{+-} - p^{++}}{S^{++} - S^{+-}} = \frac{0 - 0}{93.6 - 64.74} = 0$$

Zero shares of the underlying stock are needed for the long position in puts.

At a stock price of \$53.95,

$$n^- = \frac{p^{--} - p^{+-}}{S^{+-} - S^{--}} = \frac{15.22 - 0}{64.74 - 44.78} = 0.7625$$

Now the risk-free hedge would consist of a long position in 10,000 puts and a long position in 7,625 shares of the underlying stock.

$$c^- = \frac{0.5946(0) + 0.4054(0)}{1.05} = \$0$$

The call price today is

$$c = \frac{0.5946(13.36) + 0.4054(0)}{1.05} = \$7.57$$

D. The hedge ratios at each point in the binomial tree are calculated as follows.

At the current stock price of \$65,

$$n = \frac{c^+ - c^-}{S^+ - S^-} = \frac{13.36 - 0}{78 - 53.95} = 0.5555$$

Therefore, today at time 0, the risk-free hedge would consist of a short position in 10,000 calls and a long position in 5,555 shares of the underlying stock.

At a stock price of \$78,

$$n^+ = \frac{c^{++} - c^{+-}}{S^{++} - S^{+-}} = \frac{23.6 - 0}{93.6 - 64.74} = 0.8177$$

Now, the risk-free hedge would consist of a short position in 10,000 calls and a long position in 8,177 shares of the underlying stock.

At a stock price of \$53.95,

$$n^- = \frac{c^{+-} - c^{--}}{S^{+-} - S^{--}} = \frac{0 - 0}{64.74 - 44.78} = 0$$

Zero shares of the underlying stock are needed for the short position in calls.

13. Current stock price, S_0 = \$65
Up move, u = 1.20
Down move, d = 0.83
Risk-free rate, r = 5 percent

A. Exercise price, X = \$60

Stock prices in the binomial tree one and two periods from now are

$$S^+ = Su = 65(1.20) = \$78$$

$$S^- = Sd = 65(0.83) = \$53.95$$

$$S^{++} = Su^2 = 65(1.20)(1.20) = \$93.6$$

$$S^{+-} = Sud = 65(1.20)(0.83) = \$64.74$$

$$S^{--} = Sd^2 = 65(0.83)(0.83) = \$44.78$$

Put option values at expiration two periods from now are

$$p^{++} = \text{Max}(0,60 - 93.6) = \$0$$

$$p^{+-} = \text{Max}(0,60 - 64.74) = \$0$$

$$p^{--} = \text{Max}(0,60 - 44.78) = \$15.22$$

The risk-neutral probability is

$$\pi = \frac{1.05 - 0.83}{1.20 - 0.83} = 0.5946 \text{ and } 1 - \pi = 0.4054$$

The call price today is

$$c = \frac{0.5946(20.86) + 0.4054(2.68)}{1.05} = \$12.85$$

B. The hedge ratios at each point in the binomial tree are calculated as follows:

At the current stock price of \$65,

$$n = \frac{c^+ - c^-}{S^+ - S^-} = \frac{20.86 - 2.68}{78 - 53.95} = 0.7559$$

Therefore, today at time 0, the risk-free hedge would consist of a short position in 10,000 calls and a long position in 7,559 shares of the underlying stock.

At a stock price of \$78,

$$n^+ = \frac{c^{++} - c^{+-}}{S^{++} - S^{+-}} = \frac{33.6 - 4.74}{93.6 - 64.74} = 1$$

Now the risk-free hedge would consist of a short position in 10,000 calls and a long position in 10,000 shares of the underlying stock.

At a stock price of \$53.95,

$$\frac{n^- = c^{+-} - c^{--}}{S^{+-} - S^{--}} = \frac{4.74 - 0}{64.74 - 44.78} = 0.2375$$

Now the risk-free hedge would consist of a short position in 10,000 calls and a long position in 2,375 shares of the underlying stock.

C. Exercise price, X = \$70

Stock prices in the binomial tree one and two periods from now are

$$S^+ = Su = 65(1.20) = \$78$$

$$S^- = Sd = 65(0.83) = \$53.95$$

$$S^{++} = Su^2 = 65(1.20)(1.20) = \$93.6$$

$$S^{+-} = Sud = 65(1.20)(0.83) = \$64.74$$

$$S^{--} = Sd^2 = 65(0.83)(0.83) = \$44.78$$

Call option values at expiration two periods from now are

$$c^{++} = \text{Max}(0,93.6 - 70) = \$23.6$$

$$c^{+-} = \text{Max}(0,64.74 - 70) = \$0$$

$$c^{--} = \text{Max}(0,44.78 - 70) = \$0$$

The risk-neutral probability is

$$\pi = \frac{1.05 - 0.83}{1.20 - 0.83} = 0.5946, \text{ and } 1 - \pi = 0.4054$$

Now find the option prices at time 1:

$$c^+ = \frac{0.5946(23.6) + 0.4054(0)}{1.05} = \$13.36$$

C. If the current put price is \$11, it is underpriced. In order to create a hedge portfolio, we should buy the put and buy the underlying stock. The hedge ratio is

$$n = \frac{p^+ - p^-}{S^+ - S^-} = \frac{0 - 22.5}{199.5 - 127.5} = -0.3125$$

For every option purchased we should buy 0.3125 shares of stock. If we buy 10,000 puts we should buy 3,125 shares of stock.

Buy 10,000 puts at 11	= −110,000
Buy 3,125 shares at 150	= −468,750
Net cash flow	= −578,750

That is, we invest \$578,750.

At expiration, the value of this combination will be

$$3{,}125(199.5) + 10{,}000(0) = \$623{,}437 \text{ if } S_T = \$199.5$$

$$3{,}125(127.5) + 10{,}000(22.5) = \$623{,}437 \text{ if } S_T = \$127.5$$

We invested \$578,750 for a payoff of \$623,437. The rate of return is $\frac{623{,}437}{578{,}750} - 1 =$ 0.0772. This rate is higher than the risk-free rate of 0.045.

12. Current stock price, S = \$65
Up move, u = 1.20
Down move, d = 0.83
Risk-free rate, r = 5 percent

A. Exercise price, X = \$60

Stock prices in the binomial tree one and two periods from now are

$$S^+ = Su = 65(1.20) = \$78$$

$$S^- = Sd = 65(0.83) = \$53.95$$

$$S^{++} = Su^2 = 65(1.20)(1.20) = \$93.60$$

$$S^{+-} = Sud = 65(1.20)(0.83) = \$64.74$$

$$S^{--} = Sd^2 = 65(0.83)(0.83) = \$44.78$$

Call option values at expiration two periods from now are

$$c^{++} = \text{Max}(0, 93.60 - 60) = \$33.6$$

$$c^{+-} = \text{Max}(0, 64.74 - 60) = \$4.74$$

$$c^{--} = \text{Max}(0, 44.78 - 60) = \$0$$

The risk-neutral probability is

$$\pi = \frac{1.05 - 0.83}{1.20 - 0.83} = 0.5946, \text{ and } 1 - \pi = 0.4054$$

Now find the option prices at time 1:

$$c^+ = \frac{0.5946(33.6) + 0.4054(4.74)}{1.05} = \$20.86$$

$$c^- = \frac{0.5946(4.74) + 0.4054(0)}{1.05} = \$2.68$$

At expiration the value of this combination will be

$$100(20) - 80(110) = -\$6{,}800 \text{ if } S_T = 110$$

$$100(0) - 80(85) = -\$6{,}800 \text{ if } S_T = 85$$

We generated \$6,600 up front and pay back \$6,800. The rate of return is (6,800/6,600) − 1 = 0.0303. This borrowing rate is lower than the risk-free rate of 0.065.

11. Current stock price, $S = \$150$
Up move, $u = 1.33$
Down move, $d = 0.85$
Exercise price, $X = \$150$
Risk-free rate, $r = 4.5$ percent

A. Stock prices one period from now are

$$S^+ = Su = 150(1.33) = \$199.5$$

$$S^- = Sd = 150(0.85) = \$127.5$$

Put option values at expiration one period from now are

$$p^+ = \text{Max}(0, 150 - 199.5) = \$0$$

$$p^- = \text{Max}(0, 150 - 127.5) = \$22.5$$

The risk-neutral probability is

$$\pi = \frac{1.045 - 0.85}{1.33 - 0.85} = 0.4063, \text{ and } 1 - \pi = 0.5937$$

The put price today is

$$\pi = \frac{0.4063(0) + 0.5937(22.50)}{1.045} = 12.78$$

B. If the current put price is \$14, it is overpriced. In order to create a hedge portfolio, we should sell the put and short the underlying stock. The hedge ratio is

$$n = \frac{p^+ - p^-}{S^+ - S^-} = \frac{0 - 22.5}{199.5 - 127.5} = -0.3125$$

For every option sold, we should sell 0.3125 shares of stock. If we sell 10,000 puts, we should sell 3,125 shares of stock.

Sell 10,000 puts at 14	= 140,000
Sell 3,125 shares at 150	= 468,750
Net cash flow	= 608,750

Thus, we generate \$608,750 up front.

At expiration, the value of this combination will be

$$-3{,}125(199.5) - 10{,}000(0) = -\$623{,}437 \text{ if } S_T = \$199.5$$

$$-3{,}125(127.5) - 10{,}000(22.5) = -\$623{,}437 \text{ if } S_T = \$127.5$$

We generated \$608,750 up front and pay back \$623,437. The rate of return is

$$\frac{623{,}437}{608{,}750} - 1 = 0.0241$$

This borrowing rate is lower than the risk-free rate of 0.045.

10. Current stock price, S = \$100
Up move, u = 1.1
Down move, d = 0.85
Exercise price, X = \$90
Risk-free rate, r = 6.5 percent

A. Stock prices one period from now are

$$S^+ = Su = 100(1.1) = \$110$$

$$S^- = Sd = 100(0.85) = \$85$$

Call option values at expiration one period from now are

$$c^+ = \text{Max}(0,110 - 90) = \$20$$

$$c^- = \text{Max}(0,85 - 90) = \$0$$

The risk-neutral probability is

$$\pi = \frac{1.065 - 0.85}{1.1 - 0.85} = 0.86 \text{ and } 1 - \pi = 0.14$$

The call price today is

$$c = \frac{0.86(20) + 0.14(0)}{1.065} = 16.15$$

B. If the current call price is \$17.50, it is overpriced. Therefore, we should sell the call and buy the underlying stock. The hedge ratio is

$$n = \frac{20 - 0}{110 - 85} = 0.8$$

For every option sold we should purchase 0.8 shares of stock. If we sell 100 calls we should buy 80 shares of stock.

Sell 100 calls at 17.50	=	1,750
Buy 80 shares at 100	=	−8,000
Net cash flow	=	−6,250

At expiration the value of this combination will be

$$80(110) - 100(20) = \$6,800 \text{ if } S_T = 110$$

$$80(85) - 100(0) = \$6,800 \text{ if } S_T = 85$$

We invested \$6,250 for a payoff of \$6,800. The rate of return is (6,800/6,250) − 1 = 0.088. This rate is higher than the risk-free rate of 0.065.

C. If the current call price is \$14, it is underpriced. Therefore, we should buy the call and sell the underlying stock. The hedge ratio is

$$n = \frac{20 - 0}{110 - 85} = 0.8$$

For every option purchased we should sell 0.8 shares of stock. If we buy 100 calls we should sell 80 shares of stock.

Buy 100 calls at 14	=	−1,400
Sell 80 shares at 100	=	8,000
Net cash flow	=	6,600

Thus, we generate \$6,600 up front.

Time to expiration = 139/365 = 0.3808
Current stock price, S_0 = \$67.32
Bond price = $X/(1 + r)^T = 70/(1 + 0.05)^{0.3808}$ = \$68.71

A. Synthetic call = $p_0 + S_0 - X/(1 + r)^T = 6.8 + 67.32 - 68.71 = \$5.41$
Synthetic put = $c_0 + X/(1 + r)^T - S_0 = 4.5 + 68.71 - 67.32 = \$5.89$
Synthetic bond = $p_0 + S_0 - c_0 = 6.8 + 67.32 - 4.5 = \$69.62$
Synthetic underlying = $c_0 + X/(1 + r)^T - p_0 = 4.5 + 68.71 - 6.8 = \$66.41$

B.

Instrument	Actual Price	Synthetic Price	Mispricing/Profit
Call	4.50	5.41	0.91
Put	6.80	5.89	0.91
Bond	68.71	69.62	0.91
Stock	67.32	66.41	0.91

Thus, the mispricing is the same regardless of the instrument used to look at it.

C. The actual call is cheaper than the synthetic call. Therefore, an arbitrage transaction where you buy the call (underpriced) and sell the synthetic call (overpriced) will yield a risk-free profit of \$5.41 − \$4.50 = \$0.91.

As shown below, at expiration no cash will be received or paid out.

	Value at Expiration	
Transaction	$S_T < 70$	$S_T > 70$
Buy call	0	$S_T - 70$
Sell synthetic call		
Short put	$-(70 - S_T)$	0
Short stock	$-S_T$	$-S_T$
Long bond	70	70
Total	0	0

D. The actual put is more expensive than the synthetic put. Therefore, an arbitrage transaction in which you buy the synthetic put (underpriced) and sell the put (overpriced) will yield a risk-free profit of \$6.80 − \$5.89 = \$0.91. As shown below, at expiration no cash will be received or paid out.

	Value at Expiration	
Transaction	$S_T < 70$	$S_T > 70$
Sell put	$-(70 - S_T)$	0
Buy synthetic put		
Long call	0	$S_T - 70$
Long bond	70	70
Short stock	$-S_T$	$-S_T$
Total	0	0

Maximum value for the call: $C_0 = S_0 = \$1.05$
Lower bound for the call: $C_0 = \text{Max}[0, 1.05 - 0.95/(1.055)^{0.1644}] = \$0.11$
Maximum value for the put: $P_0 = X = \$0.95$
Lower bound for the put: $P_0 = \text{Max}(0, 0.95 - 1.05) = \$0$

ii. $X = \$1.10$
Maximum value for the call: $C_0 = S_0 = \$1.05$
Lower bound for the call: $C_0 = \text{Max}[0, 1.05 - 1.10/(1.055)^{0.1644}] = \$0$
Maximum value for the put: $P_0 = X = \$1.10$
Lower bound for the put: $P_0 = \text{Max}(0, 1.10 - 1.05) = \$0.05$

B. $S_0 = 1.05$, $T = 60/365 = 0.1644$, $X = 0.95$ or 1.10, European-style options

i. $X = \$0.95$
Maximum value for the call: $c_0 = S_0 = \$1.05$
Lower bound for the call: $c_0 = \text{Max}[0, 1.05 - 0.95/(1.055)^{0.1644}] = \$0.11$
Maximum value for the put: $p_0 = 0.95/(1.055)^{0.1644} = \$0.94$
Lower bound for the put: $p_0 = \text{Max}[0, 0.95/(1.055)^{0.1644} - 1.05] = \$0$

ii. $X = \$1.10$
Maximum value for the call: $c_0 = S_0 = \$1.05$
Lower bound for the call: $c_0 = \text{Max}[0, 1.05 - 1.10/(1.055)^{0.1644}] = \$0$
Maximum value for the put: $p_0 = 1.10/(1.055)^{0.1644} = \$1.09$
Lower bound for the put: $p_0 = \text{Max}[0, 1.10/(1.055)^{0.1644} - 1.05] = \$0.04$

8. We can illustrate put–call parity by showing that for the fiduciary call and the protective put, the current values and values at expiration are the same.
Call price, $c_0 = \$6.64$
Put price, $p_0 = \$2.75$
Exercise price, $X = \$30$
Risk-free rate, $r = 4$ percent
Time to expiration = 219/365 = 0.6
Current stock price, $S_0 = \$33.19$
Bond price, $X/(1 + r)^T = 30/(1 + 0.04)^{0.6} = \$29.3$

		Value at Expiration	
Transaction	Current Value	$S_T = 20$	$S_T = 40$
Fiduciary call			
Buy call	6.64	0	40 − 30 = 10
Buy bond	29.30	30	30
Total	35.94	30	40
Protective put			
Buy put	2.75	30 − 20 = 10	0
Buy stock	33.19	20	40
Total	35.94	30	40

The values in the table above show that the current values and values at expiration for the fiduciary call and the protective put are the same. That is, $c_0 + X/(1 + r)^T = p_0 + S_0$.

9. Call price, $c_0 = \$4.50$
Put price, $p_0 = \$6.80$
Exercise price, $X = \$70$
Risk-free rate, $r = 5$ percent

SOLUTIONS

1. A. $S_T = 579.32$
   i. Call payoff, X = 450: Max(0,579.32 − 450) × 100 = \$12,932
   ii. Call payoff, X = 650: Max(0,579.32 − 650) × 100 = 0

   B. $S_T = 579.32$
   i. Put payoff, X = 450: Max(0,450 − 579.32) × 100 = 0
   ii. Put payoff, X = 650: Max(0,650 − 579.32) × 100 = \$7,068

2. A. $S_T = \$0.95$
   i. Call payoff, X = 0.85: Max(0,0.95 − 0.85) × 100,000 = \$10,000
   ii. Call payoff, X = 1.15: Max(0,0.95 − 1.15) × 100,000 = \$0

   B. $S_T = \$0.95$
   i. Put payoff, X = 0.85: Max(0,0.85 − 0.95) × 100,000 = \$0
   ii. Put payoff, X = 1.15: Max(0,1.15 − 0.95) × 100,000 = \$20,000

3. A. $S_T = 0.0653$
   i. Call payoff, X = 0.05: Max(0,0.0653 − 0.05) × (180/360) × 10,000,000 = \$76,500
   ii. Call payoff, X = 0.08: Max(0,0.0653 − 0.08) × (180/360) × 10,000,000 = 0

   B. $S_T = 0.0653$
   i. Put payoff, X = 0.05: Max(0,0.05 − 0.0653) × (180/360) × 10,000,000 = 0
   ii. Put payoff, X = 0.08: Max(0,0.08 − 0.0653) × (180/360) × 10,000,000 = \$73,500

4. A. $S_T = \$1.438$
   i. Call payoff, X = 1.35: Max(0,1.438 − 1.35) × 125,000 = \$11,000
   ii. Call payoff, X = 1.55: Max(0,1.438 − 1.55) × 125,000 = \$0

   B. $S_T = \$1.438$
   i. Put payoff, X = 1.35: Max(0,1.35 − 1.438) × 125,000 = \$0
   ii. Put payoff, X = 1.55: Max(0,1.55 − 1.438) × 125,000 = \$14,000

5. A. $S_T = 1136.76$
   i. Call payoff, X = 1130: Max(0,1136.76 − 1130) × 1,000 = \$6,760
   ii. Call payoff, X = 1140: Max(0,1136.76 − 1140) × 1,000 = 0

   B. $S_T = 1136.76$
   i. Put payoff, X = 1130: Max(0,1130 − 1136.76) × 1,000 = 0
   ii. Put payoff, X = 1140: Max(0,1140 − 1136.76) × 1,000 = \$3,240

6. A. $S_0 = 1240.89$, T = 75/365 = 0.2055, X = 1225 or 1255, call options
   i. X = 1225
   Maximum value for the call: $c_0 = S_0 = 1240.89$
   Lower bound for the call: $c_0 = \text{Max}[0,1240.89 - 1225/(1.03)^{0.2055}] = 23.31$
   ii. X = 1255
   Maximum value for the call: $c_0 = S_0 = 1240.89$
   Lower bound for the call: $c_0 = \text{Max}[0,1240.89 - 1255/(1.03)^{0.2055}] = 0$

   B. $S_0 = 1240.89$, T = 75/365 = 0.2055, X = 1225 or 1255, put options
   i. X = 1225
   Maximum value for the put: $p_0 = 1225/(1.03)^{0.2055} = 1217.58$
   Lower bound for the put: $p_0 = \text{Max}[0,1225/(1.03)^{0.2055} - 1240.89] = 0$
   ii. X = 1255
   Maximum value for the put: $p_0 = 1255/(1.03)^{0.2055} = 1247.40$
   Lower bound for the put: $p_0 = \text{Max}[0,1255/(1.03)^{0.2055} - 1240.89] = 6.51$

7. A. $S_0 = 1.05$, T = 60/365 = 0.1644, X = 0.95 or 1.10, American-style options
   i. X = \$0.95

20. A. An interest rate call option based on a 90-day underlying rate has an exercise rate of 7.5 percent and expires in 180 days. The forward rate is 7.25 percent, and the volatility is 0.04. The continuously compounded risk-free rate is 5 percent. Calculate the price of the interest rate call option using the Black model.

B. An interest rate put option based on a 90-day underlying rate has an exercise rate of 7.5 percent and expires in 180 days. The forward rate is 7.25 percent, and the volatility is 0.04. The continuously compounded risk-free rate is 5 percent. Calculate the price of the interest rate put option using the Black model.

A. From the Black–Scholes–Merton model, obtain the approximate values of the call delta and put delta if the underlying asset price is $28.

B. Using the call delta and put delta obtained in Part A and the call and put prices given in the problem for the asset price of $28, calculate the approximate new call and put prices for a

i. $1 increase in the price of the underlying asset

ii. $5 increase in the price of the underlying asset

C. Based on a comparison of your answers in Part B with the actual call and put prices given in the problem, what can you say about the approximations based on delta?

17. Consider an asset that trades at $100 today. Call and put options on this asset are available with an exercise price of $100. The options expire in 275 days, and the volatility is 0.45. The continuously compounded risk-free rate is 3 percent.

A. Calculate the value of European call and put options using the Black–Scholes–Merton model. Assume that the present value of cash flows on the underlying asset over the life of the options is $4.25.

B. Calculate the value of European call and put options using the Black–Scholes–Merton model. Assume that the continuously compounded dividend yield is 1.5 percent.

18. Consider the following information on put and call options on an asset:

Call price, c_0 = $3.50

Put price, p_0 = $9

Exercise price, X = $50

Forward price, F(0,T) = $45

Days to option expiration = 175

Risk-free rate, r = 4 percent

A. Use put–call–forward parity to calculate prices of the following:

i. Synthetic call option

ii. Synthetic put option

iii. Synthetic forward contract

B. For each of the synthetic instruments in Part A, identify any mispricing by comparing the actual price with the synthetic price.

C. Based on the mispricing in Part B, illustrate how to earn a risk-free profit using a synthetic call.

D. Based on the mispricing in Part B, illustrate how to earn a risk-free profit using a synthetic put.

19. A forward contract is priced at 145. European options on the forward contract have an exercise price of 150 and expire in 65 days. The continuously compounded risk-free rate is 3.75 percent, and volatility is 0.33.

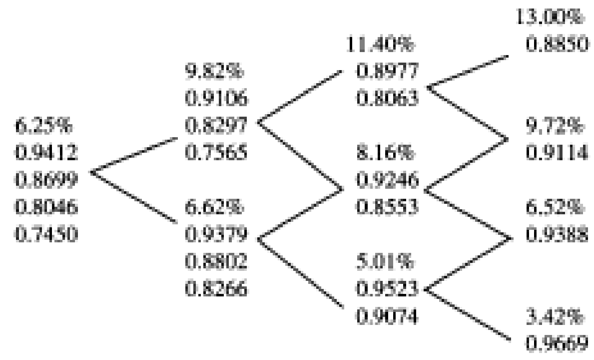
A. Calculate the price of a call option on the forward contract using the Black model.

B. Calculate the price of the underlying asset. Calculate the price of a call option on the underlying asset using the Black–Scholes–Merton model. Compare your answer here with the answer in Part A.

C. Calculate the price of a put option on the forward contract using the Black model.

D. Now calculate the price of a put option on the underlying asset using the Black–Scholes–Merton model. Compare your answer here with the answer in Part C.

the one-period interest rate and prices of zero-coupon bonds with maturities of one period, two periods, and three periods. At t = 2, you are provided with the one-period interest rate and prices of zero-coupon bonds with maturities of one period and two periods. At t = 3, you are provided with the one-period interest rate and prices of zero-coupon bonds with maturity of one period.



A. Calculate the value of a European call option on a four-period zero-coupon bond. The call option expires in two periods and has an exercise price of 0.85.

B. Calculate the value of a European call option on a four-period 8 percent coupon bond with a 1.0 face value. The call option expires in two periods and has an exercise price of 0.95.

C. Calculate the value of a European put option on a four-period zero-coupon bond. The call option expires in two periods and has an exercise price of 0.85.

D. Calculate the value of a European put option on a four-period 8 percent coupon bond with a 1.0 face value. The put option expires in two periods and has an exercise price of 1.

15. Assume that the evolution of one-period spot rates is still the same as in Problem 14; that is, the same three-period binomial interest rate tree still applies.

A. Calculate the price of a two-period cap with an exercise rate of 8 percent. The underlying is the one-period rate.

B. Calculate the price of a two-period floor with an exercise rate of 9 percent. The underlying is the one-period rate.

16. Call and put options on an asset are available with an exercise price of $30. The options expire in 75 days, and the volatility is 0.40. The continuously compounded risk-free rate is 3.5 percent, and there are no cash flows on the underlying. The precise Black–Scholes–Merton values of these options at different underlying prices are as follows.

Asset Price	Call Price	Put Price
$28	1.3144	3.0994
$29	1.7538	2.5388
$33	4.2270	1.0120

D. Based on the mispricing in Part B, illustrate an arbitrage transaction using a synthetic put.

10. A stock currently trades at a price of $100. The stock price can go up 10 percent or down 15 percent. The risk-free rate is 6.5 percent.

A. Use a one-period binomial model to calculate the price of a call option with an exercise price of $90.

B. Suppose the call price is currently $17.50. Show how to execute an arbitrage transaction that will earn more than the risk-free rate. Use 100 call options.

C. Suppose the call price is currently $14. Show how to execute an arbitrage transaction that replicates a loan that will earn less than the risk-free rate. Use 100 call options.

11. Suppose a stock currently trades at a price of $150. The stock price can go up 33 percent or down 15 percent. The risk-free rate is 4.5 percent.

A. Use a one-period binomial model to calculate the price of a put option with exercise price of $150.

B. Suppose the put price is currently $14. Show how to execute an arbitrage transaction that will earn more than the risk-free rate. Use 10,000 put options.

C. Suppose the put price is currently $11. Show how to execute an arbitrage transaction that will earn more than the risk-free rate. Use 10,000 put options.

12. Consider a two-period binomial model in which a stock currently trades at a price of $65. The stock price can go up 20 percent or down 17 percent each period. The risk-free rate is 5 percent.

A. Calculate the price of a call option expiring in two periods with an exercise price of $60.

B. Based on your answer in Part A, calculate the number of units of the underlying stock that would be needed at each point in the binomial tree to construct a risk-free hedge. Use 10,000 calls.

C. Calculate the price of a call option expiring in two periods with an exercise price of $70.

D. Based on your answer in Part C, calculate the number of units of the underlying stock that would be needed at each point in the binomial tree to construct a risk-free hedge. Use 10,000 calls.

13. Consider a two-period binomial model in which a stock currently trades at a price of $65. The stock price can go up 20 percent or down 17 percent each period. The risk-free rate is 5 percent.

A. Calculate the price of a put option expiring in two periods with exercise price of $60.

B. Based on your answer in Part A, calculate the number of units of the underlying stock that would be needed at each point in the binomial tree in order to construct a risk-free hedge. Use 10,000 puts.

C. Calculate the price of a put option expiring in two periods with an exercise price of $70.

D. Based on your answer in Part C, calculate the number of units of the underlying stock that would be needed at each point in the binomial tree in order to construct a risk-free hedge. Use 10,000 puts.

14. The three-period binomial interest rate tree provided below gives one-period interest rates and prices of zero-coupon bonds. Starting at t = 0, you are provided with the one-period interest rate and prices of zero-coupon bonds with maturities of one period, two periods, three periods, and four periods. At t = 1, you are provided with

6. Consider a stock index option that expires in 75 days. The stock index is currently at 1240.89 and makes no cash payments during the life of the option. Assume that the stock index has a multiplier of 1. The risk-free rate is 3 percent.

A. Calculate the lowest and highest possible prices for European-style call options on the above stock index with exercise prices of

i. 1225

ii. 1255

B. Calculate the lowest and highest possible prices for European-style put options on the above stock index with exercise prices of

i. 1225

ii. 1255

7. A. Consider American-style call and put options on a bond. The options expire in 60 days. The bond is currently at \$1.05 per \$1 par and makes no cash payments during the life of the option. The risk-free rate is 5.5 percent. Assume that the contract is on \$1 face value bonds. Calculate the lowest and highest possible prices for the calls and puts with exercise prices of

i. \$0.95

ii. \$1.10

B. Consider European style call and put options on a bond. The options expire in 60 days. The bond is currently at \$1.05 per \$1 par and makes no cash payments during the life of the option. The risk-free rate is 5.5 percent. Assume that the contract is on \$1 face value bonds. Calculate the lowest and highest possible prices for the calls and puts with exercise prices of

i. \$0.95

ii. \$1.10

8. You are provided with the following information on put and call options on a stock:

Call price, c_0 = \$6.64

Put price, p_0 = \$2.75

Exercise price, X = \$30

Days to option expiration = 219

Current stock price, S_0 = \$33.19

Put–call parity shows the equivalence of a call/bond portfolio (fiduciary call) and a put/underlying portfolio (protective put). Illustrate put–call parity assuming stock prices at expiration (S_T) of \$20 and of \$40. Assume that the risk-free rate, r, is 4 percent.

9. Consider the following information on put and call options on a stock:

Call price, c_0 = \$4.50

Put price, p_0 = \$6.80

Exercise price, X = \$70

Days to option expiration = 139

Current stock price, S_0 = \$67.32

Risk-free rate, r = 5 percent

A. Use put–call parity to calculate prices of the following:

i. Synthetic call option

ii. Synthetic put option

iii. Synthetic bond

iv. Synthetic underlying stock

B. For each of the synthetic instruments in Part A, identify any mispricing by comparing the actual price with the synthetic price.

C. Based on the mispricing in Part B, illustrate an arbitrage transaction using a synthetic call.

PROBLEMS

1. **A.** Calculate the payoff at expiration for a call option on the S&P 100 stock index in which the underlying price is 579.32 at expiration, the multiplier is 100, and the exercise price is
   - i. 450
   - ii. 650

   B. Calculate the payoff at expiration for a put option on the S&P 100 in which the underlying is at 579.32 at expiration, the multiplier is 100, and the exercise price is
   - i. 450
   - ii. 650

2. **A.** Calculate the payoff at expiration for a call option on a bond in which the underlying is at $0.95 per $1 par at expiration, the contract is on $100,000 face value bonds, and the exercise price is
   - i. $0.85
   - ii. $1.15

   B. Calculate the payoff at expiration for a put option on a bond in which the underlying is at $0.95 per $1 par at expiration, the contract is on $100,000 face value bonds, and the exercise price is
   - i. $0.85
   - ii. $1.15

3. **A.** Calculate the payoff at expiration for a call option on an interest rate in which the underlying is a 180-day interest rate at 6.53 percent at expiration, the notional principal is $10 million, and the exercise price is
   - i. 5 percent
   - ii. 8 percent

   B. Calculate the payoff at expiration for a put option on an interest rate in which the underlying is a 180-day interest rate at 6.53 percent at expiration, the notional principal is $10 million, and the exercise price is
   - i. 5 percent
   - ii. 8 percent

4. **A.** Calculate the payoff at expiration for a call option on the British pound in which the underlying is at $1.438 at expiration, the options are on 125,000 British pounds, and the exercise price is
   - i. $1.35
   - ii. $1.55

   B. Calculate the payoff at expiration for a put option on the British pound where the underlying is at $1.438 at expiration, the options are on 125,000 British pounds, and the exercise price is
   - i. $1.35
   - ii. $1.55

5. **A.** Calculate the payoff at expiration for a call option on a futures contract in which the underlying is at 1136.76 at expiration, the options are on a futures contract for $1,000, and the exercise price is
   - i. 1130
   - ii. 1140

   B. Calculate the payoff at expiration for a put option on a futures contract in which the underlying is at 1136.76 at expiration, the options are on a futures contract for $1000, and the exercise price is
   - i. 1130
   - ii. 1140

APPENDIX 4A Cumulative Probabilities for a Standard Normal Distribution
$P(X \le x) = N(x)$ for $x \ge 0$ or $1 - N(-x)$ for $x < 0$

x	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.00	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.10	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.20	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.30	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.40	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.50	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.60	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.70	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.80	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.90	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.00	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.10	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.20	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.30	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.40	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.50	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.60	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.70	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.80	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.90	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.00	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.10	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.20	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.30	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.40	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.50	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.60	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.70	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.80	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.90	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.00	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

change in the underlying price. To construct a delta-hedged position, a short (long) position in each option is matched with a long (short) position in delta units of the underlying. Changes in the underlying price will generate offsetting changes in the value of the option position, provided the changes in the underlying price are small and occur over a short time period. A delta-hedged position should be adjusted as the delta changes and time passes.

- If changes in the price of the underlying are large or the delta hedge is not adjusted over a longer time period, the hedge may not be effective. This effect is due to the instability of the delta and is called the gamma effect. If the gamma effect is large, option price changes will not be very close to the changes as approximated by the delta times the underlying price change.
- Cash flows on the underlying are accommodated in option pricing models by reducing the price of the underlying by the present value of the cash flows over the life of the option.
- Volatility can be estimated by calculating the standard deviation of the continuously compounded returns from a sample of recent data for the underlying. This is called the historical volatility. An alternative measure, called the implied volatility, can be obtained by setting the Black–Scholes–Merton model price equal to the market price and inferring the volatility. The implied volatility is a measure of the volatility the market is using to price the option.
- The payoffs of a call on a forward contract and an appropriately chosen zero-coupon bond are equivalent to the payoffs of a put on the forward contract and the forward contract. Thus, their current values must be the same. For this equality to occur, the call price plus the bond price must equal the put price. The appropriate zero-coupon bond is one with a face value equal to the exercise price minus the forward price. This relationship is called put–call–forward (or futures) parity.
- There is no justification for exercising American options on forward contracts early, so they are equivalent to European options on forwards. American options on futures, both calls and puts, can sometimes be exercised early, so they are different from European options on futures and carry a higher price.
- The Black model can be used to price European options on forwards or futures by entering the forward price, exercise price, risk-free rate, time to expiration, and volatility into a formula that will also require the determination of two normal probabilities.
- The Black model can be used to price European options on interest rates by entering the forward interest rate into the model for the forward or futures price and the exercise rate for the exercise price.
- Options are useful in financial markets because they provide a way to limit losses to the premium paid while permitting potentially large gains. They can be used for hedging purposes, especially in the case of puts, which can be used to limit the loss on a long position in an asset. Options also provide information on the volatility of the underlying asset. Options can be standardized and exchange-traded or customized in the over-the-counter market.

- Cash flows on the underlying affect an option's boundary conditions and put–call parity by lowering the underlying price by the present value of the cash flows over the life of the option.
- A higher interest rate increases a call option's price and decreases a put option's price.
- In a one-period binomial model, the underlying asset can move up to one of two prices. A portfolio consisting of a long position in the underlying and a short position in a call option can be made risk-free and, therefore, must return the risk-free rate. Under this condition, the option price can be obtained by inferring it from a formula that uses the other input values. The option price is a weighted average of the two option prices at expiration, discounted back one period at the risk-free rate.
- If an option is trading for a price higher than that given in the binomial model, one can sell the option and buy a specific number of units of the underlying, as given by the model. This combination is risk free but will earn a return higher than the risk-free rate. If the option is trading for a price lower than the price given in the binomial model, a short position in a specific number of units of the underlying and a long position in the option will create a risk-free loan that costs less than the risk-free rate.
- In a two-period binomial model, the underlying can move to one of two prices in each of two periods; thus three underlying prices are possible at the option expiration. To price an option, start at the expiration and work backward, following the procedure in the one-period model in which an option price at any given point in time is a weighted average of the next two possible prices discounted at the risk-free rate.
- To calculate the price of an option on a zero-coupon bond or a coupon bond, one must first construct a binomial tree of the price of the bond over the life of the option. To calculate the price of an option on an interest rate, one should use a binomial tree of interest rates. Then the option price is found by starting at the option expiration, determining the payoff and successively working backwards by computing the option price as the weighted average of the next two option prices discounted back one period. For the case of options on bonds or interest rates, a different discount rate is used at different parts of the tree.
- For an option of a given expiration, a greater pricing accuracy is obtained by dividing the option's life into a greater number of time periods in a binomial tree. As more time periods are added, the discrete-time binomial price converges to a stable value as though the option is being modeled in a continuous-time world.
- The assumptions under which the Black–Scholes–Merton model is derived state that the underlying asset follows a geometric lognormal diffusion process, the risk-free rate is known and constant, the volatility of the underlying asset is known and constant, there are no taxes or transaction costs, there are no cash flows on the underlying, and the options are European.
- To calculate the value of an option using the Black–Scholes–Merton model, enter the underlying price, exercise price, risk-free rate, volatility, and time to expiration into a formula. The formula will require you to look up two normal probabilities, obtained from either a table or preferably a computer routine.
- The change in the option price for a change in the price of the underlying is called the delta. The change in the option price for a change in the risk-free rate is called the rho. The change in the option price for a change in the time to expiration is called the theta. The change in the option price for a change in the volatility is called the vega.
- The delta is defined as the change in the option price divided by the change in the underlying price. The option price change can be approximated by the delta times the

- The lower bound of a European call is established by constructing a portfolio consisting of a long call and risk-free bond and a short position in the underlying asset. This combination produces a non-negative value at expiration, so its current value must be non-negative. For this situation to occur, the call price has to be worth at least the underlying price minus the present value of the exercise price. The lower bound of a European put is established by constructing a portfolio consisting of a long put, a long position in the underlying, and the issuance of a zero-coupon bond. This combination produces a non-negative value at expiration so its current value must be non-negative. For this to occur, the put price has to be at least as much as the present value of the exercise price minus the underlying price. For both calls and puts, if this lower bound is negative, we invoke the rule that an option price can be no lower than zero.
- The lowest price of a European call is referred to as the lower bound. The lowest price of an American call is also the lower bound of a European call. The lowest price of a European put is also referred to as the lower bound. The lowest price of an American put, however, is its intrinsic value.
- Buying a call with a given exercise price and selling an otherwise identical call with a higher exercise price creates a combination that always pays off with a non-negative value. Therefore, its current value must be non-negative. For this to occur, the call with the lower exercise price must be worth at least as much as the other call. A similar argument holds for puts, except that one would buy the put with the higher exercise price. This line of reasoning shows that the put with the higher exercise price must be worth at least as much as the one with the lower exercise price.
- A longer-term European or American call must be worth at least as much as a corresponding shorter-term European or American call. A longer-term American put must be worth at least as much as a shorter-term American put. A longer-term European put, however, can be worth more or less than a shorter-term European put.
- A fiduciary call, consisting of a European call and a zero-coupon bond, produces the same payoff as a protective put, consisting of the underlying and a European put. Therefore, their current values must be the same. For this equivalence to occur, the call price plus bond price must equal the underlying price plus put price. This relationship is called put–call parity and can be used to identify combinations of instruments that synthesize another instrument by rearranging the equation to isolate the instrument you are trying to create. Long positions are indicated by positive signs, and short positions are indicated by negative signs. One can create a synthetic call, a synthetic put, a synthetic underlying, and a synthetic bond, as well as synthetic short positions in these instruments for the purpose of exploiting mispricing in these instruments.
- Put–callparity violations exist when one side of the equation does not equal the other. An arbitrageur buys the lower-priced side and sells the higher-priced side, thereby earning the difference in price, and the positions offset at expiration. The combined actions of many arbitrageurs performing this set of transactions would increase the demand and price for the underpriced instruments and decrease the demand and price for the overpriced instruments, until the put–call parity relationship is upheld.
- American option prices must always be no less than those of otherwise equivalent European options. American call options, however, are never exercised early unless there is a cash flow on the underlying, so they can sell for the same as their European counterparts in the absence of such a cash flow. American put options nearly always have a possibility of early exercise, so they ordinarily sell for more than their European counterparts.

In Chapter 2, we covered forward contracts; in Chapter 3, we covered futures contracts; and in this chapter we covered option contracts. We have one more major class of derivative instruments, swaps, which we now turn to in Chapter 5. We shall return to options in Chapter 7, where we explore option trading strategies.

KEY POINTS

- Options are rights to buy or sell an underlying at a fixed price, the exercise price, for a period of time. The right to buy is a call; the right to sell is a put. Options have a definite expiration date. Using the option to buy or sell is the action of exercising it. The buyer or holder of an option pays a price to the seller or writer for the right to buy (a call) or sell (a put) the underlying instrument. The writer of an option has the corresponding potential obligation to sell or buy the underlying.
- European options can be exercised only at expiration; American options can be exercised at any time prior to expiration. Moneyness refers to the characteristic that an option has positive intrinsic value. The payoff is the value of the option at expiration. An option's intrinsic value is the value that can be captured if the option is exercised. Time value is the component of an option's price that reflects the uncertainty of what will happen in the future to the price of the underlying.
- Options can be traded as standardized instruments on an options exchange, where they are protected from default on the part of the writer, or as customized instruments on the over-the-counter market, where they are subject to the possibility of the writer defaulting. Because the buyer pays a price at the start and does not have to do anything else, the buyer cannot default.
- The underlying instruments for options are individual stocks, stock indices, bonds, interest rates, currencies, futures, commodities, and even such random factors as the weather. In addition, a class of options called real options is associated with the flexibility in capital investment projects.
- Like FRAs, which are forward contracts in which the underlying is an interest rate, interest rate options are options in which the underlying is an interest rate. However, FRAs are commitments to make one interest payment and receive another, whereas interest rate options are rights to make one interest payment and receive another.
- Option payoffs, which are the values of options when they expire, are determined by the greater of zero or the difference between underlying price and exercise price, if a call, or the greater of zero or the difference between exercise price and underlying price, if a put. For interest rate options, the exercise price is a specified rate and the underlying price is a variable interest rate.
- Interest rate options exist in the form of caps, which are call options on interest rates, and floors, which are put options on interest rates. Caps consist of a series of call options, called caplets, on an underlying rate, with each option expiring at a different time. Floors consist of a series of put options, called floorlets, on an underlying rate, with each option expiring at a different time.
- The minimum value of European and American calls and puts is zero. The maximum value of European and American calls is the underlying price. The maximum value of a European put is the present value of the exercise price. The maximum value of an American put is the exercise price.