CHAPTER

3 Futures Markets and Contracts

LEARNING OUTCOMES

After completing this chapter, you will be able to do the following:

- Identify the institutional features that distinguish futures contracts from forward contracts.
- Understand the origins of modern futures markets.
- List the primary characteristics of futures contracts.
- Explain the difference between margin in the securities markets and margin in the futures markets.
- Describe how a futures trade takes place.
- Describe how a futures position may be closed out prior to expiration.
- Define initial margin, maintenance margin, variation margin, and settlement price.
- Describe the process of marking to market.
- Compute the margin balance given the previous day's balance and the new futures price.
- Explain price limits, limit move, limit up, limit down, and locked limit.
- Describe how a futures contract can be terminated by either a closeout at expiration, delivery, an equivalent cash settlement, or an exchange for physicals.
- Explain delivery options in futures contracts.
- Distinguish among scalpers, day traders, and position traders.
- Describe the primary characteristics of the following types of futures contracts: Treasury bill, Eurodollar, Treasury bond, stock index, and currency.
- Explain why the futures price must converge to the spot price at expiration.
- Explain how to determine the value of a futures contract.
- Explain how forward and futures prices differ.
- Describe how an arbitrage transaction is constructed to derive the futures price.
- Identify the different types of monetary and nonmonetary benefits and costs associated with holding the underlying asset, and explain how they affect the futures price.
- Define backwardation and contango.
- Discuss whether futures prices equal expected spot prices.

- Describe and illustrate how to price Treasury bill futures.
- Explain the concept of an implied repo rate.
- Describe and illustrate the difficulties in determining the price of Eurodollar futures.
- Describe and illustrate how to price Treasury bond futures.
- Describe and illustrate how to price stock index futures.
- Describe and illustrate how to price currency futures.
- Discuss the role of futures markets and exchanges in financial systems and in society.

1 INTRODUCTION

In Chapter 1, we undertook a general overview of derivative markets. In Chapter 2, we focused on forward markets. Now we explore futures markets in a similar fashion. Although we shall see a clear similarity between forward and futures contracts, critical distinctions nonetheless exist between the two.

In Chapter 1 we learned that, like a forward contract, *a futures contract is an agreement between two parties in which one party, the buyer, agrees to buy from the other party, the seller, an underlying asset or other derivative, at a future date at a price agreed on today.* Unlike a forward contract, however, a futures contract is not a private and customized transaction but rather a public transaction that takes place on an organized futures exchange. In addition, a futures contract is standardized—the exchange, rather than the individual parties, sets the terms and conditions, with the exception of price. As a result, futures contracts have a secondary market, meaning that previously created contracts can be traded. Also, parties to futures contracts are guaranteed against credit losses resulting from the counterparty's inability to pay. A clearinghouse provides this guarantee via a procedure in which it converts gains and losses that accrue on a daily basis into actual cash gains and losses. Futures contracts are regulated at the federal government level; as we noted in Chapter 2, forward contracts are essentially unregulated. Futures contracts are created on organized trading facilities referred to as futures exchanges, whereas forward contracts are not created in any specific location but rather initiated between any two parties who wish to enter into such a contract. Finally, each futures exchange has a division or subsidiary called a clearinghouse that performs the specific responsibilities of paying and collecting daily gains and losses as well as guaranteeing to each party the performance of the other.

In a futures transaction, one party, the long, is the buyer and the other party, the short, is the seller. The buyer agrees to buy the underlying at a later date, the expiration, at a price agreed on at the start of the contract. The seller agrees to sell the underlying to the buyer at the expiration, at the price agreed on at the start of the contract. Every day, the futures contract trades in the market and its price changes in response to new information. Buyers benefit from price increases, and sellers benefit from price decreases. On the expiration day, the contract terminates and no further trading takes place. Then, either the buyer takes delivery of the underlying from the seller, or the two parties make an equivalent cash settlement. We shall explore each of these characteristics of futures contracts in more detail. First, however, it is important to take a brief look at how futures markets came into being.

1.1 A Brief History of Futures Markets

Although vestiges of futures markets appear in the Japanese rice markets of the 18th century and perhaps even earlier, the mid-1800s marked the first clear origins of modern futures markets. For example, in the United States in the 1840s, Chicago was becoming a major transportation and distribution center for agricultural commodities. Its central location and access to the Great Lakes gave Chicago a competitive advantage over other U.S. cities. Farmers from the Midwest would harvest their grain and take it to Chicago for sale. Grain production, however, is seasonal. As a result, grain prices would rise sharply just prior to the harvest but then plunge when the grain was brought to the market. Too much grain at one time and too little at another resulted in severe problems. Grain storage facilities in Chicago were inadequate to accommodate the oversupply. Some farmers even dumped their grain in the Chicago River because prices were so low that they could not afford to take their grain to another city to sell.

To address this problem, in 1848 a group of businessmen formed an organization later named the Chicago Board of Trade (CBOT) and created an arrangement called a "to-arrive" contract. These contracts permitted farmers to sell their grain before delivering it. In other words, farmers could harvest the grain and enter into a contract to deliver it at a much later date at a price already agreed on. This transaction allowed the farmer to hold the grain in storage at some other location besides Chicago. On the other side of these contracts were the businessmen who had formed the Chicago Board of Trade.

It soon became apparent that trading in these to-arrive contracts was more important and useful than trading in the grain itself. Soon the contracts began trading in a type of secondary market, which allowed buyers and sellers to discharge their obligations by passing them on, for a price, to other parties. With the addition of the clearinghouse in the 1920s, which provided a guarantee against default, modern futures markets firmly established their place in the financial world. It was left to other exchanges, such as today's Chicago Mercantile Exchange, the New York Mercantile Exchange, Eurex, and the London International Financial Futures Exchange, to develop and become, along with the Chicago Board of Trade, the global leaders in futures markets.

We shall now explore the important features of futures contracts in more detail.

1.2 Public Standardized Transactions

A private transaction is not generally reported in the news or to any price-reporting service. Forward contracts are private contracts. Just as in most legal contracts, the parties do not publicly report that they have engaged in a contract. In contrast, a futures transaction is reported to the futures exchange, the clearinghouse, and at least one regulatory agency. The price is recorded and available from price reporting services and even on the Internet.[1]

We noted that a futures transaction is not customized. Recall from Chapter 2 that in a forward contract, the two parties establish all of the terms of the contract, including the identity of the underlying, the expiration date, and the manner in which the contract is settled (cash or actual delivery) as well as the price. The terms are customized to meet the needs of both parties. In a futures contract, the price is the only term established by the two parties; the exchange establishes all other terms. Moreover, the terms that are established by the exchange are standardized, meaning that the exchange selects a number of choices for underlyings, expiration dates, and a variety of other contract-specific items. These standardized terms are well known to all parties. If a party wishes to trade a futures contract, it must accept these terms. The only alternative would be to create a similar but customized contract on the forward market.

[1] The information reported to the general public does not disclose the identity of the parties to transactions but only that a transaction took place at a particular price.

With respect to the underlying, for example, a given asset has a variety of specifications and grades. Consider a futures contract on U.S. Treasury bonds. There are many different Treasury bonds with a variety of characteristics. The futures exchange must decide which Treasury bond or group of bonds the contract covers. One of the most actively traded commodity futures contracts is oil, but there are many different types of oil.[2] To which type of oil does the contract apply? The exchange decides at the time it designs the contract.

The parties to a forward contract set its expiration at whatever date they want. For a futures contract, the exchange establishes a set of expiration dates. The first specification of the expiration is the month. An exchange might establish that a given futures contract expires only in the months of March, June, September, and December. The second specification determines how far the expirations go out into the future. For example, in January of a given year, there may be expirations of March, June, September, and December. Expirations might also be available for March, June, September, and December of the following year, and perhaps some months of the year after that. The exchange decides which expiration months are appropriate for trading, based on which expirations they believe would be actively traded. Treasury bond futures have expirations going out only about a year. Eurodollar futures, however, have expirations that go out about 10 years.[3] The third specification of the expiration is the specific day of expiration. Many, but not all, contracts expire some time during the third week of the expiration month.

The exchange determines a number of other contract characteristics, including the contract size. For example, one Eurodollar futures contract covers $1 million of a Eurodollar time deposit. One U.S. Treasury bond futures contract covers $100,000 face value of Treasury bonds. One futures contract on crude oil covers 1,000 barrels. The exchange also decides on the price quotation unit. For example, Treasury bond futures are quoted in points and 32nds of par of 100. Hence, you will see a price like 104 21/32, which means 104.65625. With a contract size of $100,000, the actual price is $104,656.25.

The exchange also determines what hours of the day trading takes place and at what physical location on the exchange the contract will be traded. Many futures exchanges have a trading floor, which contains octagonal-shaped pits. A contract is assigned to a certain pit. Traders enter the pits and express their willingness to buy and sell by calling out and/or indicating by hand signals their bids and offers. Some exchanges have electronic trading, which means that trading takes place on computer terminals, generally located in companies' offices. Some exchanges have both floor trading and electronic trading; some have only one or the other.

1.3 Homogenization and Liquidity

By creating contracts with generally accepted terms, the exchange standardizes the instrument. In contrast, forward contracts are quite heterogeneous because they are customized. Standardizing the instrument makes it more acceptable to a broader group of participants, with the advantage being that the instrument can then more easily trade in a type of secondary market. Indeed, the ability to sell a previously purchased contract or purchase a previously sold contract is one of the important features of futures contracts. A futures contract is therefore said to have liquidity in contrast to a forward contract, which does not generally trade after it has been created.[4] This ability to trade a previously opened contract

[2] Some of the main types are Saudi Arabian light crude, Brent crude, and West Texas intermediate crude.

[3] You may be wondering why some Eurodollar futures contracts have such long expirations. Dealers in swaps and forward rate agreements use Eurodollar futures to hedge their positions. Many of those over-the-counter contracts have very long expirations.

[4] The notion of liquidity here is only that a market exists for futures contracts, but this does not imply a high degree of liquidity. There may be little trading in a given contract, and the bid–ask spread can be high. In contrast, some forward markets can be very liquid, allowing forward contracts to be offset, as described in Chapter 2.

allows participants in this market to offset the position before expiration, thereby obtaining exposure to price movements in the underlying without the actual requirement of holding the position to expiration. We shall discuss this characteristic further when we describe futures trading in Section 2.

1.4 The Clearinghouse, Daily Settlement, and Performance Guarantee

Another important distinction between futures and forwards is that the futures exchange guarantees to each party the performance of the other party, through a mechanism known as the clearinghouse. This guarantee means that if one party makes money on the transaction, it does not have to worry about whether it will collect the money from the other party because the clearinghouse ensures it will be paid. In contrast, each party to a forward contract assumes the risk that the other party will default.

An important and distinguishing feature of futures contracts is that the gains and losses on each party's position are credited and charged on a daily basis. This procedure, called **daily settlement** or **marking to market**, essentially results in paper gains and losses being converted to cash gains and losses each day. It is also equivalent to terminating a contract at the end of each day and reopening it the next day at that settlement price. In some sense, a futures contract is like a strategy of opening up a forward contract, closing it one day later, opening up a new contract, closing it one day later, and continuing in that manner until expiration. The exact manner in which the daily settlement works will be covered in more detail later in Section 3.

1.5 Regulation

In most countries, futures contracts are regulated at the federal government level. State and regional laws may also apply. In the United States, the Commodity Futures Trading Commission regulates the futures market. In the United Kingdom, the Securities and Futures Authority regulates both the securities and futures markets.

Federal regulation of futures markets generally arises out of a concern to protect the general public and other futures market participants, as well as through a recognition that futures markets affect all financial markets and the economy. Regulations cover such matters as ensuring that prices are reported accurately and in a timely manner, that markets are not manipulated, that professionals who offer their services to the public are qualified and honest, and that disputes are resolved. In the United States, the government has delegated some of these responsibilities to an organization called the National Futures Association (NFA). An industry self-regulatory body, the NFA was created with the objective of having the industry regulate itself and reduce the federal government's burden.

2 FUTURES TRADING

In this section, we look more closely at how futures contracts are traded. As noted above, futures contracts trade on a futures exchange either in a pit or on a screen or electronic terminal.

We briefly mentioned pit trading, also known as floor-based trading, in Section 1.2. Pit trading is a very physical activity. Traders stand in the pit and shout out their orders in the form of prices they are willing to pay or accept. They also use hand signals to indicate their bids and offers.[5] They engage in transactions with other traders in the pits by simply agreeing on a price and number of contracts to trade. The activity is fast, furious, exciting, and stressful. The average pit trader is quite young, owing to the physical demands of the job and the toll it takes on body and mind. In recent years, more trading has come off of the exchange floor to electronic screens or terminals. In electronic or screen-based trading,

[5] Hand signals facilitate trading with someone who is too far away in the pit for verbal communication.

exchange members enter their bids and offers into a computer system, which then displays this information and allows a trader to consummate a trade electronically. In the United States, pit trading is dominant, owing to its long history and tradition. Exchange members who trade on the floor enjoy pit trading and have resisted heavily the advent of electronic trading. Nonetheless, the exchanges have had to respond to market demands to offer electronic trading. In the United States, both pit trading and electronic trading are used, but in other countries, electronic trading is beginning to drive pit trading out of business.[6]

A person who enters into a futures contract establishes either a long position or a short position. Similar to forward contracts, long positions are agreements to buy the underlying at the expiration at a price agreed on at the start. Short positions are agreements to sell the underlying at a future date at a price agreed on at the start. When the position is established, each party deposits a small amount of money, typically called the margin, with the clearinghouse. Then, as briefly described in Section 1.4, the contract is marked to market, whereby the gains are distributed to and the losses collected from each party. We cover this marking-to-market process in more detail in the next section. For now, however, we focus only on the opening and closing of the position.

A party that has opened a long position collects profits or incurs losses on a daily basis. At some point in the life of the contract prior to expiration, that party may wish to re-enter the market and close out the position. This process, called **offsetting**, is the same as selling a previously purchased stock or buying back a stock to close a short position. The holder of a long futures position simply goes back into the market and offers the identical contract for sale. The holder of a short position goes back into the market and offers to buy the identical contract. It should be noted that when a party offsets a position, it does not necessary do so with the same counterparty to the original contract. In fact, rarely would a contract be offset with the same counterparty. Because of the ability to offset, futures contracts are said to be fungible, which means that any futures contract with any counterparty can be offset by an equivalent futures contract with another counterparty. Fungibility is assured by the fact that the clearinghouse inserts itself in the middle of each contract and, therefore, becomes the counterparty to each party.

For example, suppose in early January a futures trader purchases an S&P 500 stock index futures contract expiring in March. Through 15 February, the trader has incurred some gains and losses from the daily settlement and decides that she wants to close the position out. She then goes back into the market and offers for sale the March S&P 500 futures. Once she finds a buyer to take the position, she has a long and short position in the same contract. The clearinghouse considers that she no longer has a position in that contract and has no remaining exposure, nor any obligation to make or take delivery at expiration. Had she initially gone short the March futures, she might re-enter the market in February offering to buy it. Once she finds a seller to take the opposite position, she becomes long and short the same contract and is considered to have offset the contract and therefore have no net position.

3 THE CLEARINGHOUSE, MARGINS, AND PRICE LIMITS

As briefly noted in the previous section, when a trader takes a long or short position in a futures, he must first deposit sufficient funds in a margin account. This amount of money is traditionally called the margin, a term derived from the stock market practice in which an investor borrows a portion of the money required to purchase a certain amount of stock.

[6] For example, in France electronic trading was introduced while pit trading continued. Within two weeks, all of the volume had migrated to electronic trading and pit trading was terminated.

Margin in the stock market is quite different from margin in the futures market. In the stock market, "margin" means that a loan is made. The loan enables the investor to reduce the amount of his own money required to purchase the securities, thereby generating leverage or gearing, as it is sometimes known. If the stock goes up, the percentage gain to the investor is amplified. If the stock goes down, however, the percentage loss is also amplified. The borrowed money must eventually be repaid with interest. The margin percentage equals the market value of the stock minus the market value of the debt divided by the market value of the stock—in other words, the investor's own equity as a percentage of the value of the stock. For example, in the United States, regulations permit an investor to borrow up to 50 percent of the initial value of the stock. This percentage is called the initial margin requirement. On any day thereafter, the equity or percentage ownership in the account, measured as the market value of the securities minus the amount borrowed, can be less than 50 percent but must be at least a percentage known as the maintenance margin requirement. A typical maintenance margin requirement is 25 to 30 percent.

In the futures market, by contrast, the word **margin** is commonly used to describe the amount of money that must be put into an account by a party opening up a futures position, but the term is misleading. When a transaction is initiated, a futures trader puts up a certain amount of money to meet the **initial margin requirement**; however, the remaining money is not borrowed. The amount of money deposited is more like a down payment for the commitment to purchase the underlying at a later date. Alternatively, one can view this deposit as a form of good faith money, collateral, or a performance bond: The money helps ensure that the party fulfills his or her obligation.[7] Moreover, both the buyer and the seller of a futures contract must deposit margin.

In securities markets, margin requirements are normally set by federal regulators. In the United States, maintenance margin requirements are set by the securities exchanges and the NASD. In futures markets, margin requirements are set by the clearinghouses. In further contrast to margin practices in securities markets, futures margins are traditionally expressed in dollar terms and not as a percentage of the futures price. For ease of comparison, however, we often speak of the futures margin in terms of its relationship to the futures price. In futures markets, the initial margin requirement is typically much lower than the initial margin requirement in the stock market. In fact, futures margins are usually less than 10 percent of the futures price.[8] Futures clearinghouses set their margin requirements by studying historical price movements. They then establish minimum margin levels by taking into account normal price movements and the fact that accounts are marked to market daily. The clearinghouses thus collect and disburse margin money every day. Moreover, they are permitted to do so more often than daily, and on some occasions they have used that privilege. By carefully setting margin requirements and collecting margin money every day, clearinghouses are able to control the risk of default.

In spite of the differences in margin practices for futures and securities markets, the effect of leverage is similar for both. By putting up a small amount of money, the trader's gains and losses are magnified. Given the tremendously low margin requirements of futures markets, however, the magnitude of the leverage effect is much greater in futures markets. We shall see how this works as we examine the process of the daily settlement.

[7] In fact, the Chicago Mercantile Exchange uses the term "performance bond" instead of "margin." Most other exchanges use the term "margin."

[8] For example, the margin requirement of the Eurodollar futures contract at the Chicago Mercantile Exchange has been less than one-tenth of one percent of the futures price. An exception to this requirement, however, is individual stock futures, which in the United States have margin requirements comparable to those of the stock market.

As previously noted, each day the clearinghouse conducts an activity known as the daily settlement, also called marking to market. This practice results in the conversion of gains and losses on paper into actual gains and losses. As margin account balances change, holders of futures positions must maintain balances above a level called the **maintenance margin requirement**. The maintenance margin requirement is lower than the initial margin requirement. On any day in which the amount of money in the margin account at the end of the day falls below the maintenance margin requirement, the trader must deposit sufficient funds to bring the balance back up to the initial margin requirement. Alternatively, the trader can simply close out the position but is responsible for any further losses incurred if the price changes before a closing transaction can be made.

To provide a fair mark-to-market process, the clearinghouse must designate the official price for determining daily gains and losses. This price is called the **settlement price** and represents an average of the final few trades of the day. It would appear that the closing price of the day would serve as the settlement price, but the closing price is a single value that can potentially be biased high or low or perhaps even manipulated by an unscrupulous trader. Hence, the clearinghouse takes an average of all trades during the closing period (as defined by each exchange).

Exhibit 3-1 provides an example of the marking-to-market process that occurs over a period of six trading days. We start with the assumption that the futures price is $100 when the transaction opens, the initial margin requirement is $5, and the maintenance margin requirement is $3. In Panel A, the trader takes a long position of 10 contracts on Day 0, depositing $50 ($5 times 10 contracts) as indicated in Column 3. At the end of the day, his ending balance is $50.[9] Although the trader can withdraw any funds in excess of the initial margin requirement, we shall assume that he does not do so.[10]

EXHIBIT 3-1 Mark-to-Market Example

Initial futures price = $100, Initial margin requirement = $5, Maintenance margin requirement = $3

A. *Holder of Long Position of 10 Contracts*

Day (1)	Beginning Balance (2)	Funds Deposited (3)	Settlement Price (4)	Futures Price Change (5)	Gain/ Loss (6)	Ending Balance (7)
0	0	50	100.00			50
1	50	0	99.20	−0.80	−8	42
2	42	0	96.00	−3.20	−32	10
3	10	40	101.00	5.00	50	100
4	100	0	103.50	2.50	25	125
5	125	0	103.00	−0.50	−5	120
6	120	0	104.00	1.00	10	130

[9] Technically, we are assuming that the position was opened at the settlement price on Day 0. If the position is opened earlier during the day, it would be marked to the settlement price at the end of the day.

[10] Virtually all professional traders are able to deposit interest-earning assets, although many other account holders are required to deposit cash. If the deposit earns interest, there is no opportunity cost and no obvious necessity to withdraw the money to invest elsewhere.

B. *Holder of Short Position of 10 Contracts*

Day (1)	Beginning Balance (2)	Funds Deposited (3)	Settlement Price (4)	Futures Price Change (5)	Gain/ Loss (6)	Ending Balance (7)
0	0	50	100.00			50
1	50	0	99.20	−0.80	8	58
2	58	0	96.00	−3.20	32	90
3	90	0	101.00	5.00	−50	40
4	40	0	103.50	2.50	−25	15
5	15	35	103.00	−0.50	5	55
6	55	0	104.00	1.00	−10	45

The ending balance on Day 0 is then carried forward to the beginning balance on Day 1. On Day 1, the futures price moves down to 99.20, as indicated in Column 4 of Panel A. The futures price change, Column 5, is −0.80 (99.20 − 100). This amount is then multiplied by the number of contracts to obtain the number in Column 6 of −0.80 × 10 = −$8. The ending balance, Column 7, is the beginning balance plus the gain or loss. The ending balance on Day 1 of $42 is above the maintenance margin requirement of $30, so no funds need to be deposited on Day 2.

On Day 2 the settlement price goes down to $96. Based on a price decrease of $3.20 per contract and 10 contracts, the loss is $32, lowering the ending balance to $10. This amount is $20 below the maintenance margin requirement. Thus, the trader will get a margin call the following morning and must deposit $40 to bring the balance up to the initial margin level of $50. This deposit is shown in Column 3 on Day 3.

Here, we must emphasize two important points. First, additional margin that must be deposited is the amount sufficient to bring the ending balance up to the initial margin requirement, not the maintenance margin requirement.[11] This additional margin is called the **variation margin**. In addition, the amount that must be deposited the following day is determined regardless of the price change the following day, which might bring the ending balance well above the initial margin requirement, as it does here, or even well below the maintenance margin requirement. Thus, another margin call could occur. Also note that when the trader closes the position, the account is marked to market to the final price at which the transaction occurs, not the settlement price that day.

Over the six-day period, the trader in this example deposited $90. The account balance at the end of the sixth day is $130—nearly a 50 percent return over six days; not bad. But look at Panel B, which shows the position of a holder of 10 short contracts over that same period. Note that the short gains when prices decrease and loses when prices increase. Here the ending balance falls below the maintenance margin requirement on Day 4, and the short must deposit $35 on Day 5. At the end of Day 6, the short has deposited $85 and the balance is $45, a loss of $40 or nearly 50 percent, which is the same $40 the long made. Both cases illustrate the leverage effect that magnifies gains and losses.

When establishing a futures position, it is important to know the price level that would trigger a margin call. In this case, it does not matter how many contracts one has. The price change would need to fall for a long position (or rise for a short position) by the difference between the initial and maintenance margin requirements. In this example, the

[11] In the stock market, one must deposit only the amount necessary to bring the balance up to the maintenance margin requirement.

difference between the initial and maintenance margin requirements is $5 − $3 = $2. Thus, the price would need to fall from $100 to $98 for a long position (or rise from $100 to $102 for a short position) to trigger a margin call.

As described here, when a trader receives a margin call, he is required to deposit funds sufficient to bring the account balance back up to the initial margin level. Alternatively, the trader can choose to simply close out the position as soon as possible. For example, consider the position of the long at the end of the second day when the margin balance is $10. This amount is $20 below the maintenance level, and he is required to deposit $40 to bring the balance up to the initial margin level. If he would prefer not to deposit the additional funds, he can close out the position as soon as possible the following day. Suppose, however, that the price is moving quickly at the opening on Day 3. If the price falls from $96 to $95, he has lost $10 more, wiping out the margin account balance. In fact, if it fell any further, he would have a negative margin account balance. He is still responsible for these losses. Thus, the trader could lose more than the amount of money he has placed in the margin account. The total amount of money he could lose is limited to the price per contract at which he bought, $100, times the number of contracts, 10, or $1,000. Such a loss would occur if the price fell to zero, although this is not likely. This potential loss may not seem like a lot, but it is certainly large relative to the initial margin requirement of $50. For the holder of the short position, there is no upper limit on the price and the potential loss is theoretically infinite.

PRACTICE PROBLEM 1

Consider a futures contract in which the current futures price is $82. The initial margin requirement is $5, and the maintenance margin requirement is $2. You go long 20 contracts and meet all margin calls but do not withdraw any excess margin. Assume that on the first day, the contract is established at the settlement price, so there is no mark-to-market gain or loss on that day.

A. Complete the table below and provide an explanation of any funds deposited.

Day	Beginning Balance	Funds Deposited	Futures Price	Price Change	Gain/Loss	Ending Balance
0			82			
1			84			
2			78			
3			73			
4			79			
5			82			
6			84			

B. Determine the price level that would trigger a margin call.

SOLUTIONS

A.

Day	Beginning Balance	Funds Deposited	Futures Price	Price Change	Gain/Loss	Ending Balance
0	0	100	82			100
1	100	0	84	2	40	140

2	140	0	78	−6	−120	20
3	20	80	73	−5	−100	0
4	0	100	79	6	120	220
5	220	0	82	3	60	280
6	280	0	84	2	40	320

On Day 0, you deposit $100 because the initial margin requirement is $5 per contract and you go long 20 contracts. At the end of Day 2, the balance is down to $20, which is $20 below the $40 maintenance margin requirement ($2 per contract times 20 contracts). You must deposit enough money to bring the balance up to the initial margin requirement of $100 ($5 per contract times 20 contracts). So on Day 3, you deposit $80. The price change on Day 3 causes a gain/loss of −$100, leaving you with a balance of $0 at the end of Day 3. On Day 4, you must deposit $100 to return the balance to the initial margin level.

B. A price decrease to $79 would trigger a margin call. This calculation is based on the fact that the difference between the initial margin requirement and the maintenance margin requirement is $3. If the futures price starts at $82, it can fall by $3 to $79 before it triggers a margin call.

Some futures contracts impose limits on the price change that can occur from one day to the next. Appropriately, these are called **price limits**. These limits are usually set as an absolute change over the previous day. Using the example above, suppose the price limit was $4. This would mean that each day, no transaction could take place higher than the previous settlement price plus $4 or lower than the previous settlement price minus $4. So the next day's settlement price cannot go beyond the price limit and thus no transaction can take place beyond the limits.

If the price at which a transaction would be made exceeds the limits, then price essentially freezes at one of the limits, which is called a **limit move**. If the price is stuck at the upper limit, it is called **limit up**; if stuck at the lower limit, it is called **limit down**. If a transaction cannot take place because the price would be beyond the limits, this situation is called **locked limit**. By the end of the day, unless the price has moved back within the limits, the settlement price will then be at one of the limits. The following day, the new range of acceptable prices is based on the settlement price plus or minus limits. The exchanges have different rules that provide for expansion or contraction of price limits under some circumstances. In addition, not all contracts have price limits.

Finally, we note that the exchanges have the power to mark contracts to market whenever they deem it necessary. Thus, they can do so during the trading day rather than wait until the end of the day. They sometimes do so when abnormally large market moves occur.

The daily settlement procedure is designed to collect losses and distribute gains in such a manner that losses are paid before becoming large enough to impose a serious risk of default. Recall that the clearinghouse guarantees to each party that it need not worry about collecting from the counterparty. The clearinghouse essentially positions itself in the middle of each contract, becoming the short counterparty to the long and the long counterparty to the short. The clearinghouse collects funds from the parties incurring losses in this daily settlement procedure and distributes them to the parties incurring gains. By doing so each day, the clearinghouse ensures that losses cannot build up. Of course, this process offers no guarantee that counterparties will not default. Some defaults do occur, but the counterparty is defaulting to the clearinghouse, which has never failed to pay off the opposite party. In the unlikely event that the clearinghouse were unable to pay, it would turn to a reserve fund or to the exchange, or it would levy a tax on exchange members to cover losses.

4 DELIVERY AND CASH SETTLEMENT

As previously described, a futures trader can close out a position before expiration. If the trader holds a long position, she can simply enter into a position to go short the same futures contract. From the clearinghouse's perspective, the trader holds both a long and short position in the same contract. These positions are considered to offset and, therefore, there is no open position in place. Most futures contracts are offset before expiration. Those that remain in place are subject to either delivery or a final cash settlement. Here we explore this process, which determines how a futures contract terminates at expiration.

When the exchange designs a futures contract, it specifies whether the contract will terminate with delivery or cash settlement. If the contract terminates in delivery, the clearinghouse selects a counterparty, usually the holder of the oldest long contract, to accept delivery. The holder of the short position then delivers the underlying to the holder of the long position, who pays the short the necessary cash for the underlying. Suppose, for example, that two days before expiration, a party goes long one futures contract at a price of $50. The following day (the day before expiration), the settlement price is $52. The trader's margin account is then marked to market by crediting it with a gain of $2. Then suppose that the next day the contract expires with the settlement price at $53. As the end of the trading day draws near, the trader has two choices. She can attempt to close out the position by selling the futures contract. The margin account would then be marked to market at the price at which she sells. If she sells close enough to the expiration, the price she sold at would be very close to the final settlement price of $53. Doing so would add $1 to her margin account balance.

The other choice is to leave the position open at the end of the trading day. Then she would have to take delivery. If that occurred, she would be required to take possession of the asset and pay the short the settlement price of the previous day. Doing so would be equivalent to paying $52 and receiving the asset. She could then sell the asset for its price of $53, netting a $1 gain, which is equivalent to the final $1 credited to her margin account if she had terminated the position at the settlement price of $53, as described above.[12]

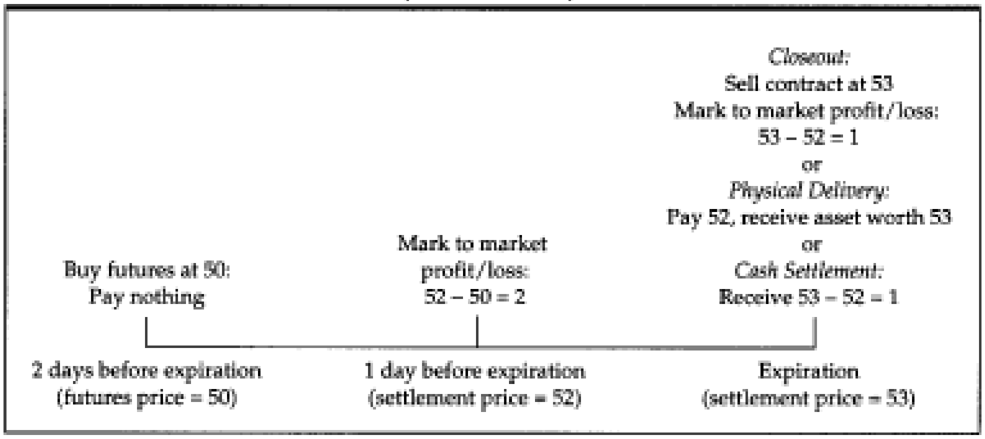
An alternative settlement procedure, which we described in Chapter 2 on forward contracts, is cash settlement. The exchange designates certain futures contracts as cash-settled contracts. If the contract used in this example were cash settled, then the trader would not need to close out the position close to the end of the expiration day. She could simply leave the position open. When the contract expires, her margin account would be marked to market for a gain on the final day of $1. Cash settlement contracts have some advantages over delivery contracts, particularly with respect to significant savings in transaction costs.[13]

[12] The reason she pays the settlement price of the previous day is because on the previous day when her account was marked to market, she essentially created a new futures position at a price of $52. Thus, she committed to purchase the asset at expiration, just one day later, at a price of $52. The next day when the contract expires, it is then appropriate that she buy the underlying for $52.

[13] Nonetheless, cash settlement has been somewhat controversial in the United States. If a contract is designated as cash settlement, it implies that the buyer of the contract never intended to actually take possession of the underlying asset. Some legislators and regulators feel that this design is against the spirit of the law, which views a futures contract as a commitment to buy the asset at a later date. Even though parties often offset futures contracts prior to expiration, the possibility of actual delivery is still present in contracts other than those settled by cash. This controversy, however, is relatively minor and has caused no serious problems or debates in recent years.

Exhibit 3-2 illustrates the equivalence of these three forms of delivery. Note, however, that because of the transaction costs of delivery, parties clearly prefer a closeout or cash settlement over physical delivery, particularly when the underlying asset is a physical commodity.

EXHIBIT 3-2 Closeout versus Physical Delivery versus Cash Settlement



Contracts designated for delivery have a variety of features that can complicate delivery. In most cases, delivery does not occur immediately after expiration but takes place over several days. In addition, many contracts permit the short to choose when delivery takes place. For many contracts, delivery can be made any business day of the month. The delivery period usually includes the days following the last trading day of the month, which is usually in the third week of the month.

In addition, the short often has other choices regarding delivery, a major one being exactly which underlying asset is delivered. For example, a futures contract on U.S. Treasury bonds trading at the Chicago Board of Trade permits the short to deliver any of a number of U.S. Treasury bonds.[14] The wheat futures contract at the Chicago Board of Trade permits delivery of any of several types of wheat. Futures contracts calling for physical delivery of commodities often permit delivery at different locations. A given commodity delivered to one location is not the same as that commodity delivered to another because of the costs involved in transporting the commodity. The short holds the sole right to make decisions about what, when, and where to deliver, and the right to make these decisions can be extremely valuable. The right to make a decision concerning these aspects of delivery is called a **delivery option**.

Some futures contracts that call for delivery require delivery of the actual asset, and some use only a book entry. For example, in this day and age, no one physically handles U.S. Treasury bonds in the form of pieces of paper. Bonds are transferred electronically over the Federal Reserve's wire system. Other contracts, such as oil or wheat, do actually involve the physical transfer of the asset. Physical delivery is more common when the underlying is a physical commodity, whereas book entry is more common when the underlying is a financial asset.

Futures market participants use one additional delivery procedure, which is called **exchange for physicals (EFP)**. In an EFP transaction, the long and short arrange an alternative delivery procedure. For example, the Chicago Board of Trade's wheat futures

[14] We shall cover this feature in more detail in Sections 6.2 and 7.2.3.

contracts require delivery on certain dates at certain locations either in Chicago or in a few other specified locations in the Midwest. If the long and short agree, they could effect delivery by having the short deliver the wheat to the long in, for example, Omaha. The two parties would then report to the Chicago Board of Trade that they had settled their contract outside of the exchange's normal delivery procedures, which would be satisfactory to the exchange.

5 FUTURES EXCHANGES

A futures exchange is a legal corporate entity whose shareholders are its members. The members own memberships, more commonly called **seats**. Exchange members have the privilege of executing transactions on the exchange. Each member acts as either a **floor trader** or a **broker**. Floor traders are typically called **locals**; brokers are typically called **futures commission merchants (FCMs)**. Locals are market makers, standing ready to buy and sell by quoting a bid and an ask price. They are the primary providers of liquidity to the market. FCMs execute transactions for other parties off the exchange.

The locals on the exchange floor typically trade according to one of several distinct styles. The most common is called scalping. A **scalper** offers to buy or sell futures contracts, holding the position for only a brief period of time, perhaps just seconds. Scalpers attempt to profit by buying at the bid price and selling at the higher ask price. A **day trader** holds a position open somewhat longer but closes all positions at the end of the day.[15] A **position trader** holds positions open overnight. Day traders and position traders are quite distinct from scalpers in that they attempt to profit from the anticipated direction of the market; scalpers are trying simply to buy at the bid and sell at the ask.

Recall that futures exchanges have trading either on the floor or off the floor on electronic terminals, or in some cases, both. As previously described, floor trading in the United States takes place in pits, which are octagonal, multi-tiered areas where floor traders stand and conduct transactions. Traders wear jackets of specific colors and badges to indicate such information as what type of trader (FCM or local) they are and whom they represent.[16] As noted, to indicate a willingness to trade, a trader shouts and uses a set of standard hand signals. A trade is consummated by two traders agreeing on a price and a number of contracts. These traders might not actually say anything to each other; they may simply use a combination of hand signals and/or eye contact to agree on a transaction. When a transaction is agreed on, the traders fill out small paper forms and turn them over to clerks, who then see that the transactions are entered into the system and reported.

Each trader is required to have an account at a clearing firm. The clearing firms are the actual members of the clearinghouse. The clearinghouse deals only with the clearing firms, which then deal with their individual and institutional customers.

In electronic trading, the principles remain essentially the same but the traders do not stand in the pits. In fact, they do not see each other at all. They sit at computer terminals, which enable them to see the bids and offers of other traders. Transactions are executed by the click of a computer mouse or an entry from a keyboard.

Exhibit 3-3 lists the world's 20 leading futures exchanges in 2001, ranked by trading volume. Recall from Chapter 1 that trading volume can be a misleading measure of the

[15] The term "day trader" has been around the futures market for a long time but has recently acquired a new meaning in the broader financial markets. The term is now used to describe individual investors who trade stocks, often over the Internet, during the day for a living or as a hobby. In fact, the term has even been used in a somewhat pejorative manner, in that day traders are often thought of as naïve investors speculating wildly with money they can ill afford to lose.

[16] For example, an FCM or local could be trading for himself or could represent a company.

size of a futures markets; nonetheless, it is the measure primarily used. The structure of global futures exchanges has changed considerably in recent years. Exchanges in the United States, primarily the Chicago Board of Trade and the Chicago Mercantile Exchange, were clearly the world leaders in the past. Note that the volume leader now, however, is Eurex, the combined German–Swiss exchange. Eurex has been so successful partly because of its decision to be an all-electronic futures exchange, whereas the Chicago exchanges are still primarily pit-trading exchanges. Note the popularity of futures trading in Japan; four of the 20 leading exchanges are Japanese.

EXHIBIT 3-3 The World's 20 Leading Futures Exchanges

Exchange and Location	Volume in 2001 (Number of Contracts)
Eurex (Germany and Switzerland)	435,141,707
Chicago Mercantile Exchange (United States)	315,971,885
Chicago Board of Trade (United States)	209,988,002
London International Financial Futures and Options Exchange (United Kingdom)	161,522,775
Bolsa de Mercadorias & Futuros (Brazil)	94,174,452
New York Mercantile Exchange (United States)	85,039,984
Tokyo Commodity Exchange (Japan)	56,538,245
London Metal Exchange (United Kingdom)	56,224,495
Paris Bourse SA (France)	42,042,673
Sydney Futures Exchange (Australia)	34,075,508
Korea Stock Exchange (Korea)	31,502,184
Singapore Exchange (Singapore)	30,606,546
Central Japan Commodity Exchange (Japan)	27,846,712
International Petroleum Exchange (United Kingdom)	26,098,207
OM Stockholm Exchange (Sweden)	23,408,198
Tokyo Grain Exchange (Japan)	22,707,808
New York Board of Trade (United States)	14,034,168
MEFF Renta Variable (Spain)	13,108,293
Tokyo Stock Exchange (Japan)	12,465,433
South African Futures Exchange (South Africa)	11,868,242

Source: Futures Industry, January/February 2002

6 TYPES OF FUTURES CONTRACTS

The different types of futures contracts are generally divided into two main groups: commodity futures and financial futures. Commodity futures cover traditional agricultural, metal, and petroleum products. Financial futures include stocks, bonds, and currencies. Exhibit 3-4 gives a broad overview of the most active types of futures contracts traded on global futures exchanges. These contracts are those covered by the *Wall Street Journal* on the date indicated.

EXHIBIT 3-4 Most-Active Global Futures Contracts as Covered by the *Wall Street Journal,* 18 June 2002

Commodity Futures	Financial Futures	
Corn (CBOT)	Treasury Bonds (CBOT)	Euro (CME)
Oats (CBOT)	Treasury Notes (CBOT)	Euro–Sterling (NYBOT)
Soybeans (CBOT)	10-Year Agency Notes (CBOT)	Euro–U.S. Dollar (NYBOT)
Soybean Meal (CBOT)	10-Year Interest Rate Swaps (CBOT)	Euro–Yen (NYBOT)
Soybean Oil (CBOT)	2-Year Agency Notes (CBOT)	Dow Jones Industrial Average (CBOT)
Wheat (CBOT, KCBT, MGE)	5-Year Treasury Notes (CBOT)	Mini Dow Jones Industrial Average (CBOT)
Canola (WPG)	2-Year Treasury Notes (CBOT)	S&P 500 Index (CME)
Barley (WPG)	Federal Funds (CBOT)	Mini S&P 500 Index (CME)
Feeder Cattle (CME)	Municipal Bond Index (CBOT)	S&P Midcap 400 Index (CME)
Live Cattle (CME)	Treasury Bills (CME)	Nikkei 225 (CME)
Lean Hogs (CME)	1-Month LIBOR (CME)	Nasdaq 100 Index (CME)
Pork Bellies (CME)	Eurodollar (CME)	Mini Nasdaq Index (CME)
Milk (CME)	Euroyen (CME, SGX)	Goldman Sachs Commodity Index (CME)
Lumber (CME)	Short Sterling (LIFFE)	Russell 1000 Index (CME)
Cocoa (NYBOT)	Long Gilt (LIFFE)	Russell 2000 Index (CME)
Coffee (NYBOT)	3-Month Euribor (LIFFE)	NYSE Composite Index (NYBOT)
World Sugar (NYBOT)	3-Month Euroswiss (LIFFE)	U.S. Dollar Index (NYBOT)
Domestic Sugar (NYBOT)	Canadian Bankers Acceptance (ME)	Share Price Index (SFE)
Cotton (NYBOT)	10-Year Canadian Government Bond (ME)	CAC 40 Stock Index (MATIF)
Orange Juice (NYBOT)	10-Year Euro Notional Bond (MATIF)	Xetra Dax (EUREX)
Copper (NYMEX)	3-Month Euribor (MATIF)	FTSE 200 Index (LIFFE)
Gold (NYMEX)	3-Year Commonwealth T-Bonds (SFE)	Dow Jones Euro Stoxx 50 Index (EUREX)
Platinum (NYMEX)	5-Year German Euro Government Bond (EUREX)	Dow Jones Stoxx 50 Index (EUREX)
Palladium (NYMEX)	10-Year German Euro Government Bond (EUREX)	
Silver (NYMEX)	2-Year German Euro Government Bond (EUREX)	
Crude Oil (NYMEX)	Japanese Yen (CME)	
No. 2 Heating Oil (NYMEX)	Canadian Dollar (CME)	
Unleaded Gasoline (NYMEX)	British Pound (CME)	
Natural Gas (NYMEX)	Swiss Franc (CME)	
Brent Crude Oil (IPEX)	Australian Dollar (CME)	
Gas Oil (IPEX)	Mexican Peso (CME)	

Exchange codes: CBOT (Chicago Board of Trade), CME (Chicago Mercantile Exchange), LIFFE (London International Financial Futures Exchange), WPG (Winnipeg Grain Exchange), EUREX (Eurex), NYBOT (New York Board of Trade), IPEX (International Petroleum Exchange), MATIF (Marché a Terme International de France), ME (Montreal Exchange), MGE (Minneapolis Grain Exchange), SFE (Sydney Futures Exchange), SGX (Singapore Exchange), KCBT (Kansas City Board of Trade), NYMEX (New York Mercantile Exchange)

Note: These are not the only global futures contracts but are those covered in the *Wall Street Journal* on the date given and represent the most active contracts at that time.

Our primary focus in this book is on financial and currency futures contracts. Within the financials group, our main interest is on interest rate and bond futures, stock index futures, and currency futures. We may occasionally make reference to a commodity futures contract, but that will primarily be for illustrative purposes. In the following subsections, we introduce the primary contracts we shall focus on. These are U.S. contracts, but they resemble most types of futures contracts found on exchanges throughout the world. Full contract specifications for these and other contracts are available on the Web sites of the futures exchanges, which are easy to locate with most Internet search engines.

6.1 Short-Term Interest Rate Futures Contracts

The primary short-term interest rate futures contracts are those on U.S. Treasury bills and Eurodollars on the Chicago Mercantile Exchange.

6.1.1 Treasury Bill Futures

The Treasury bill contract, launched in 1976, was the first interest rate futures contract. It is based on a 90-day U.S. Treasury bill, one of the most important U.S. government debt instruments (described in Chapter 2, Section 3.2.1). The Treasury bill, or T-bill, is a discount instrument, meaning that its price equals the face value minus a discount representing interest. The discount equals the face value multiplied by the quoted rate times the days to maturity divided by 360. Thus, using the example from Chapter 2, if a 180-day T-bill is selling at a discount of 4 percent, its price per $1 par is $1 - 0.04(180/360) = 0.98$. An investor who buys the bill and holds it to maturity would receive $1 at maturity, netting a gain of $0.02.

The futures contract is based on a 90-day $1,000,000 U.S. Treasury bill. Thus, on any given day, the contract trades with the understanding that a 90-day T-bill will be delivered at expiration. While the contract is trading, its price is quoted as 100 minus the rate quoted as a percent priced into the contract by the futures market. This value, 100 − Rate, is known as the IMM Index; IMM stands for International Monetary Market, a division of the Chicago Mercantile Exchange. The IMM Index is a reported and publicly available price; however, it is not the actual futures price, which is

$$100 - (\text{Rate}/100)(90/360)$$

For example, suppose on a given day the rate priced into the contract is 6.25 percent. Then the quoted price will be $100 - 6.25 = 93.75$. The actual futures price would be

$$\$1{,}000{,}000[1 - 0.0625(90/360)] = \$984{,}375$$

Recall, however, that except for the small margin deposit, a futures transaction does not require any cash to be paid up front. As trading takes place, the rate fluctuates with market interest rates and the associated IMM Index price changes accordingly. The actual futures price, as calculated above, also fluctuates according to the above formula, but interestingly, that price is not very important. The same information can be captured more easily by referencing the IMM Index than by calculating the actual price.

Suppose, for example, that a trader had his account marked to market to the above price, 6.25 in terms of the rate, 93.75 in terms of the IMM Index, and $984,375 in terms of the actual futures price. Now suppose the rate goes to 6.50, an increase of 0.25 or 25 basis points. The IMM Index declines to 93.50, and the actual futures price drops to

$$\$1{,}000{,}000[1 - 0.065(90/360)] = \$983{,}750$$

Thus, the actual futures price decreased by $984,375 − $983,750 = $625. A trader who is long would have a loss of $625; a trader who is short would have a gain of $625.

This $625 gain or loss can be arrived at more directly, however, by simply noting that each basis point move is equivalent to $25.[17] This special design of the contract makes it easy for floor traders to do the necessary arithmetic in their heads. For example, if floor traders observe the IMM Index move from 93.75 to 93.50, they immediately know that it has moved down 25 basis points and that 25 basis points times $25 per basis point is a loss of $625. The minimum tick size is one-half basis point or $12.50.

T-bill futures contracts have expirations of the current month, the next month, and the next four months of March, June, September, and December. Because of the small trading volume, however, only the closest expiration has much trading volume, and even that one is only lightly traded. T-bill futures expire specifically on the Monday of the week of the third Wednesday each month and settle in cash rather than physical delivery of the T-bill, as described in Section 4.

As important as Treasury bills are in U.S. financial markets, however, today this futures contract is barely active. The Eurodollar contract is considered much more important because it reflects the interest rate on a dollar borrowed by a high-quality private borrower. The rates on T-bills are considered too heavily influenced by U.S. government policies, budget deficits, government funding plans, politics, and Federal Reserve monetary policy. Although unquestionably Eurodollar rates are affected by those factors, market participants consider them much less directly influenced. But in spite of this relative inactivity, T-bill futures are useful instruments for illustrating certain principles of futures market pricing and trading. Accordingly, we shall use them on some occasions. For now, however, we turn to the Eurodollar futures contract.

6.1.2 Eurodollar Futures

Recall that in Chapter 2, we devoted a good bit of effort to understanding Eurodollar forward contracts, known as FRAs. These contracts pay off based on LIBOR on a given day. The Eurodollar futures contract of the Chicago Mercantile Exchange is based on $1 million notional principal of 90-day Eurodollars. Specifically, the underlying is the rate on a 90-day dollar-denominated time deposit issued by a bank in London. As we described in Chapter 2, this deposit is called a Eurodollar time deposit, and the rate is referred to as LIBOR (London Interbank Offer Rate). On a given day, the futures contract trades based on the understanding that at expiration, the official Eurodollar rate, as compiled by the British Bankers Association (BBA), will be the rate at which the final settlement of the contract is made. While the contract is trading, its price is quoted as 100 minus the rate priced into the contract by futures traders. Like its counterpart in the T-bill futures market, this value, 100 − Rate, is also known as the IMM Index.

As in the T-bill futures market, on a given day, if the rate priced into the contract is 5.25 percent, the quoted price will be 100 − 5.25 = 94.75. With each contract based on $1 million notional principal of Eurodollars, the actual futures price is

$$\$1{,}000{,}000[1 - 0.0525(90/360)] = \$986{,}875$$

Like the T-bill contract, the actual futures price moves $25 for every basis point move in the rate or IMM Index price.

[17] Expressed mathematically, $\$1{,}000{,}000[0.0001(90/360)] = \$25$. In other words, any move in the last digit of the rate (a basis point) affects the actual futures price by $25.

As with all futures contracts, the price fluctuates on a daily basis and margin accounts are marked to market according to the exchange's official settlement price. At expiration, the final settlement price is the official rate quoted on a 90-day Eurodollar time deposit by the BBA. That rate determines the final settlement. Eurodollar futures contracts do not permit actual delivery of a Eurodollar time deposit; rather, they settle in cash, as described in Section 4.

The Eurodollar futures contract is one of the most active in the world. Because its rate is based on LIBOR, it is widely used by dealers in swaps, FRAs, and interest rate options to hedge their positions taken in dollar-denominated over-the-counter interest rate derivatives. Such derivatives usually use LIBOR as the underlying rate.

It is important to note, however, that there is a critical distinction between the manner in which the interest calculation is built into the Eurodollar futures contract and the manner in which interest is imputed on actual Eurodollar time deposits. Recall from Chapter 2 that when a bank borrows \$1 million at a rate of 5 percent for 90 days, the amount it will owe in 90 days is

$$\$1{,}000{,}000[1 + 0.05(90/360)] = \$1{,}012{,}500$$

Interest on Eurodollar time deposits is computed on an add-on basis to the principal. As described in this section, however, it appears that in computing the futures price, interest is deducted from the principal so that a bank borrowing \$1,000,000 at a rate of 5 percent would receive

$$\$1{,}000{,}000[1 - 0.05(90/360)] = \$987{,}500$$

and would pay back \$1,000,000. This procedure is referred to as discount interest and is used in the T-bill market.

The discount interest computation associated with Eurodollar futures is merely a convenience contrived by the futures exchange to facilitate quoting prices in a manner already familiar to its traders, who were previously trading T-bill futures. This inconsistency between the ways in which Eurodollar futures and Eurodollar spot transactions are constructed causes some pricing problems, as we shall see in Section 7.2.2.

The minimum tick size for Eurodollar futures is 1 basis point or \$25. The available expirations are the next two months plus March, June, September, and December. The expirations go out about 10 years, a reflection of their use by over-the-counter derivatives dealers to hedge their positions in long-term interest rate derivatives. Eurodollar futures expire on the second business day on which London banks are open before the third Wednesday of the month and terminate with a cash settlement.

6.2 Intermediate- and Long-Term Interest Rate Futures Contracts

In U.S. markets, the primary interest-rate-related instruments of intermediate and long maturities are U.S. Treasury notes and bonds. The U.S. government issues both instruments: Treasury notes have an original maturity of 2 to 10 years, and Treasury bonds have an original maturity of more than 10 years. Futures contracts on these instruments are very actively traded on the Chicago Board of Trade. For the most part, there are no real differences in the contract characteristics for Treasury note and Treasury bond futures; the underlying bonds differ slightly, but the futures contracts are qualitatively the same. We shall focus here on one of the most active instruments, the U.S. Treasury bond futures contract.

The contract is based on the delivery of a U.S. Treasury bond with any coupon but with a maturity of at least 15 years. If the deliverable bond is callable, it cannot be callable for at least 15 years from the delivery date.[18] These specifications mean that there are potentially a large number of deliverable bonds, which is exactly the way the Chicago Board of Trade, the Federal Reserve, and the U.S. Treasury want it. They do not want a potential run on a single issue that might distort prices. By having multiple deliverable issues, however, the contract must be structured with some fairly complicated procedures to adjust for the fact that the short can deliver whatever bond he chooses from among the eligible bonds. This choice gives the short a potentially valuable option and puts the long at a disadvantage. Moreover, it complicates pricing the contract, because the identity of the underlying bond is not clear. Although when referring to a futures contract on a 90-day Eurodollar time deposit we are relatively clear about the underlying instrument, a futures contract on a long-term Treasury bond does not allow us the same clarity.

To reduce the confusion, the exchange declares a standard or hypothetical version of the deliverable bond. This hypothetical deliverable bond has a 6 percent coupon. When a trader holding a short position at expiration delivers a bond with a coupon greater (less) than 6 percent, she receives an upward (a downward) adjustment to the price paid for the bond by the long. The adjustment is done by means of a device called the **conversion factor**. In brief, the conversion factor is the price of a $1.00 par bond with a coupon and maturity equal to those of the deliverable bond and a yield of 6 percent. Thus, if the short delivers a bond with a coupon greater (less) than 6 percent, the conversion factor exceeds (is less than) 1.0.[19] The amount the long pays the short is the futures price at expiration multiplied by the conversion factor. Thus, delivery of a bond with coupon greater (less) than the standard amount, 6 percent, results in the short receiving an upward (a downward) adjustment to the amount received. A number of other technical considerations are also involved in determining the delivery price.[20]

The conversion factor system is designed to put all bonds on equal footing. Ideally, application of the conversion factor would result in the short finding no preference for delivery of any one bond over any other. That is not the case, however, because the complex relationships between bond prices cannot be reduced to a simple linear adjustment, such as the conversion factor method. As a result, some bonds are cheaper to deliver than others. When making the delivery decision, the short compares the cost of buying a given bond on the open market with the amount she would receive upon delivery of that bond. The former will always exceed the latter; otherwise, a clear arbitrage opportunity would be available. The most attractive bond for delivery would be the one in which the amount received for delivering the bond is largest relative to the amount paid on the open market for the bond. The bond that minimizes this loss is referred to as the **cheapest-to-deliver** bond.

At any time during the life of a Treasury bond futures contract, traders can identify the cheapest-to-deliver bond. Determining the amount received at delivery is straightforward; it equals the current futures price times the conversion factor for a given bond. To determine the amount the bond would cost at expiration, one calculates the forward price of the bond, positioned at the delivery date. Of course, this is just a forward computation; circumstances could change by the expiration date. But this forward calculation gives a picture of circumstances as they currently stand and identifies which bond is currently the cheapest to deliver. That bond is then considered the bond most likely to be delivered.

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[18] The U.S. government no longer issues callable bonds but has done so in the past.

[19] This statement is true regardless of the maturity of the deliverable bond. Any bond with a coupon in excess of its yield is worth more than its par value.

[20] For example, the actual procedure for delivery of U.S. Treasury bonds is a three-day process starting with the short notifying the exchange of intention to make delivery. Delivery actually occurs several days later. In addition, as is the custom in U.S. bond markets, the quoted price does not include the accrued interest. Accordingly, an adjustment must be made.

Recall that one problem with this futures contract is that the identity of the underlying bond is unclear. Traders traditionally treat the cheapest to deliver as the bond that underlies the contract. As time passes and interest rates change, however, the cheapest-to-deliver bond can change. Thus, the bond underlying the futures contract can change, adding an element of uncertainty to the pricing and trading of this contract.

With this complexity associated with the U.S. Treasury bond futures contract, one might suspect that it is less actively traded. In fact, the opposite is true: Complexity creates extraordinary opportunities for gain for those who understand what is going on and can identify the cheapest bond to deliver.

The Chicago Board of Trade's U.S. Treasury bond futures contract covers $100,000 par value of U.S. Treasury bonds. The expiration months are March, June, September, and December. They expire on the seventh business day preceding the last business day of the month and call for actual delivery, through the Federal Reserve's wire system, of the Treasury bond. Prices are quoted in points and 32nds, meaning that you will see prices like 98 18/32, which equals 98.5625. For a contract covering $100,000 par value, for example, the price is $98,562.50. The minimum tick size is 1/32, which is $31.25.

In addition to the futures contract on the long-term government bond, there are also very similar futures contracts on intermediate-term government bonds. The Chicago Board of Trade's contracts on 2-, 5-, and 10-year Treasury notes are very actively traded and are almost identical to its long-term bond contract, except for the exact specification of the underlying instrument. Intermediate and long-term government bonds are important instruments in every country's financial markets. They give the best indication of the long-term default-free interest rate and are often viewed as a benchmark bond for various comparisons in financial markets.[21] Accordingly, futures contracts on such bonds play an important role in a country's financial markets and are almost always among the most actively traded contracts in futures markets around the world.

If the underlying instrument is not widely available and not actively traded, the viability of a futures contract on it becomes questionable. The reduction seen in U.S. government debt in the late 1990s has led to a reduction in the supply of intermediate and long-term government bonds, and some concern has arisen over this fact. In the United States, some efforts have been made to promote the long-term debt of Fannie Mae and Freddie Mac as substitute benchmark bonds.[22] It remains to be seen whether such efforts will be necessary and, if so, whether they will succeed.

6.3 Stock Index Futures Contracts

One of the most successful types of futures contracts of all time is the class of futures on stock indices. Probably the most successful has been the Chicago Mercantile Exchange's contract on the Standard and Poor's 500 Stock Index. Called the S&P 500 Stock Index futures, this contract premiered in 1982 and has benefited from the widespread acceptance of the S&P 500 Index as a stock market benchmark. The contract is quoted in terms of a

[21] For example, the default risk of a corporate bond is often measured as the difference between the corporate bond yield and the yield on a Treasury bond or note of comparable maturity. Fixed rates on interest rate swaps are usually quoted as a spread over the rate on a Treasury bond or note of comparable maturity.

[22] Fannie Mae is the Federal National Mortgage Association, and Freddie Mac is the Federal Home Loan Mortgage Corporation. These institutions were formerly U.S. government agencies that issued debt to raise funds to buy and sell mortgages and mortgage-backed securities. These institutions are now publicly traded corporations but are considered to have extremely low default risk because of their critical importance in U.S. mortgage markets. It is believed that an implicit Federal government guarantee is associated with their debt. Nonetheless, it seems unlikely that the debt of these institutions could take over that of the U.S. government as a benchmark. The Chicago Board of Trade has offered futures contracts on the bonds of these organizations, but the contracts have not traded actively.

price on the same order of magnitude as the S&P 500 itself. For example, if the S&P 500 Index is at 1183, a two-month futures contract might be quoted at a price of, say, 1187. We shall explain how to determine a stock index futures price in Section 7.3.

The contract implicitly contains a multiplier, which is (appropriately) multiplied by the quoted futures price to produce the actual futures price. The multiplier for the S&P 500 futures is $250. Thus, when you hear of a futures price of 1187, the actual price is 1187($250) = $296,750.

S&P 500 futures expirations are March, June, September, and December and go out about two years, although trading is active only in the nearest two to three expirations. With occasional exceptions, the contracts expire on the Thursday preceding the third Friday of the month. Given the impracticality of delivering a portfolio of the 500 stocks in the index combined according to their relative weights in the index, the contract is structured to provide for cash settlement at expiration.

The S&P 500 is not the only active stock index futures contract. In fact, the Chicago Mercantile Exchange has a smaller version of the S&P 500 contract, called the Mini S&P 500, which has a multiplier of $50 and trades only electronically. Other widely traded contracts in the United States are on the Dow Jones Industrials, the S&P Midcap 400, and the Nasdaq 100. Virtually every developed country has a stock index futures contract based on the leading equities of that country. Well-known stock index futures contracts around the world include the United Kingdom's FTSE 100 (pronounced "Footsie 100"), Japan's Nikkei 225, France's CAC 40, and Germany's DAX 30.

6.4 Currency Futures Contracts

In Chapter 2 we described forward contracts on foreign currencies. There are also futures contracts on foreign currencies. Although the forward market for foreign currencies is much more widely used, the futures market is still quite active. In fact, currency futures were the first futures contracts not based on physical commodities. Thus, they are sometimes referred to as the first financial futures contracts, and their initial success paved the way for the later introduction of interest rate and stock index futures.

Compared with forward contracts on currencies, currency futures contracts are much smaller in size. In the United States, these contracts trade at the Chicago Mercantile Exchange with a small amount of trading at the New York Board of Trade. In addition there is some trading on exchanges outside the United States. The characteristics we describe below refer to the Chicago Mercantile Exchange's contract.

In the United States, the primary currencies on which trading occurs are the euro, Canadian dollar, Swiss franc, Japanese yen, British pound, Mexican peso, and Australian dollar. Each contract has a designated size and a quotation unit. For example, the euro contract covers €125,000 and is quoted in dollars per euro. A futures price such as $0.8555 is stated in dollars and converts to a contract price of

$$125{,}000(\$0.8555) = \$106{,}937.50$$

The Japanese yen futures price is structured somewhat differently. Because of the large number of yen per dollar, the contract covers ¥12,500,000 and is quoted without two zeroes that ordinarily precede the price. For example, a price might be stated as 0.8205, but this actually represents a price of 0.008205, which converts to a contract price of

$$12{,}500{,}000(0.008205) = \$102{,}562.50$$

Alternatively, a quoted price of 0.8205 can be viewed as 1/0.008205 = ¥121.88 per dollar.

Currency futures contracts expire in the months of March, June, September, and December. The specific expiration is the second business day before the third Wednesday of the month. Currency futures contracts call for actual delivery, through book entry, of the underlying currency.

We have briefly examined the different types of futures contracts of interest to us. Of course there are a variety of similar instruments trading on futures exchanges around the world. The purpose of this book, however, is not to provide institutional details, which can be obtained at the Web sites of the world's futures exchanges, but rather to enhance your understanding of the important principles necessary to function in the world of derivatives.

Until now we have made reference to prices of futures contracts. Accordingly, let us move forward and examine the pricing of futures contracts.

7 PRICING AND VALUATION OF FUTURES CONTRACTS

In Chapter 2, we devoted considerable effort to understanding the pricing and valuation of forward contracts. We first discussed the notion of what it means to *price* a forward contract in contrast to what it means to *value* a forward contract. Recall that pricing means to assign a fixed price or rate at which the underlying will be bought by the long and sold by the short at expiration. In assigning a forward price, we set the price such that the value of the contract is zero at the start. A zero-value contract means that the present value of the payments promised by each party to the other is the same, a result in keeping with the fact that neither party pays the other any money at the start. The value of the contract to the long is the present value of the payments promised by the short to the long minus the present value of the payments promised by the long to the short. Although the value is zero at the start, during the life of the contract, the value will fluctuate as market conditions change; the original forward contract price, however, stays the same.

In Chapter 2, we presented numerous examples of how to apply the concept of pricing and valuation when dealing with forward contracts on stocks, bonds, currencies, and interest rates. To illustrate the concepts of pricing and valuation, we started with a generic forward contract. Accordingly, we do so here in the futures chapter. We assume no transaction costs.

7.1 Generic Pricing and Valuation of a Futures Contract

As we did with forward contracts, we start by illustrating the time frame within which we are working:

Today is time 0. The expiration date of the futures contract is time T. Times t − 1 and t are arbitrary times between today and the expiration and are the points at which the contract will be marked to market. Thus, we can think of the three periods depicted above, 0 to t − 1, t − 1 to t, and t to T, as three distinct trading days with times t − 1, t, and T being the end of each of the three days.

The price of the underlying asset in the spot market is denoted as S_0 at time 0, S_{t-1} at time t − 1, S_t at time t, and S_T at time T. We denote the futures contract price at time 0 as $f_0(T)$. This notation indicates that $f_0(T)$ is the price of a futures contract at time 0 that expires at time T. Unlike forward contract prices, however, futures prices fluctuate in an open and competitive market. The marking-to-market process results in each futures contract being terminated every day and reinitiated. Thus, we not only have a futures price set at time 0 but we also have a new one at time t − 1, at time t, and at time T. In other words,

$f_0(T)$ = price of a futures contract at time 0 that expires at time T

$f_{t-1}(T)$ = price of a futures contract at time t − 1 that expires at time T

$f_t(T)$ = price of a futures contract at time t that expires at time T

$f_T(T)$ = price of a futures contract at time T that expires at time T

Note, however, that $f_{t-1}(T)$ and $f_t(T)$ are also the prices of contracts newly established at times t − 1 and t for delivery at time T. Futures contracts are homogeneous and fungible. Any contract for delivery of the underlying at T is equivalent to any other contract, regardless of when the contracts were created.[23]

The value of the futures contract is denoted as $v_0(T)$. This notation indicates that $v_0(T)$ is the value at time 0 of a futures contract expiring at time T. We are also interested in the values of the contract prior to expiration, such as at time t, denoted as $v_t(T)$, as well as the value of the contract at expiration, denoted as $v_T(T)$.[24]

7.1.1 The Futures Price at Expiration

Now suppose we are at time T. The spot price is S_T and the futures price is $f_T(T)$. To avoid an arbitrage opportunity, *the futures price must converge to the spot price at expiration*:

$$f_T(T) = S_T \tag{3-1}$$

Consider what would happen if this were not the case. If $f_T(T) < S_T$, a trader could buy the futures contract, let it immediately expire, pay $f_T(T)$ to take delivery of the underlying, and receive an asset worth S_T. The trader would have paid $f_T(T)$ and received an asset worth S_T, which is greater, at no risk. If $f_T(T) > S_T$, the trader would go short the futures, buy the asset for S_T, make delivery, and receive $f_T(T)$ for the asset, for which he paid a lesser amount. Only if $f_T(T) = S_T$ does this arbitrage opportunity go away. Thus, the futures price must equal the spot price at expiration.

Another way to understand this point is to recall that by definition, a futures contract calls for the delivery of an asset at expiration at a price determined when the transaction is initiated. If expiration is right now, a futures transaction is equivalent to a spot transaction, so the futures price must equal the spot price.

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[23] As an analogy from the bond markets, consider a 9 percent coupon bond, originally issued with 10 years remaining. Three years later, that bond is a 9 percent seven-year bond. Consider a newly issued 9 percent coupon bond with seven years maturity and the same issuer. As long as the coupon dates are the same and all other terms are the same, these two bonds are fungible and are perfect substitutes for each other.

[24] It is important at this point to make some comments about notation. First, note that in Chapter 2 we use an uppercase F and V for forward contracts; here we use lowercase f and v for futures contracts. Also, we follow the pattern of using subscripts to indicate a price or value at a particular point in time. The arguments in parentheses refer to characteristics of a contract. Thus, in Chapter 2 we described the price of a forward contract as F(0,T) meaning the price of a forward contract initiated at time 0 and expiring at time T. This price does not fluctuate during the life of the contract. A futures contract, however, reprices on a daily basis. Its original time of initiation does not matter—it is reinitiated every day. Hence, futures prices are indicated by notation such as $f_0(T)$ and $f_t(T)$. We follow a similar pattern for value, using $V_0(0,T)$, $V_t(0,T)$, and $V_T(0,T)$ for forwards and $v_0(T)$, $v_t(T)$, and $v_T(T)$ for futures.

7.1.2 Valuation of a Futures

Let us consider how to determine the value of a futures contract. We already agreed that because no money changes hands, the value of a forward contract at the initiation date is zero. For the same reason, *the value of a futures contract at the initiation date is zero*. Thus,

$$v_0(T) = 0 \tag{3-2}$$

Now let us determine the value of the contract during its life. Suppose we are at the end of the second day, at time t. In our diagram above, this point would be essentially at time t, but perhaps just an instant before it. So let us call it time t−. An instant later, we call the time point t+. In both cases, the futures price is $f_t(T)$. The contract was previously marked to market at the end of day t − 1 to a price of $f_{t-1}(T)$. An instant later when the futures account is marked to market, the trader will receive a gain of $f_t(T) - f_{t-1}(T)$. We can reasonably ignore the present value difference of receiving this money an instant later. Let us now state more formally that the value of a futures contract is

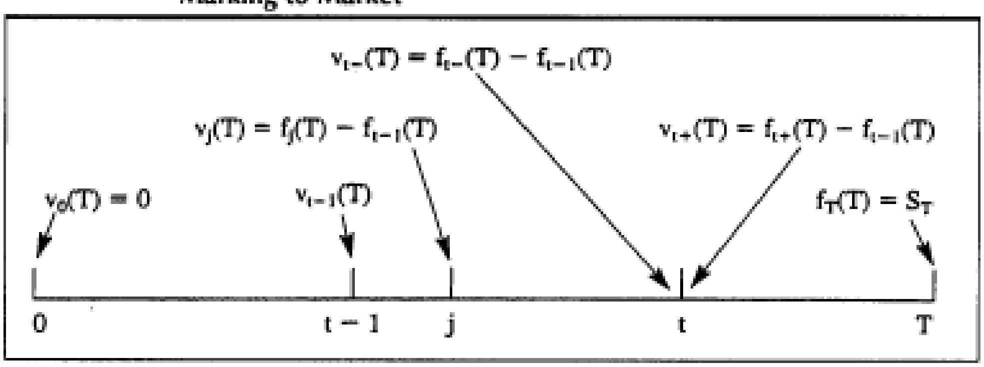
$v_{t+}(T) = f_t(T) - f_{t-1}(T)$ *an instant before the account is marked to market* **(3-3)**

$v_{t-}(T) = 0$ *as soon as the account is marked to market*

Suppose, however, that the trader is at a time j during the second trading day, between t − 1 and t. The accumulated gain or loss since the account was last marked to market is $f_j(T) - f_{t-1}(T)$. If the trader closes the position out, he would receive or be charged this amount at the end of the day. So the value at time j would be $f_j(T) - f_{t-1}(T)$ discounted back from the end of the day at time t until time j—that is, a fraction of a day. It is fairly routine to ignore this intraday interest. Thus, in general we say that *the value of a futures contract before it has been marked to market is the gain or loss accumulated since the account was last marked to market.*

So to recap, the value of a futures contract is the accumulated gain or loss since the last mark to market. The holder of a futures contract has a claim or liability on this amount. Once that claim is captured or the liability paid through the mark-to-market process, the contract is repriced to its current market price and the claim or liability goes back to a value of zero. Using these results, determining the value of a futures contract at expiration is easy. An instant before expiration, it is simply the accumulated profit since the last mark to market. At expiration, the value goes back to zero. With respect to the value of the futures, expiration is no different from any other day. Exhibit 3-5 summarizes the principles of valuation.

EXHIBIT 3-5 The Value of a Futures Contract Before and After Marking to Market



In Chapter 2, we devoted considerable effort toward understanding how forward contracts are valued. When holding positions in forward contracts, we are forced to assign values to instruments that do not trade in an open market with widely disseminated prices. Thus, it is important that we understand how forward contracts are valued. When dealing with futures contracts, the process is considerably simplified. Because futures contracts are generally quite actively traded, there is a market with reliable prices that provides all of the information we need. For futures contracts, we see that the value is simply the observable price change since the last mark to market.

7.1.3 Forward and Futures Prices

For all financial instruments, it is important to be able to determine whether the price available in the market is an appropriate one. Hence, we engage in the process of "pricing" the financial instrument. A major objective of this chapter is to determine the appropriate price of a futures contract. Given the similarity between futures and forward prices, however, we can benefit from studying forward contract pricing, which was covered in Chapter 2. But first, we must look at the similarities and differences between forward and futures contracts.

Recall that futures contracts settle daily and are essentially free of default risk. Forward contracts settle only at expiration and are subject to default risk. Yet both types of contracts allow the party to purchase or sell the underlying asset at a price agreed on in advance. It seems intuitive that futures prices and forward prices would be relatively close to each other.

The issues involved in demonstrating the relationship between futures and forward prices are relatively technical and beyond the scope of this book. We can, however, take a brief and fairly nontechnical look at the question. First let us ignore the credit risk issue. We shall assume that the forward contract participants are prime credit risks. We focus only on the technical distinction caused by the daily marking to market.

The day before expiration, both the futures contract and the forward contract have one day to go. At expiration, they will both settle. These contracts are therefore the same. At any other time prior to expiration, futures and forward prices can be the same or different. If interest rates are constant or at least known, any effect of the addition or subtraction of funds from the marking-to-market process can be shown to be neutral. If interest rates are positively correlated with futures prices, traders with long positions will prefer futures over forwards, because they will generate gains when interest rates are going up, and traders can invest those gains for higher returns. Also, traders will incur losses when interest rates are going down and can borrow to cover those losses at lower rates. Because traders holding long positions prefer the marking to market of futures over forwards when futures prices are positively correlated with interest rates, futures will carry higher prices than forwards. Conversely, when futures prices are negatively correlated with interest rates, traders will prefer not to mark to market, so forward contracts will carry higher prices.

Because interest rates and fixed-income security prices move in opposite directions, interest rate futures are good examples of cases in which forward and futures prices should be inversely related. Alternatively, when inflation is high, interest rates are high and investors oftentimes put their money in such assets as gold. Thus, gold futures prices and interest rates would tend to be positively correlated. It would be difficult to identify a situation in which futures prices are not correlated with interest rates. Zero correlation is rare in the financial world, but we can say that when the correlation is low or close to zero, the difference between forward and futures prices would be very small.

At this introductory level of treatment, we shall make the simplifying assumption that futures prices and forward prices are the same. We do so by ignoring the effects of marking a futures contract to market. In practice, some significant issues arise related to the marking-to-market process, but they detract from our ability to understand the important concepts in pricing and trading futures and forwards.

Therefore, based on the equivalence we are assuming between futures and forwards, we can assume that the value of a futures contract at expiration, before marking to market, is

$$v_T(T) = f_T(T) - f_0(T) = S_T - f_0(T)$$

with the spot price substituted for the futures price at T, given what we know about their convergence.

7.1.4 Pricing Futures Contracts

Now let us proceed to the pricing of futures contracts. As we did with forward contracts, we consider the case of a generic underlying asset priced at $100. A futures contract calls for delivery of the underlying asset in one year at a price of $108. Let us see if $108 is the appropriate price for this futures contract.

Suppose we buy the asset for $100 and sell the futures contract. We hold the position until expiration. For right now, we assume no costs are involved in holding the asset. We do, however, lose interest on the $100 tied up in the asset for one year. We assume that this opportunity cost is at the risk-free interest rate of 5 percent.

Recall that no money changes hands at the start of a futures contract. Moreover, we can reasonably ignore the rather small margin deposit that would be required. In addition, margin deposits can generally be met by putting up interest-earning securities, so there is really no opportunity cost. As discussed in the previous section, we also will assume away the daily settlement procedure; in other words, the value of the futures contract paid out at expiration is the final futures price minus the original futures price. Because the final futures price converges to the spot price, the final payout is the spot price minus the original futures price.

So at the contract expiration, we are short the futures and must deliver the asset, which we own. We do so and receive the original futures price for it. So we receive $108 for an asset purchased a year ago at $100. At a 5 percent interest rate, we lose only $5 in interest, so our return in excess of the opportunity cost is 3 percent risk free. This risk-free return in excess of the risk-free rate is clearly attractive and would induce traders to buy the asset and sell the futures. This arbitrage activity would drive the futures price down until it reaches $105.

If the futures price falls below $105, say to $102, the opposite arbitrage would occur. The arbitrageur would buy the futures, but either we would need to be able to borrow the asset and sell it short, or investors who own the asset would have to be willing to sell it and buy the futures. They would receive the asset price of $100 and invest it at 5 percent interest. Then at expiration, those investors would get the asset back upon taking delivery, paying $102. This transaction would net a clear and risk-free profit of $3, consisting of interest of $5 minus a $2 loss from selling the asset at $100 and buying it back at $102. Again, through the buying of the futures and shorting of the asset, the forces of arbitrage would cause prices to realign to $105.

Some difficulties occur with selling short certain assets. Although the financial markets make short selling relatively easy, some commodities are not easy to sell short. In such a case, it is still possible for arbitrage to occur. If investors who already own the asset sell it and buy the futures, they can reap similar gains at no risk. Because our interest is in financial instruments, we shall ignore these commodity market issues and assume that short selling can be easily executed.[25]

[25] Keep in mind that there are some restrictions on the short selling of financial instruments, such as uptick rules and margin requirements, but we will not concern ourselves with these impediments here.

If the market price is not equal to the price given by the model, it is important to note that regardless of the asset price at expiration, the above arbitrage guarantees a risk-free profit. That profit is known at the time the parties enter the transaction. Exhibit 3-6 summarizes and illustrates this point.

EXHIBIT 3-6 The Risk-Free Nature of Long and Short Futures Arbitrage

Asset is priced at \$100, futures is priced at $f_0(T)$ and expires in one year. Interest rate over the life of the futures is 5 percent.

Time	Long Asset, Short Futures Arbitrage	Short Asset, Long Futures Arbitrage
Today (time 0)	Buy asset at \$100 Sell futures at $f_0(T)$	Sell short asset for \$100 Buy futures for $f_0(T)$
Expiration (time T)	Asset price is S_T Futures price converges to asset price Deliver asset Profit on asset after accounting for the 5 percent (\$5) interest lost from \$100 tied up in the investment in the asset: $S_T - 100 - 5$ Profit on futures: $f_0(T) - S_T$ Total profit: $f_0(T) - 100 - 5$	Asset price is S_T Futures price converges to asset price Take delivery of asset Profit on asset after accounting for the 5 percent (\$5) interest earned on the \$100 received from the short sale of the asset: $100 + 5 - S_T$ Profit on futures: $S_T - f_0(T)$ Total profit: $100 + 5 - f_0(T)$

Conclusion: The asset price at expiration has no effect on the profit captured at expiration for either transaction. The profit is known today. To eliminate arbitrage, the futures price today, $f_0(T)$, must equal $100 + 5 = \$105$.

The transactions we have described are identical to those using forward contracts. We did note with forward contracts, however, that one can enter into an off-market forward contract, having one party pay cash to another to settle any difference resulting from the contract not trading at its arbitrage-free value up front. In the futures market, this type of arrangement is not permitted; all contracts are entered into without any cash payments up front.

So in general, through the forces of arbitrage, we say that *the futures price is the spot price compounded at the risk-free rate*:

$$f_0(T) = S_0(1 + r)$$

It is important, however, to write this result in a form we are more likely to use. In the above form, we specify r as the interest rate over the life of the futures contract. In financial markets, however, interest rates are nearly always specified as annual rates. Therefore, to compound the asset price over the life of the futures, we let r equal an annual rate and specify the life of the futures as T years. Then the futures price is found as

$$f_0(T) = S_0(1 + r)^T \qquad \textbf{(3-4)}$$

The futures price is the spot price compounded over the life of the contract, T years, at the annual risk-free rate, r. From this point on, we shall use this more general specification.

As an example, consider a futures contract that has a life of 182 days; the annual interest rate is 5 percent. Then T = 182/365 and r = 0.05. If the spot price is $100, the futures price would then be

$$f_0(T) = S_0(1 + r)^T$$
$$f_0(182/365) = 100(1.05)^{182/365}$$
$$= 102.46$$

If the futures is selling for more than $102.46, an arbitrageur can buy the asset for $100 and sell the futures for whatever its price is, hold the asset (losing interest on $100 at an annual rate of 5 percent) and deliver it to receive the futures price. The overall strategy will net a return in excess of 5 percent a year at no risk. If the futures is selling for less than $102.46, the arbitrageur can borrow the asset, sell it short, and buy the futures. She will earn interest on the funds obtained from the short sale and take delivery of the asset at the futures expiration, paying the original futures price. The overall transaction results in receiving $100 up front and paying back an amount less than the 5 percent risk-free rate, making the transaction like a loan that is paid back at less than the risk-free rate. If one could create such a loan, one could use it to raise funds and invest the funds at the risk-free rate to earn unlimited gains.

7.1.5 Pricing Futures Contracts When There Are Storage Costs

Except for opportunity costs, we have until now ignored any costs associated with holding the asset. In many asset markets, there are significant costs, other than the opportunity cost, to holding an asset. These costs are referred to as **storage costs** or **carrying costs** and are generally a function of the physical characteristics of the underlying asset. Some assets are easy to store; some are difficult. For example, assume the underlying is oil, which has significant storage costs but a very long storage life.[26] One would not expect to incur costs associated with a decrease in quality of the oil. Significant risks do exist, however, such as spillage, fire, or explosion. Some assets on which futures are based are at risk for damage. For example, cattle and pigs can become ill and die during storage. Grains are subject to pest damage and fire. All of these factors have the potential to produce significant storage costs, and protection such as insurance leads to higher storage costs for these assets. On the other hand, financial assets have virtually no storage costs. Of course, all assets have one significant storage cost, which is the opportunity cost of money tied up in the asset, but this effect is covered in the present value calculation.

It is reasonable to assume that the storage costs on an asset are a function of the quantity of the asset to be stored and the length of time in storage. Let us specify this cost with the variable FV(SC,0,T), which denotes the value at time T (expiration) of the storage costs (excluding opportunity costs) associated with holding the asset over the period 0 to T. By specifying these costs as of time T, we are accumulating the costs and compounding the interest thereon until the end of the storage period. We can reasonably assume that when storage is initiated, these costs are known.[27]

Revisiting the example we used previously, we would buy the asset at S_0, sell a futures contract at $f_0(T)$, store the asset and accumulate costs of FV(SC,0,T), and deliver the asset at expiration to receive the futures price. The total payoff is $f_0(T) - FV(SC,0,T)$.

[26] After all, oil has been stored by nature for millions of years.

[27] There may be reason to suggest that storage costs have an element of uncertainty in them, complicating the analysis.

This amount is risk free. To avoid an arbitrage opportunity, its present value should equal the initial outlay, S_0, required to establish the position. Thus,

$$[f_0(T) - FV(SC,0,T)]/(1 + r)^T = S_0$$

Solving for the futures price gives

$$f_0(T) = S_0(1 + r)^T + FV(SC,0,T) \quad (3\text{-}5)$$

This result says that *the futures price equals the spot price compounded over the life of the futures contract at the risk-free rate, plus the future value of the storage costs over the life of the contract.* In the previous example with no storage costs, we saw that the futures price was the spot price compounded at the risk-free rate. With storage costs, we must add the future value of the storage costs. The logic behind this adjustment should make sense. The futures price should be higher by enough to cover the storage costs when a trader buys the asset and sells a futures to create a risk-free position.[28]

Consider the following example. The spot price of the asset is \$50, the interest rate is 6.25 percent, the future value of the storage costs is \$1.35, and the futures expires in 15 months. Then T = 15/12 = 1.25. The futures price would, therefore, be

$$f_0(T) = S_0(1 + r)^T + FV(SC,0,T)$$
$$f_0(1.25) = 50(1.0625)^{1.25} + 1.35$$
$$= 55.29$$

If the futures is selling for more than \$55.29, the arbitrageur would buy the asset and sell the futures, holding the position until expiration, at which time he would deliver the asset and collect the futures price, earning a return that covers the 6.25 percent cost of the money and the storage costs of \$1.35. If the futures is selling for less than \$55.29, the arbitrageur would sell short the asset and buy the futures, reinvesting the proceeds from the short sale at 6.25 percent and saving the storage costs. The net effect would be to generate a cash inflow today plus the storage cost savings and a cash outflow at expiration that would replicate a loan with a rate less than the risk-free rate. Only if the futures sells for exactly \$55.29 do these arbitrage opportunities go away.

7.1.6 Pricing Futures Contracts When There Are Cash Flows on the Underlying Asset

In each case we have considered so far, the underlying asset did not generate any positive cash flows to the holder. For some assets, there will indeed be positive cash flows to the holder. Recall that in Chapter 2, we examined the pricing and valuation of forward contracts on stocks and bonds and were forced to recognize that stocks pay dividends, bonds pay interest, and these cash flows affect the forward price. A similar concept applies here and does so in a symmetric manner to what we described in the previous section in which the asset incurs a cash cost. As we saw in that section, a cash cost incurred from holding the asset increases the futures price. Thus, we might expect that cash generated from holding the asset would result in a lower futures price and, as we shall see in this section, that

[28] We did not cover assets that are storable at significant cost when we studied forward contracts because such contracts are less widely used for these assets. Nonetheless, the formula given here would apply for forward contracts as well, given our assumption of no credit risk on forward contracts.

is indeed the case. But in the next section, we shall also see that it is even possible for an asset to generate nonmonetary benefits that must also be taken into account when pricing a futures contract on it.

Let us start by assuming that over the life of the futures contract, the asset generates positive cash flows of FV(CF,0,T). It is no coincidence that this notation is similar to the one we used in the previous section for the storage costs of the underlying asset over the life of the futures. Cash inflows and storage costs are just different sides of the same coin. We must remember, however, that FV(CF,0,T) represents a positive flow in this case. Now let us revisit our example.

We would buy the asset at S_0, sell a futures contract at $f_0(T)$, store the asset and generate positive cash flows of FV(CF,0,T), and deliver the asset at expiration, receiving the futures price. The total payoff is $f_0(T) + FV(CF,0,T)$. This amount is risk free and known at the start. To avoid an arbitrage opportunity, its present value should equal the initial outlay, S_0, required to establish the position. Thus,

$$[f_0(T) + FV(CF,0,T)]/(1 + r)^T = S_0$$

Solving for the futures price gives

$$f_0(T) = S_0(1 + r)^T - FV(CF,0,T) \tag{3-6}$$

In the previous example that included storage costs, we saw that the futures price was the spot price compounded at the risk-free rate plus the future value of the storage costs. With positive cash flows, we must subtract the future value of these cash flows. The logic behind this adjustment should make sense. The futures price should be reduced by enough to account for the positive cash flows when a trader buys the asset and sells a futures to create a risk-free position. Otherwise, the trader would receive risk-free cash flows from the asset *and* the equivalent amount from the sale of the asset at the futures price. Reduction of the futures price by this amount avoids overcompensating the trader.

As noted, these cash flows can be in the form of dividends from a stock or coupon interest from a bond. When we specifically examine the pricing of bond and stock futures, we shall make this specification a little more precise and work an example.

7.1.7 Pricing Futures Contracts When There Is a Convenience Yield

Now consider the possibility that the asset might generate nonmonetary benefits that must also be taken into account. The notion of nonmonetary benefits that could affect futures prices might sound strange, but upon reflection, it makes perfect sense. For example, a house is a common and normally desirable investment made by individuals and families. The house generates no monetary benefits and incurs significant costs. As well as being a possible monetary investment if prices rise, the house generates some nonmonetary benefits in the form of serving as a place to live. These benefits are quite substantial; many people consider owning a residence preferable to renting, and people often sell their homes for monetary gains far less than any reasonable return on a risky asset. Clearly the notion of a nonmonetary benefit to owning an asset is one most people are familiar with.

In a futures contract on an asset with a nonmonetary gain, that gain must be taken into account. Suppose, for the purpose of understanding the effect of nonmonetary benefits on a futures contract, we create a hypothetical futures contract on a house. An individual purchases a house and sells a futures contract on it. We shall keep the arguments as simple as possible by ignoring the operating or carrying costs. What should be the futures price? If the futures is priced at the spot price plus the risk-free rate, as in the original case, the homeowner receives a guaranteed sale price, giving a return of the risk-free rate *and* the use of the home. This is clearly a good deal. Homeowners would be eager to sell futures contracts, leading to a decrease in the price of the futures. Thus, any nonmonetary benefits ought to be factored into the futures price and logically would lead to a lower futures price.

Of course, in the real world of standardized futures contracts, there are no futures contracts on houses. Nonetheless, there are futures contracts on assets that have nonmonetary benefits. Assets that are often in short supply, particularly those with seasonal and highly risky production processes, are commonly viewed as having such benefits. The nonmonetary benefits of these assets are referred to as the **convenience yield**. Formally, a convenience yield is the nonmonetary return offered by an asset when in short supply. When an asset is in short supply, its price tends to be high. Holders of the asset earn an implicit incremental return from having the asset on hand. This return enables them, as commercial enterprises, to avoid the cost and inconvenience of not having their primary product or resource input on hand. Because shortages are generally temporary, the spot price can be higher than the futures price, even when the asset incurs storage costs. If a trader buys the asset, sells a futures contract, and stores the asset, the return is risk free and will be sufficient to cover the storage costs and the opportunity cost of money, but it will be reduced by an amount reflecting the benefits of holding the asset during a period of shortage or any other nonmonetary benefits.

Now, let the notation FV(CB,0,T) represent the future value of the costs of storage minus the benefits:

$$\text{FV(CB,0,T)} = \text{Costs of storage} - \text{Nonmonetary benefits (Convenience yield)}$$

where all terms are expressed in terms of their future value at time T and are considered to be known at time 0. If the costs exceed the benefits, FV(CB,0,T) is a positive number.[29] We refer to FV(CB,0,T) as the **cost of carry**.[30] The general futures pricing formula is

$$f_0(T) = S_0(1 + r)^T + \text{FV(CB,0,T)} \qquad \textbf{(3-7)}$$

The futures price is the spot price compounded at the risk-free rate plus the cost of carry. This model is often called the **cost-of-carry model**.

Consider an asset priced at \$75; the risk-free interest rate is 5.15 percent, the net of the storage costs, interest, and convenience yield is \$3.20, and the futures expires in nine months. Thus, T = 9/12 = 0.75. Then the futures price should be

$$\begin{aligned} f_0(T) &= S_0(1 + r)^T + \text{FV(CB,0,T)} \\ f_0(0.75) &= 75(1.0515)^{0.75} + 3.20 \\ &= 81.08 \end{aligned}$$

As we have always done, we assume that this price will prevail in the marketplace. If it does not, the forces of arbitrage will drive the market price to the model price. If the futures

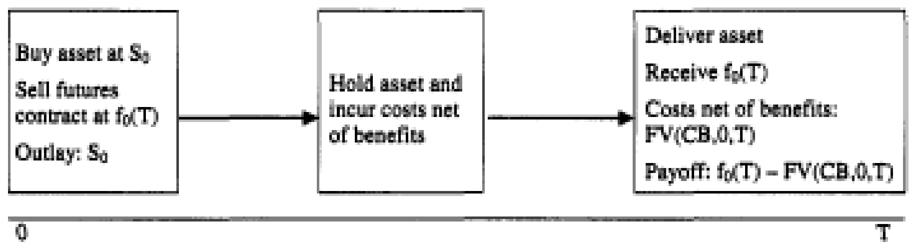
[29] In other words, FV(CB,0,T) has to be positive to refer to it as a "cost."

[30] In some cases, such as in inventory storage, it is customary to include the opportunity cost in the definition of cost of carry; but we keep it separate in this text.

price exceeds \$81.08, the arbitrageur can buy the asset and sell the futures to earn a risk-free return in excess of the risk-free rate. If the futures price is less than \$81.08, the arbitrageur can either sell the asset short or sell it if he already owns it, and then also buy the futures, creating a risk-free position equivalent to a loan that will cost less than the risk-free rate. The gains from both of these transactions will have accounted for any nonmonetary benefits. This arbitrage activity will force the market price to converge to the model price.

The above equation is the most general form of the futures pricing formula we shall encounter. Exhibit 3-7 reviews and illustrates how we obtained this formula and provides another example.

EXHIBIT 3-7 Pricing a Futures Contract



The transaction is risk-free and should be equivalent to investing S_0 dollars in a risk-free asset that pays $f_0(T)$ – FV(CB,0,T) at time T. Therefore, the payoff at T must be the future value of the initial outlay invested at the risk-free rate. For this relationship to hold, the futures price must be given as

$$f_0(T) = S_0(1 + r)^T + FV(CB,0,T)$$

Example: An asset is selling for \$225. A futures contract expires in 150 days (T = 150/365 = 0.411). The risk-free rate is 7.5 percent, and the net cost of carry is \$5.75. The futures price will be

$$f_0(T) = f_0(0.411) = \$225(1.075)^{0.411} + \$5.75 = \$237.54$$

Some variations of this general formula are occasionally seen. Sometimes the opportunity cost of interest is converted to dollars and imbedded in the cost of carry. Then we say that $f_0(T) = S_0 + FV(CB,0,T)$; the futures price is the spot price plus the cost of carry. This is a perfectly appropriate way to express the formula if the interest is imbedded in the cost of carry, but we shall not do so in this book.

Another variation of this formula is to specify the cost of carry in terms of a rate, such as y. Then we have $f_0(T) = S_0(1 + r)^T(1 + y)^T$. Again, this variation is certainly appropriate but is not the version we shall use.[31]

Note that when we get into the specifics of pricing certain types of futures contracts, we must fine-tune the formulas a little more. First, however, we explore some general characterizations of the relationship between futures and spot prices.

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[31] Yet another variation of this formula is to use $(1 + r + y)^T$ as an approximation for $(1 + r)^T(1 + y)^T$. We do not, however, consider this expression an acceptable way to compute the futures price as it is an approximation of a formula that is simple enough to use without approximating.

PRACTICE PROBLEM 2

Consider an asset priced at \$50. The risk-free interest rate is 8 percent, and a futures contract on the asset expires in 45 days. Answer the following, with questions A, B, C, and D independent of the others.

A. Find the appropriate futures price if the underlying asset has no storage costs, cash flows, or convenience yield.

B. Find the appropriate futures price if the future value of storage costs on the underlying asset at the futures expiration equals \$2.25.

C. Find the appropriate futures price if the future value of positive cash flows on the underlying asset equals \$0.75.

D. Find the appropriate futures price if the future value of the net overall cost of carry on the underlying asset equals \$3.55.

E. Using Part D above, illustrate how an arbitrage transaction could be executed if the futures contract is trading at \$60.

F. Using Part A above, determine the value of a long futures contract an instant before marking to market if the previous settlement price was \$49.

SOLUTIONS

A. First determine that T = 45/365 = 0.1233. Then the futures price is

$$f_0(0.1233) = \$50(1.08)^{0.1233} = \$50.48$$

B. Storage costs must be covered in the futures price, so we add them:

$$f_0(0.1233) = \$50(1.08)^{0.1233} + \$2.25 = \$52.73$$

C. A positive cash flow, such as interest or dividends on the underlying, reduces the futures price:

$$f_0(0.1233) = \$50(1.08)^{0.1233} - \$0.75 = \$49.73$$

D. The net overall cost of carry must be covered in the futures price, so we add it:

$$f_0(0.1233) = \$50(1.08)^{0.1233} + \$3.55 = \$54.03$$

E. Follow these steps:

- Sell the futures at \$60.
- Buy the asset at \$50.
- Because the asset price compounded at the interest rate is \$50.48, the interest forgone is \$0.48. So the asset price is effectively \$50.48 by the time of the futures expiration.
- Incur costs of \$3.55.
- At expiration, deliver the asset and receive \$60. The net investment in the asset is \$50.48 + \$3.55 = \$54.03. If the asset is sold for \$60, the net gain is \$5.97.

F. If the last settlement price was \$49.00 and the price is now \$50.48 (our answer in Part A), the value of a long futures contract equals the difference between these prices: \$50.48 − \$49.00 = \$1.48.

7.1.8 BACKWARDATION AND CONTANGO

Because the cost of carry, FV(CB,0,T), can be either positive or negative, the futures price can be greater or less than the spot price. Because the costs plus the interest tend to exceed the benefits, it is more common for the futures price to exceed the spot price, a situation called **contango**. In contrast, when the benefits exceed the costs plus the interest, the futures price will be less than the spot price, called **backwardation**. These terms are not particularly important in understanding the necessary concepts, but they are so commonly used that they are worthwhile to remember.

7.1.9 FUTURES PRICES AND EXPECTED SPOT PRICES

An important concept when examining futures prices is the relationship between futures prices and expected spot prices. In order to fully understand the issue, let us first consider the relationship between spot prices and expected spot prices. Consider an asset with no risk, but which incurs carrying costs. At time 0, the holder of the asset purchases it with the certainty that she will cover her opportunity cost and carrying cost. Otherwise, she would not purchase the asset. Thus, the spot price at time 0 is the present value of the total of the spot price at time T less costs minus benefits:

$$S_0 = \frac{S_T - FV(CB,0,T)}{(1 + r)^T}$$

$$= \frac{S_T}{(1 + r)^T} - \frac{FV(CB,0,T)}{(1 + r)^T}$$

Because FV(CB,0,T) is the future value of the carrying cost, $FV(CB,0,T)/(1 + r)^T$ is the present value of the carrying cost. So on the one hand, we can say that the spot price is the future spot price minus the future value of the carrying cost, all discounted to the present. On the other hand, we can also say that the spot price is the discounted value of the future spot price minus the present value of the carrying cost.

If, however, the future price of the asset is uncertain, as it nearly always is, we must make some adjustments. For one, we do not know at time 0 what S_T will be. We must form an expectation, which we will denote as $E_0(S_T)$. But if we simply replace S_T above with $E_0(S_T)$ we would not be acting rationally. We would be paying a price today and expecting compensation only at the risk-free rate along with coverage of our carrying cost. Indeed, one of the most important and intuitive elements of all we know about finance is that risky assets require a risk premium. Let us denote this risk premium with the symbol, $\phi_0(S_T)$. It represents a discount off of the expected value that is imbedded in the current price, S_0. Specifically, the current price is now given as

$$S_0 = \frac{E_0(S_T) - FV(CB,0,T) - \phi_0(S_T)}{(1 + r)^T}$$

where we see that the risk premium lowers the current spot price. Intuitively, investors pay less for risky assets, all other things equal.

Until now, we have worked only with the spot price, but nothing we have said so far violates the rule of no arbitrage. Hence, our futures pricing formula, $f_0(T) = S_0(1 + r)^T + FV(CB,0,T)$, still applies. If we rearrange the futures pricing formula for FV(CB,0,T), substitute this result into the formula for S_0, and solve for the futures price, $f_0(T)$, we obtain $f_0(T) = E_0(S_T) - \phi_0(S_T)$. This equation says that the futures price equals the expected future spot price minus the risk premium.

An important conclusion to draw from this formula is that the futures price does not equal the expectation of the future spot price. The futures price would be biased on the high side. If one felt that the futures price were an unbiased predictor of the future spot price, $f_0(T) = E_0(S_T)$, one could expect on average to be able to predict the future spot price of oil by looking at the futures price of oil. But that is not likely to be the case.

The intuition behind this result is easy to see. We start with the assumption that all units of the asset must be held by someone. Holders of the asset incur the risk of its future selling price. If a holder of the asset wishes to transfer that risk by selling a futures contract, she must offer a futures contract for sale. But if the futures contract is offered at a price equal to the expected spot price, the buyer of the futures contract takes on the risk but expects to earn only a price equal to the price paid for the futures. Thus, the futures trader incurs the risk without an expected gain in the form of a risk premium. On the opposite side of the coin, the holder of the asset would have a risk-free position with an expected gain in excess of the risk-free rate. Clearly, the holder of the asset would not be able to do such a transaction. Thus, she must lower the price to a level sufficient to compensate the futures trader for the risk he is taking on. This process will lead to a futures price that equals the expected spot price minus the risk premium, as shown in the above equation. In effect, the risk premium transfers from the holder of the asset to the buyer of the futures contract.

In all fairness, however, we must acknowledge that this view is not without its opponents. Some consider the futures price an unbiased predictor of the future spot price. In such a case, the futures price would tend to overshoot and undershoot the future spot price but on average would be equal to it. For such a situation to exist would require the unreasonable assumption that there is no risk or that investors are risk neutral, meaning that they are indifferent to risk. There is, however, one other situation in which the risk premium could disappear or even turn negative. Suppose holders of the asset who want to hedge their holdings could find other parties who need to purchase the asset and who would like to hedge by going long. In that case, it should be possible for the two parties to consummate a futures transaction with the futures price equal to the expected spot price. In fact, if the parties going long exerted greater pressure than the parties going short, it might even be possible for the futures price to exceed the expected spot price.

When futures prices are lower than expected spot prices, the situation is called **normal backwardation**. When futures prices are higher than expected spot prices, it is called **normal contango**. Note the contrast with the terms backwardation and contango, which we encountered in Section 7.1.8. Backwardation means that the futures price is lower than the spot price; contango means that the futures price exceeds the spot price. Normal backwardation means that the futures price is lower than the expected spot price; normal contango means that the futures price exceeds the expected spot price.

Generally speaking, we should favor the notion that futures prices are biased predictors of future spot prices because of the transferal of the risk premium from holders of the asset to buyers of futures. Intuitively, this is the more likely case, but the other interpretations are possible. Fortunately, for our purposes, it is not critical to resolve the issue, but we do need to be aware of it.

7.2 Pricing Interest Rate Futures

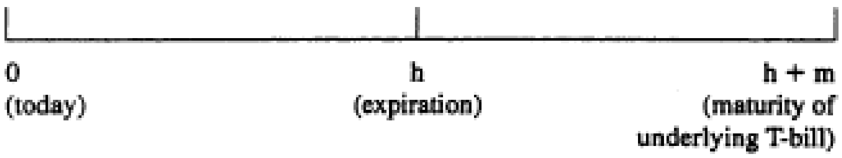
We shall examine the pricing of three classes of interest rate futures contracts: Treasury bill futures, Eurodollar futures, and Treasury bond futures. In Section 6.1, we described the characteristics of these instruments and contracts. Now we look at their pricing, keeping in mind that we established the general foundations for pricing—the cost-of-carry model—in the previous section. Recall that in the cost-of-carry model, we buy the underlying asset, sell a futures contract, store the asset (which incurs costs and could generate benefits), and deliver the asset at expiration. To prevent arbitrage, the futures price is found in general as

$$\text{Futures price} = \text{Spot price of underlying asset} \times \text{Compounding factor} + \text{Costs net of monetary and nonmonetary benefits}$$

When the underlying is a financial instrument, there will be no nonmonetary benefits and no costs other than the opportunity cost.

7.2.1 Pricing T-Bill Futures

Consider the following time line of our problem:



Time 0 is today, and time h is the expiration day of the futures contract. The T-bill underlying the contract is an m-day T-bill. Thus, when the futures expires, the T-bill is required to have m days to go before maturity. So from our perspective today, the underlying T-bill is an (h + m)-day T-bill.[32] As in Chapter 2 for FRAs, h and m represent a particular number of days. In accordance with common practice, m is traditionally 90. We now introduce some necessary notation. First, where necessary, we use a simple expression, r, for the risk-free interest rate. But when pricing Treasury bill futures, we need a more flexible notation. Here we need the rates for T-bills maturing on day h and on day h + m. In addition, because interest rates can change from day 0 to day h, we need notation that distinguishes rates for different maturities and rates at different points in time.[33]

To find the spot price of the underlying asset, we need the discount rate on an (h + m)-day T-bill. Suppose we have

$r_0^d(h)$, $r_0^d(h + m)$ = Discount rates in effect on day 0 of h-day and (h + m)-day T-bills

As described in Section 6, these are discount rates and convert to prices by the following formula: $B_0(j) = 1 - r_0^d(j)(j/360)$, where in this case j will either be h or h + m. Thus, the prices of h- and (h + m)-day spot T-bills on day 0 (assuming \$1 face amounts) are

$$B_0(h) = 1 - r_0^d(h)\left(\frac{h}{360}\right)$$

$$B_0(h + m) = 1 - r_0^d(h + m)\left(\frac{h + m}{360}\right)$$

In other words, the h- or (h + m)-day discount rate is multiplied by the number of days in the life of the T-bill over 360 and subtracted from the face value of \$1.

Now let us turn to the futures market. We define

$r_0^{df}(h)$ = implied discount rate on day 0 of futures contract expiring on day h, where the deliverable instrument is an m-day T-bill

$f_0(h)$ = price on day 0 of futures contract expiring on day h

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[32] It is common practice in the T-bill futures market to refer to the underlying as an m-day T-bill, but at time 0, the underlying must be an (h + m)-day T-bill in order for it to be an m-day T-bill at time h.

[33] When we assume that the interest rates are the same for all maturities and cannot change over time, which is considered acceptable when working with stock index and currency futures, we can use the simpler notation of r for the rate.

The relationship between $r_0^{df}(h)$ and $f_0(h)$ is

$$f_0(h) = 1 - r_0^{df}(h)\left(\frac{m}{360}\right)$$

It is important to note that the futures price, not the implied discount rate, is the more important variable. Like any price, the futures price is determined in a market of buyers and sellers. Any rate is simply a way of transforming a price into a number that can be compared with rates on various other fixed-income instruments.[34] Do not think that a futures contract pays an interest rate. It is more appropriate to think of such a rate imbedded in a futures price as an *implied rate*, hence our use of the term *implied discount rate*. Although knowing this rate does not tell us any more than knowing the futures price, traders often refer to the futures contract in terms of the rate rather than the price.

Finally, let us note that at expiration, the futures price is the price of the underlying T-bill

$$\begin{aligned} f_h(h) &= B_h(h + m) \\ &= 1 - r_h^d(h + m)\left(\frac{m}{360}\right) \end{aligned}$$

where $B_h(h + m)$ is the price on day h of the T-bill maturing on day h + m, and $r_h^d(h + m)$ is the discount rate on day h on the T-bill maturing on day h + m.

We now derive the futures price by constructing a risk-free portfolio that permits no arbitrage profits to be earned. This transaction is referred to as a cash-and-carry strategy, because the trader buys the asset in the cash (spot) market and carries (holds) it.

On day 0, we buy the (h + m)-day T-bill, investing $B_0(h + m)$. We simultaneously sell a futures contract at the price $f_0(h)$. On day h, we are required to deliver an m-day T-bill. The bill we purchased, which originally had h + m days to maturity, now has m days to maturity. We therefore deliver that bill and receive the original futures price. We can view this transaction as having paid $B_0(h + m)$ on day 0 and receiving $f_0(h)$. Because $f_0(h)$ is known on day 0, this transaction is risk free. It should thus earn the same return per dollar invested as would a T-bill purchased on day 0 that matures on day h. The return per dollar invested from the arbitrage transaction would be $f_0(h)/B_0(h + m)$, and the return per dollar invested in an h-day T-bill would be $1/B_0(h)$.[35] Consequently, we set these values equal:

$$\frac{f_0(h)}{B_0(h + m)} = \frac{1}{B_0(h)}$$

Solving for the futures price, we obtain

$$f_0(h) = \frac{B_0(h + m)}{B_0(h)}$$

[34] To further reinforce the notion that an interest rate is just a transformation of a price, consider a zero-coupon bond selling at $95 and using 360 days as a year. The price can be transformed into a rate in the manner of 1/0.95 − 1 = 0.0526 or 5.26 percent. But using the convention of the Treasury bill market, the rate is expressed as a discount rate. Then 0.95 = 1 − Rate × (360/360), and the rate would be 0.05 or 5 percent. A price can be converted into a rate in a number of other ways, such as by assuming different compound periods. The price of any asset is determined in a market-clearing process. The rate is just a means of transforming the price so that interest rate instruments and their derivatives can be discussed in a more comparable manner.

[35] For example, if a one-year $1 face value T-bill is selling for $0.90, the return per dollar invested is $1/$0.90 = 1.1111.

In words, the futures price is the ratio of the longer-term bill price to the shorter-term bill price. This price is, in fact, the same as the forward price from the term structure. In fact, as we noted above, futures prices and forward prices will be equal under the assumptions we have made so far and will follow throughout this book.

Recall that we previously demonstrated that the futures price should equal the spot price plus the cost of carry. Yet the above formula looks nothing like this result. In fact, however, it is consistent with the cost-of-carry formula. First, the above formula can be written as

$$f_0(h) = B_0(h + m)\left[\frac{1}{B_0(h)}\right]$$

As noted above, the expression $1/B_0(h)$ can be identified as the return per dollar invested over h days, which simplifies to $[1 + r_0(h)]^{h/365}$, which is essentially a compound interest factor for h days at the rate $r_0(h)$. Note that h is the number of days, assuming 365 in a year. For the period ending at day h, the above formula becomes

$$f_0(h) = B_0(h + m)[1 + r_0(h)]^{h/365} \tag{3-8}$$

and the futures price is seen to equal the spot price of the underlying compounded at the interest rate, which simply reflects the opportunity cost of the money tied up for h days.

Note that what we have been doing is deriving the appropriate price for a futures contract. In a market with no arbitrage opportunities, the actual futures price would be this theoretical price. Let us suppose for a moment, however, that the actual futures price is something else, say $f_0(h)^*$. The spot price is, of course, $B_0(h + m)$. Using these two numbers, we can infer the implied rate of return from a transaction involving the purchase of the T-bill and sale of the futures. We have

$$f_0(h)^* = B_0(h + m)[1 + r_0(h)^*]^{h/365}$$

where $r_0(h)^*$ is the implied rate of return. Solving for $r_0(h)^*$ we obtain

$$r_0(h)^* = \left[\frac{f_0(h)^*}{B_0(h + m)}\right]^{365/h} - 1 \tag{3-9}$$

This rate of return, $r_0(h)^*$, has a special name, the **implied repo rate.** It is the rate of return from a cash-and-carry transaction that is implied by the futures price relative to the spot price. Traders who engage in such transactions often obtain the funds to do so in the repurchase agreement (repo) market. The implied repo rate tells the trader what rate of return to expect from the strategy. If the financing rate available in the repo market is less than the implied repo rate, the strategy is worthwhile and would generate an arbitrage profit. If the trader could lend in the repo market at greater than the implied repo rate, the appropriate strategy would be to reverse the transaction—selling the T-bill short and buying the futures—turning the strategy into a source of financing that would cost less than the rate at which the funds could be lent in the repo market.[36]

The implied repo rate is the rate of return implied by the strategy of buying the asset and selling the futures. As noted above, the futures price is often expressed in terms of an implied discount rate. Remember that the buyer of a futures contract is committing to buy

[36] The concepts of a cash-and-carry strategy and the implied repo rate are applicable to any type of futures contract, but we cover them only with respect to T-bill futures.

a T-bill at the price $f_0(h)$. In the convention of pricing a T-bill by subtracting a discount rate from par value, the implied discount rate would be

$$r_0^{df}(h) = [1 - f_0(h)]\left(\frac{360}{m}\right) \tag{3-10}$$

We can also determine this implied discount rate from the discount rates on the h- and (h + m)-day T-bills as follows:[37]

$$r_0^{df}(h) = \left\{1 - \left[\frac{1 - r_0^d(h+m)\left(\frac{h+m}{360}\right)}{1 - r_0^d(h)\left(\frac{h}{360}\right)}\right]\right\}\left(\frac{360}{m}\right)$$

Now let us look at an example. We are interested in pricing a futures contract expiring in 30 days. A 30-day T-bill has a discount rate of 6 percent, and a 120-day T-bill has a discount rate of 6.6 percent. With h = 30 and h + m = 120, we have

$$r_0^d(h) = r_0^d(30) = 0.06$$

$$r_0^d(h+m) = r_0^d(120) = 0.066$$

The prices of these T-bills will, therefore, be

$$B_0(h) = 1 - r_0^d(h)\left(\frac{h}{360}\right)$$

$$B_0(30) = 1 - 0.06\left(\frac{30}{360}\right) = 0.9950$$

$$B_0(h+m) = 1 - r_0^d(h+m)\left(\frac{h+m}{360}\right)$$

$$B_0(120) = 1 - 0.066\left(\frac{120}{360}\right) = 0.9780$$

Using the formula we derived, we have the price of a futures expiring in 30 days as

$$f_0(h) = \frac{B_0(h+m)}{B_0(h)}$$

$$f_0(30) = \frac{B_0(120)}{B_0(30)} = \frac{0.9780}{0.9950} = 0.9829$$

The discount rate implied by the futures price would be

$$r_0^{df}(h) = [1 - f_0(h)]\left(\frac{360}{m}\right)$$

$$r_0^{df}(30) = (1 - 0.9829)\left(\frac{360}{90}\right) = 0.0684$$

[37] This formula is found by substituting $1 - r_0^d(h+m)[(h+m)/360]$ for $B_0(h+m)$ and $1 - r_0^d(h)(h/360)$ for $B_0(h)$ in the above equation for $r_0^{df}(h)$. This procedure expresses the spot prices in terms of their respective discount rates.

In other words, in the T-bill futures market, the rate would be stated as 6.84 percent, which would imply a futures price of 0.9829.[38] Alternatively, the implied futures discount rate could be obtained from the spot discount rates as

$$r_0^{df}(h) = \left\{1 - \left[\frac{1 - r_0^d(h+m)\left(\frac{h+m}{360}\right)}{1 - r_0^d(h)\left(\frac{h}{360}\right)}\right]\right\}\left(\frac{360}{m}\right)$$

$$r_0^{df}(30) = \left\{1 - \left[\frac{1 - 0.066\left(\frac{120}{360}\right)}{1 - 0.06\left(\frac{30}{360}\right)}\right]\right\}\left(\frac{360}{90}\right) = 0.0683$$

with a slight difference due to rounding.

To verify this result, one would buy the 120-day T-bill for 0.9780 and sell the futures at a price of 0.9829. Then, 30 days later, the T-bill would be a 90-day T-bill and would be delivered to settle the futures contract. The trader would receive the original futures price of 0.9829. The return per dollar invested would be

$$\frac{0.9829}{0.9780} = 1.0050$$

If, instead, the trader had purchased a 30-day T-bill at the price of 0.9950 and held it for 30 days, the return per dollar invested would be

$$\frac{1}{0.9950} = 1.0050$$

Thus, the purchase of the 120-day T-bill with its price in 30 days hedged by the sale of the futures contract is equivalent to purchasing a 30-day T-bill and holding it to maturity. Each transaction has the same return per dollar invested and is free of risk.

Suppose in the market, the futures price is 0.9850. The implied repo rate would be

$$r_0(h)^* = \left[\frac{f_0(h)^*}{B_0(h+m)}\right]^{365/h} - 1$$
$$= \left(\frac{0.9850}{0.9870}\right)^{365/30} - 1 = 0.0906$$

Buying the 120-day T-bill for 0.9780 and selling a futures for 0.9850 generates a rate of return of 0.9850/0.9780 − 1 = 0.007157. Annualizing this rate, $(1.007157)^{365/30} - 1 =$ 0.0906. If financing could be obtained in the repo market for less than this annualized rate, the strategy would be attractive. If the trader could lend in the repo market at higher than this rate, he should buy the futures and sell short the T-bill to implicitly borrow at 9.06 percent and lend in the repo market at a higher rate.

Let us now recap the pricing of Treasury bill futures. We buy an (h + m)-day bond and sell a futures expiring on day h, which calls for delivery of an m-day T-bill. The futures price should be the price of the (h + m)-day T-bill compounded at the h-day risk-free rate.

[38] We should also probably note that the IMM Index would be 100 − 6.84 = 93.16. Thus, the futures price would be quoted in the market as 93.16.

That rate is the rate of return on an h-day bill. The futures price can also be obtained as the ratio of the price of the (h + m)-day T-bill to the price of the h-day T-bill. Alternatively, we can express the futures price in terms of an implied discount rate, and we can derive the price in terms of the discount rates on the (h + m)-day T-bill and the h-day T-bill. Finally, remember that the actual futures price in the market relative to the price of the (h + m)-day T-bill implies a rate of return called the implied repo rate. The implied repo rate can be compared with the rate in the actual repo market to determine the attractiveness of an arbitrage transaction.

Exhibit 3-8 summarizes the important formulas involved in the pricing of T-bill futures. We then turn to the pricing of another short-term interest rate futures contract, the Eurodollar futures.

EXHIBIT 3-8 Pricing Formulas for T-Bill Futures Contract

Futures price = Underlying T-bill price compounded at risk-free rate

Futures price in terms of spot T-bills:

$$f_0(h) = \frac{B_0(h+m)}{B_0(h)}$$

Futures price as spot price compounded at risk-free rate:

$$f_0(h) = B_0(h+m)[1 + r_0(h)]^{h/365}$$

Discount rate implied by futures price:

$$r_0^{df}(h) = [1 - f_0(h)]\left(\frac{360}{m}\right) = \left\{1 - \left[\frac{1 - r_0^d(h+m)\left(\frac{h+m}{360}\right)}{1 - r_0^d(h)\left(\frac{h}{360}\right)}\right]\right\}\left(\frac{360}{m}\right)$$

Implied repo rate:

$$r_0(h)^* = \left[\frac{f_0(h)^*}{B_0(h+m)}\right]^{365/h} - 1$$

PRACTICE PROBLEM 3

A futures contract on a Treasury bill expires in 50 days. The T-bill matures in 140 days. The discount rates on T-bills are as follows:

50-day bill:	5.0 percent
140-day bill:	4.6 percent

A. Find the appropriate futures price by using the prices of the 50- and 140-day T-bills.

B. Find the futures price in terms of the underlying spot price compounded at the appropriate risk-free rate.

C. Convert the futures price to the implied discount rate on the futures.

D. Now assume that the futures contract is trading in the market at an implied discount rate 10 basis points lower than is appropriate, given the pricing model and the rule of no arbitrage. Demonstrate how an arbitrage transaction could be executed and show the outcome. Calculate the implied repo rate and discuss how it would be used to determine the profitability of the arbitrage.

SOLUTIONS

A. First, find the prices of the 50- and 140-day bonds:

$$B_0(50) = 1 - 0.05(50/360) = 0.9931$$
$$B_0(140) = 1 - 0.046(140/360) = 0.9821$$

The futures price is, therefore,

$$f_0(50) = \frac{0.9821}{0.9931} = 0.9889$$

B. First, find the rate at which to compound the spot price of the 140-day T-bill. This rate is obtained from the 50-day T-bill:

$$[1 + r_0(h)]^{h/365} = \frac{1}{0.9931} = 1.0069$$

We actually do not need to solve for $r_0(h)$. The above says that based on the rate $r_0(h)$, every dollar invested should grow to a value of 1.0069. Thus, the futures price should be the spot price (the price of the 140-day T-bill) compounded by the factor 1.0069:

$$f_0(50) = 0.9821(1.0069) = 0.9889$$

Annualized, this rate would equal $(1.0069)^{365/50} - 1 = 0.0515$.

C. Given the futures price of 0.9889, the implied discount rate is

$$r_0^{df}(50) = (1 - 0.9889)\left(\frac{360}{90}\right)$$
$$= 0.0444$$

D. If the futures is trading for 10 basis points lower, it trades at a rate of 4.34 percent, so the futures price would be

$$f_0(50) = 1 - 0.0434\left(\frac{90}{360}\right)$$
$$= 0.9892$$

Do the following:
- Buy the 140-day bond at 0.9821
- Sell the futures at 0.9892

This strategy provides a return per dollar invested of

$$\frac{0.9892}{0.9821} = 1.0072$$

which compares favorably with a return per dollar invested of 1.0069 if the futures is correctly priced.

The implied repo rate is simply the annualization of this rate: $(1.0072)^{365/50} - 1 = 0.0538$. The cash-and-carry transaction would, therefore, earn 5.38 percent. Because the futures appears to be mispriced, we could likely obtain financing in the repo market at less than this rate.

7.2.2 Pricing Eurodollar Futures

Based on the T-bill case, it is tempting to argue that the interest rate implied by the Eurodollar futures price would be the forward rate in the term structure of LIBOR. Unfortunately, that is not quite the case. In fact, the unusual construction of the Eurodollar futures contract relative to the Eurodollar spot market means that no risk-free combination of a Eurodollar time deposit and a Eurodollar futures contract can be constructed. Recall that the Eurodollar time deposit is an add-on instrument. Using $L_0(j)$ as the rate (LIBOR) on a j-day Eurodollar time deposit on day 0, if one deposits $1, the deposit will grow to a value of $1 + L_0(j)(j/360)$ j days later. So, the present value of $1 in j days is $1/[1 + L_0(j)(j/360)]$. The Eurodollar futures contract, however, is structured like the T-bill contract—as though the underlying were a discount instrument. So its price is stated in the form of $1 - L_0(j)(j/360)$. If we try the same arbitrage with Eurodollars that we did with T-bills, we cannot get the LIBOR that determines the spot price of a Eurodollar at expiration to offset the LIBOR that determines the futures price at expiration.

In other words, suppose that on day 0 we buy an (h + m)-day Eurodollar deposit that pays $1 on day (h + m) and sell a futures at a price of $f_0(h)$. On day h, the futures expiration, the Eurodollar deposit has m days to go and is worth $1/[1 + L_h(m)(m/360)]$. The futures price at expiration is $f_h(h) = 1 - L_h(m)(m/360)$. The profit from the futures is $f_0(h) - [1 - L_h(m)(m/360)]$. Adding this amount to the value of the m-day Eurodollar deposit we are holding gives a total position value of

$$\frac{1}{1 + L_h(m)\left(\frac{m}{360}\right)} + f_0(h) - [1 - L_h(m)]\left(\frac{m}{360}\right)$$

Although $f_0(h)$ is known when the transaction is initiated, $L_h(m)$ is not determined until the futures expiration. There is no way for the $L_h(m)$ terms to offset. This problem does not occur in the T-bill market because the spot price is a discount instrument and the futures contract is designed as a discount instrument.[39] It is, nonetheless, common for participants in the futures market to treat the Eurodollar rate as equivalent to the implied forward rate. Such an assumption would require the ability to conduct the risk-free arbitrage, which, as we have shown, is impossible. The differences are fairly small, but we shall not assume that the Eurodollar futures rate should equal the implied forward rate. In that case, it would take a more advanced model to solve the pricing problem. The essential points in pricing interest rate futures on short-term instruments can be understood by studying the T-bill futures market.

This mismatch in the design of spot and futures instruments in the Eurodollar market would appear to make the contract difficult to use as a hedging instrument. Although we cover hedging futures and forwards in Chapter 6, we should note that in the above equation for the payoff of the portfolio combining a spot Eurodollar time deposit and a short Eurodollar futures contract, an increase (decrease) in LIBOR lowers (raises) the value of the spot Eurodollar deposit and raises (lowers) the payoff from the short Eurodollar futures. Thus, the Eurodollar futures contract can still serve as a hedging tool. The hedge will not be perfect but can still be quite effective. Indeed, the Eurodollar futures contract is a major hedging tool of dealers in over-the-counter derivatives.

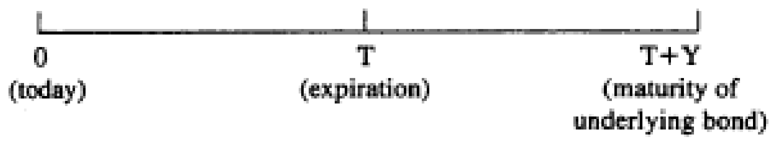
[39] It is not clear why the Chicago Mercantile Exchange designed the Eurodollar contract as a discount instrument when the underlying Eurodollar deposit is an add-on instrument. The most likely reason is that the T-bill futures contract was already trading, was successful, and its design was well understood and accepted by traders. The CME most likely felt that this particular design was successful and should be continued with the Eurodollar contract. Ironically, the Eurodollar contract became exceptionally successful and the T-bill contract now has virtually no trading volume.

We have now completed the treatment of futures contracts on short-term interest rate instruments. Now let us look at the pricing of Treasury bond futures.

7.2.3 Pricing Treasury Note and Bond Futures

Recall that in Section 6.2, we described the bond futures contract as one in which there are a number of deliverable bonds. When a given bond is delivered, the long pays the short the futures price times an adjustment term called the conversion factor. The conversion factor is the price of a \$1 bond with coupon equal to that of the deliverable bond and yield equal to 6 percent, with calculations based on semiannual compounding. Bonds with a coupon greater (less) than 6 percent will have a conversion factor greater (less) than 1. Before we delve into the complexities added by this feature, however, let us start off by assuming a fairly generic type of contract: one in which the underlying is a single, specific bond.

When examining bond forward contracts in Chapter 2, we specified a time line and notation. We return to that specific time line and notation, which differs from those we used for examining short-term interest rate futures.



Recall our notation from Chapter 2:

$B_0^c(T + Y)$ = price at time 0 of coupon bond that matures at time T + Y. The bond has a maturity of Y at the futures expiration.

CI_i = coupon at time t_i, where the coupons occur at times $t_1, t_2, \ldots, t_n$. Note that we care only about coupons prior to the futures expiration at T.

$f_0(T)$ = price at time 0 of futures expiring at time T.

$B_0(T)$ = price at time 0 of zero-coupon bond maturing at T.

We also need to know at time T the accumulated value of all coupons received over the period from 0 to T. We need the compound value from 0 to T of any coupons paid during the time the futures contract is alive. This value is denoted as FV(CI,0,T). We introduced this variable in Chapter 2 and showed how to compute it, so you may wish to review that material. It is traditionally assumed that the interest rate at which these coupons are reinvested is known. We also assume that this interest rate applies to the period from 0 to T for money borrowed or lent. We denote this rate as

$r_0(T)$ = Interest rate at time 0 for period until time T

As described in the section on T-bill futures pricing, this is the rate that determines the price of a zero-coupon bond maturing at T.[40] Hence,

$$B_0(T) = \frac{1}{[1 + r_0(T)]^T}$$

[40] Keep in mind, however, that this rate is not the discount rate that determines the price of the zero-coupon bond maturing at T. It is the rate of return, expressed as an annual rate. When working with T-bills, the symbol "T" represented Days/365, which is consistent with its use here with T-bonds.

The futures price at expiration is the price of the deliverable bond at expiration:

$$f_T(T) = B_T(T + Y)$$

Now we are ready to price this bond futures contract. On day 0, we buy the bond at the price $B_0^c(T + Y)$ and sell the futures at the price $f_0(T)$. Because the futures does not require any cash up front, its initial value is zero. The current value of the overall transaction is, therefore, just the value of the bond, $B_0^c(T + Y)$. This value represents the amount of money we must invest to engage in this transaction.

We hold this position until the futures expiration. During this time, we collect and reinvest the coupons. On day T, the futures expires. We deliver the bond and receive the futures price, $f_0(T)$. We also have the reinvested coupons, which have a value at T of FV(CI,0,T). These two amounts, $f_0(T)$ and FV(CI,0,T), are known when the transaction was initiated at time 0, so the transaction is risk-free. Therefore, the current value of the transaction, $B_0^c(T + Y)$, should be the discounted value of its value at T of $f_0(T)$ + FV(CI,0,T):

$$B_0^c(T + Y) = \frac{f_0(T) + FV(CI, 0, T)}{[1 + r_0(T)]^T}$$

Note that we are simply discounting the known future value at T of the transaction at the risk-free rate of $r_0(T)$.[41]

We are, of course, more interested in the futures price, which is the only unknown in the above equation. Solving, we obtain

$$f_0(T) = B_0^c(T + Y)[1 + r_0(T)]^T - FV(CI,0,T) \qquad \textbf{(3-11)}$$

This equation is a variation of our basic cost-of-carry formula. The spot price, $B_0^c(T + Y)$, is compounded at the risk-free interest rate. We then subtract the compound future value of the reinvested coupons over the life of the contract. The coupon interest is like a negative cost of carry; it is a positive cash flow associated with holding the underlying bond.

Now let us work an example. Consider a $1 face value Treasury bond that pays interest at 7 percent semiannually. Thus, each coupon is $0.035. The bond has exactly five years remaining, so during that time it will pay 10 coupons, each six months apart. The yield on the bond is 8 percent. The price of the bond is found by calculating the present value of both the 10 coupons and the face value: The price is $0.9594.

Now consider a futures contract that expires in one year and three months: T = 1.25. The risk-free rate, $r_0(T)$, is 6.5 percent. The accumulated value of the coupons and the interest on them is

$$\$0.035(1.065)^{0.75} + \$0.035(1.065)^{0.25} = \$0.0722$$

The first coupon is paid in one-half a year and reinvests for three-quarters of a year. The second coupon is paid in one year and reinvests for one-quarter of a year.

Now the futures price is obtained as

$$f_0(T) = B_0^c(T + Y)[1 + r_0(T)]^T - FV(CI,0,T)$$

$$f_0(1.25) = \$0.9594(1.065)^{1.25} - \$0.0722 = \$0.9658$$

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[41] We shall not take up the topic of the implied repo rate again, but note that if the futures is selling for $f_0(T)$, then $r_0(T)$ would be the implied repo rate.

This is the price at which the futures should trade, given current market conditions. To verify this result, buy the five-year bond for \$0.9594 and sell the futures for \$0.9658. Hold the position for 15 months until the futures expiration. Collect and reinvest the coupons. When the futures expires, deliver the bond and receive the futures price of \$0.9658. Then add the reinvested coupons of \$0.0722 for a total of \$0.9658 + \$0.0722 = \$1.0380. If we invest \$0.9594 and end up with \$1.0380 15 months later, the return is \$1.0380/\$0.9594 = 1.0819. For comparison purposes, we should determine the annual equivalent of this rate, which is found as $(1.0819)^{1/1.25} - 1 = 0.065$. This is the same 6.5 percent risk-free rate. If the futures contract trades at a higher price, the above transaction would result in a return greater than 6.5 percent. The amount available at expiration would be higher, clearly leading to a rate of return higher than 6.5 percent. If the futures trades at a lower price, the arbitrageur would sell short the bond and buy the futures, which would generate a cash inflow today. The amount paid back would be at less than the risk-free rate of 6.5 percent.[42]

Unfortunately, we now must complicate the matter a little by moving to the more realistic case with a delivery option. Bond futures contracts traditionally permit the short to choose which bond to deliver. This feature reduces the possibility of unusual price behavior of the deliverable bond caused by holders of short positions scrambling to buy a single deliverable bond at expiration. By allowing more than one bond to be deliverable, such problems are avoided. The contract is structured as though there is a standard hypothetical deliverable bond, which has a given coupon rate. The Chicago Board of Trade's contract uses a 6 percent rate. If the short delivers a bond with a higher (lower) coupon rate, the price received at delivery is adjusted upward (downward). The conversion factor is defined and calculated as the price of a \$1 face value bond with a coupon and maturity equal to that of the deliverable bond and a yield of 6 percent. Each deliverable bond has its own conversion factor. The short designates which bond he will deliver, and that bond's conversion factor is multiplied by the final futures price to determine the amount the long will pay the short for the bond.

The availability of numerous deliverable bonds creates some confusion in pricing the futures contract, arising from the fact that the underlying cannot be uniquely identified, at least not on the surface. This confusion has given rise to the concept that one bond is always the best one to deliver. If a trader buys a given bond and sells the futures, he creates a risk-free hedge. If there are no arbitrage opportunities, the return from that hedge cannot exceed the risk-free rate. That return can, however, be *less* than the risk-free rate. How can this be? In all previous cases, if a return from a risk-free transaction is less than the risk-free rate, it should be a simple matter to reverse the transaction and capture an arbitrage profit. In this case, however, a reverse transaction would not work. If the arbitrageur sells short the bond and buys the futures, she must be assured that the short will deliver the bond from which the potential arbitrage profit was computed. But the short makes the delivery decision and in all likelihood would not deliver that particular bond.

Thus, the short can be long a bond and short futures and earn a return less than the risk-free rate. One bond, however, results in a return closest to the risk-free rate. Clearly that bond is the best bond to deliver. The terminology in the business is that this bond is the cheapest to deliver.

The cheapest-to-deliver bond is determined by selecting a given bond and computing the rate of return from buying that bond and selling the futures to hedge its delivery at expiration. This calculation is performed for all bonds. The one with the highest rate of return is the cheapest to deliver.[43] The cheapest-to-deliver bond can change, however,

[42] Again, as in the section on T-bill futures, this analysis could be conducted in terms of the implied repo rate.

[43] As noted, this rate of return will not exceed the risk-free rate but will be the highest rate below the risk-free rate.

which can benefit the short and not the long. We ignore the details of determining the cheapest-to-deliver bond and assume that it has been identified. From here, we proceed to price the futures.

Let CF(T) be the conversion factor for the bond we have identified as the cheapest to deliver. Now we go back to the arbitrage transaction described for the case where there is only one deliverable bond. Recall that we buy the bond, sell a futures, and reinvest the coupons on the bond. At expiration, we deliver the bond, receive the futures price $f_0(T)$, and have the reinvested coupons, which are worth FV(CI,0,T). Now, in the case where the futures contract has many deliverable bonds, we must recognize that when the bond is delivered, the long pays $f_0(T)$ times CF(T). This adjustment does not add any risk to this risk-free transaction. Thus, the present value of the amount received at delivery, $f_0(T)CF(T) + FV(CI,0,T)$, should still equal the original price of the bond, which was the amount we invested to initiate the transaction:

$$B_0^c(T + Y) = \frac{f_0(T)CF(T) + FV(CI,0,T)}{[1 + r_0(T)]^T}$$

Solving for the futures price, we obtain

$$f_0(T) = \frac{B_0^c(T + Y)[1 + r_0(T)]^T - FV(CI,0,T)}{CF(T)} \quad \textbf{(3-12)}$$

Note that when we had only one deliverable bond, the formula did not have the CF(T) term, but a better way to look at it is that for only one deliverable bond, the conversion factor is effectively 1, so Equation 3-12 would still apply.

Consider the same example we previously worked, but now we need a conversion factor. As noted above, the conversion factor is the price of a \$1 bond with coupon and maturity equal to that of the deliverable bond on the expiration day and yield of 6 percent, with all calculations made assuming semiannual interest payments. As noted, we shall skip the specifics of this calculation here; it is simply a present value calculation. For this example, the 7 percent bond with maturity of three and three-quarter years on the delivery day would have a conversion factor of 1.0505. Thus, the futures price would be

$$f_0(T) = \frac{B_0^c(T + Y)[1 + r_0(T)]^T - FV(CI,0,T)}{CF(T)}$$

$$f_0(1.25) = \frac{0.9594(1.065)^{1.25} - 0.0722}{1.0505} = 0.9193$$

If the futures is priced higher than 0.9193, one can buy the bond and sell the futures to earn more than the risk-free rate. If the futures price is less than 0.9193, one can sell short the bond and buy the futures to end up borrowing at less than the risk-free rate. As noted previously, however, this transaction has a complication: If one goes short the bond and long the futures, this bond must remain the cheapest to deliver. Otherwise, the short will not deliver this particular bond and the arbitrage will not be successful.

Exhibit 3-9 reviews the important formulas for pricing Treasury bond futures contracts.

EXHIBIT 3-9 Pricing Formulas for Treasury Bond Futures Contract

Futures price = Underlying T-bond price compounded at risk-free rate less Compound future value of reinvested coupons.

Futures price if underlying bond is the only deliverable bond:

$$f_0(T) = B_0^c(T + Y)[1 + r_0(T)]^T - FV(CI,0,T)$$

Futures price when there are multiple deliverable bonds:

$$f_0(T) = \frac{B_0^c(T + Y)[1 + r_0(T)]^T - FV(CI,0,T)}{CF(T)}$$

PRACTICE PROBLEM 4

Consider a three-year \$1 par Treasury bond with a 7.5 percent annual yield and 8 percent semiannual coupon. Its price is \$1.0132. A futures contract calling for delivery of this bond only expires in one year. The one-year risk-free rate is 7 percent.

A. Find the future value in one year of the coupons on this bond. Assume a reinvestment rate of 3.75 percent per six-month period.

B. Find the appropriate futures price.

C. Now suppose the bond is one of many deliverable bonds. The contract specification calls for the use of a conversion factor to determine the price paid for a given deliverable bond. Suppose the bond described here has a conversion factor of 1.0372. Now determine the appropriate futures price.

SOLUTIONS

A. One coupon of 0.04 will be invested for half a year at 3.75 percent (half of the rate of 7.5 percent). The other coupon is not reinvested but is still counted. Thus, $FV(CI,0,1) = 0.04(1.0375) + 0.04 = 0.0815$.

B. $f_0(1) = 1.0132(1.07) - 0.0815 = 1.0026$

C. $f_0(1) = \frac{1.0132(1.07) - 0.0815}{1.0372} = 0.9667$

7.3 Pricing Stock Index Futures

Now let the underlying be either a portfolio of stocks or an individual stock.[44] The former are normally referred to as stock index futures, in which the portfolio is identical in composition to an underlying index of stocks. In this material, we focus on the pricing of stock index futures, but the principles are the same if the underlying is an individual stock.

In pricing stock index futures, we must account for the fact that the underlying stocks pay dividends.[45] Recall that in our previous discussions about the generic pricing of

[44] Futures on individual stocks have taken a long time to develop, primarily because of regulatory hurdles. They were introduced in the United States in late 2002 and, as of the publication date of this book, have achieved only modest trading volume. They currently trade in a few other countries such as the United Kingdom and Australia.

[45] Even if not all of the stocks pay dividends, at least some of the stocks almost surely pay dividends.

futures, we demonstrated that the futures price is lower as a result of the compound future value of any cash flows paid on the asset. Such cash flows consist of coupon interest payments if the underlying is a bond, or storage costs if the underlying incurs costs to store.[46] Dividends work exactly like coupon interest.

Consider the same time line we used before. Today is time 0, and the futures expires at time T. During the life of the futures, there are n dividends of D_j, $j = 1, 2, \ldots, n$. We assume these dividends are all known when the futures contract is initiated. Let

FV(D,0,T) = the compound value over the period of 0 to T of all dividends collected and reinvested

We introduced this variable in Chapter 2 and showed how to compute it, so you may wish to review that material. The other notation is the same we have previously used:

S_0 = current value of the stock index

$f_0(T)$ = futures price today of a contract that expires at T

r = risk-free interest rate over the period 0 to T

Now that we are no longer working with interest rate futures, we do not need the more flexible notation for interest rates on bonds of different maturities or interest rates at different time points. So we can use the simple notation of r as the risk-free interest rate, but we must keep in mind that it is the risk-free rate for the time period from 0 to T.

We undertake the following transaction: On day 0, we buy the stock portfolio that replicates the index. This transaction will require that we invest the amount S_0. We simultaneously sell the futures at the price $f_0(T)$.

On day T, the futures expires. We deliver the stock and receive the original futures price $f_0(T)$.[47] We also have the accumulated value of the reinvested dividends, FV(D,0,T) for a total of $f_0(T) + FV(D,0,T)$. Because this amount is known at time 0, the transaction is risk free. Therefore, we should discount its value at the risk-free rate and set this equal to the initial value of the portfolio, S_0, as follows:

$$S_0 = \frac{f_0(T) + FV(D,0,T)}{(1 + r)^T}$$

Solving for the futures price gives

$$f_0(T) = S_0(1 + r)^T - FV(D,0,T) \quad \textbf{(3-13)}$$

which is the cost-of-carry formula for stock index futures. Notice that it is virtually identical to that for bond futures. Ignoring the conversion factor necessitated by the delivery option, the only difference is that we use the compound future value of the dividends instead of the compound future value of the coupon interest.

Consider the following example. A stock index is at 1,452.45, and a futures contract on the index expires in three months. Thus, T = 3/12 = 0.25. The risk-free interest rate is

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[46] We also allowed for the possibility of noncash costs, which we called the convenience yield, but there are no implicit costs or benefits associated with stock index futures.

[47] Virtually all stock index futures contracts call for cash settlement at expiration. See the explanation of the equivalence of delivery and cash settlement in Section 4 and Exhibit 3-2.

5.5 percent. The value of the dividends reinvested over the life of the futures is 7.26. The futures price should, therefore, be

$$f_0(T) = S_0(1 + r)^T - FV(D,0,T)$$
$$f_0(0.25) = 1{,}452.45(1.055)^{0.25} - 7.26$$
$$= 1{,}464.76$$

Thus, if the futures contract is selling for more than this price, an arbitrageur can buy the stocks and sell the futures. The arbitrageur would collect and reinvest the dividends and at expiration would receive a gain that would exceed the risk-free rate of 5.5 percent, a result of receiving more than 1,464.76 for the stocks. If the futures contract is selling for less than this price, the arbitrageur can sell short the stocks and buy the futures. After paying the dividends while holding the stocks,[48] the arbitrageur will end up buying back the stocks at a price that implies that he has borrowed money and paid it back at a rate less than the risk-free rate. The combined activities of all arbitrageurs will force the futures price to 1,464.76.

The stock index futures pricing formula has a number of variations. Suppose we define $FV(D,0,T)/(1 + r)^T$ as the present value of the dividends, $PV(D,0,T)$:

$$FV(D,0,T) = PV(D,0,T)(1 + r)^T$$

Substituting in the futures pricing formula above for $FV(D,0,T)$, we obtain

$$f_0(T) = [S_0 - PV(D,0,T)](1 + r)^T \tag{3-14}$$

Notice here that the stock price is reduced by the present value of the dividends. This adjusted stock price is then compounded at the risk-free rate over the life of the futures.

In the problem we worked above, the present value of the dividends is found as

$$PV(D,0,T) = \frac{FV(D,0,T)}{(1 + r)^T}$$

$$PV(D,0,0.25) = \frac{7.26}{(1.055)^{0.25}} = 7.16$$

Then the futures price would be

$$f_0(T) = [S_0 - PV(D,0,T)](1 + r)^T$$
$$f_0(0.25) = (1{,}452.45 - 7.16)(1.055)^{0.25}$$
$$= 1{,}464.76$$

Another variation of the formula defines the yield as δ in the following manner:

$$\frac{1}{(1 + \delta)^T} = 1 - \frac{FV(D,0,T)}{S_0(1 + r)^T}$$

The exact solution for δ is somewhat complex, so we shall just leave it in the form above. Using this specification, we find that the futures pricing formula would be

$$f_0(T) = \left(\frac{S_0}{(1 + \delta)^T}\right)(1 + r)^T \tag{3-15}$$

[48] Remember that a short seller must make restitution for any dividends paid while the position is short.

The stock price is, thus, discounted at the dividend yield, and this adjusted stock price is then compounded at the risk-free rate over the life of the futures.[49]

In the example above, the yield calculation is

$$\frac{1}{(1+\delta)^T} = 1 - \frac{FV(D,0,T)}{S_0(1+r)^T}$$

$$\frac{1}{(1+\delta)^T} = 1 - \frac{7.26}{1,452.45(1.055)^{0.25}} = 0.9951$$

Then $(1 + \delta)^T$ is 1/0.9951 = 1.0049 and the futures price is

$$f_0(T) = \left(\frac{S_0}{(1+\delta)^T}\right)(1+r)^T$$

$$f_0(0.25) = \left(\frac{1,452.45}{1.0049}\right)(1.055)^{0.25}$$

$$= 1,464.84$$

The difference between this and the answer we previously obtained is strictly caused by a rounding error.

Another variation of this formula is to express the yield as

$$\delta^* = \frac{PV(D,0,T)}{S_0} = \frac{FV(D,0,T)/(1+r)^T}{S_0}$$

This means that $FV(D,0,T) = S_0(1 + r)^T\delta^*$. Substituting into our futures pricing formula for FV(D,0,T), we obtain

$$f_0(T) = S_0(1 - \delta^*)(1 + r)^T \qquad \textbf{(3-16)}$$

Here again, the stock price is reduced by the yield, and this "adjusted" stock price is compounded at the risk-free rate.

In the problem we worked above, the yield would be found as

$$\delta^* = \frac{PV(D,0,T)}{S_0}$$

$$\delta^* = \frac{7.16}{1,452.45} = 0.0049$$

Then the futures price would be

$$f_0(T) = S_0(1 - \delta^*)(1 + r)^T$$

$$f_0(0.25) = 1,452.45(1 - 0.0049)(1.055)^{0.25}$$

$$= 1,464.81$$

Again, the difference between the two prices comes from rounding.

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[49] Sometimes the futures price is written as $f_0(T) = S_0(1 + r - \delta)^T$ where the dividend yield is simply subtracted from the risk-free rate to give a net cost of carry. This formula is a rough approximation that we do not consider acceptable.

A common variation uses the assumption of continuous compounding. The continuously compounded risk-free rate is defined as $r^c = \ln(1 + r)$. The continuously compounded dividend yield is $\delta^c = \ln(1 + \delta)$. When working with discrete dividends, we obtained the relationship

$$\frac{1}{(1+\delta)^T} = 1 - \frac{FV(D,0,T)}{S_0(1+r)^T}$$

We calculated $(1 + \delta)^T$. To obtain δ^c, we take the natural log of this value and divide by T: $\delta^c = (1/T)\ln[(1 + \delta)^T]$. The formula for the futures price is

$$f_0(T) = S_0 e^{(r^c - \delta^c)T}$$

In the above formula, the opportunity cost, expressed as the interest rate, is reduced by the dividend yield. Thus, the formula compounds the spot price by the interest cost less the dividend benefits. An equivalent variation of the above formula is

$$f_0(T) = (S_0 e^{-\delta^c T})e^{r^c T} \quad \textbf{(3-17)}$$

The expression in parentheses is the stock price discounted at the dividend yield rate. The result is an adjusted stock price with the present value of the dividends removed. This adjusted stock price is then compounded at the risk-free rate. So, as we have previously seen, the stock price less the present value of the dividends is compounded at the risk-free rate to obtain the futures price.

In the previous problem, $(1 + \delta)^T = 1.0049$. Then $\delta^c = (1/0.25)\ln(1.0049) = 0.0196$. The continuously compounded risk-free rate is $\ln(1.055) = 0.0535$. The futures price is, therefore, $f_0(0.25) = (1452.45e^{-0.0196(0.25)})e^{0.0535(0.25)} = 1464.81$; again the difference comes from rounding.

Exhibit 3-10 summarizes the formulas for pricing stock index futures contracts. Each of these formulas is consistent with the general formula for pricing futures. They are each based on the notion that a futures price is the spot price compounded at the risk-free rate, plus the compound future value of any other costs minus any cash flows and benefits. Alternatively, one can convert the compound future value of the costs net of benefits or cash flows of holding the asset to their current value and subtract this amount from the spot price before compounding the spot price at the interest rate. In this manner, the spot price adjusted for any costs or benefits is then compounded at the risk-free interest rate to give the futures price. These costs, benefits, and cash flows thus represent the linkage between spot and futures prices.

EXHIBIT 3-10 Pricing Formulas for Stock Index Futures Contract

Futures price = Stock index compounded at risk-free rate − Future value of dividends, or (Stock index − Present value of dividends) compounded at risk-free rate.

Futures price as stock index compounded at risk-free rate − Future value of dividends:

$$f_0(T) = S_0(1 + r)^T - FV(D,0,T)$$

Futures price as stock index − Present value of dividends compounded at risk-free rate:

$$f_0(T) = [S_0 - PV(D,0,T)](1 + r)^T$$

Futures price as stock index discounted at dividend yield, compounded at risk-free rate:

$$f_0(T) = \left(\frac{S_0}{(1+\delta)^T}\right)(1+r)^T \qquad \text{or}$$

$$f_0(T) = S_0(1-\delta^*)^T(1+r)^T$$

Futures price in terms of continuously compounded rate and yield:

$$f_0(T) = S_0 e^{(r^c-\delta^c)T} \qquad \text{or}$$

$$f_0(T) = (S_0 e^{-\delta^c T})e^{r^c T}$$

PRACTICE PROBLEM 5

A stock index is at 755.42. A futures contract on the index expires in 57 days. The risk-free interest rate is 6.25 percent. At expiration, the value of the dividends on the index is 3.94.

A. Find the appropriate futures price, using both the future value of the dividends and the present value of the dividends.

B. Find the appropriate futures price in terms of the two specifications of the dividend yield.

C. Using your answer in Part B, find the futures price under the assumption of continuous compounding of interest and dividends.

SOLUTIONS

A. T = 57/365 = 0.1562

$$f_0(0.1562) = 755.42(1.0625)^{0.1562} - 3.94 = 758.67$$

Alternatively, we can find the present value of the dividends:

$$PV(D,0,0.1562) = \frac{3.94}{(1.0625)^{0.1562}} = 3.90$$

Then we can find the futures price as $f_0(0.1562) = (755.42 - 3.90)(1.0625)^{0.1562} = 758.67$.

B. Under one specification of the yield, we have

$$\frac{1}{(1+\delta)^T} = 1 - \frac{3.94}{755.42(1.0625)^{0.1562}} = 0.9948$$

We need the inverse of this amount, which is 1/0.9948 = 1.0052. Then the futures price is

$$f_0(0.1562) = \left(\frac{755.42}{1.0052}\right)(1.0625)^{0.1562} = 758.66$$

Under the other specification of the dividend yield, we have

$$\delta^* = \frac{3.90}{755.42} = 0.0052$$

The futures price is $f_0(0.1562) = 755.42(1 - 0.0052)(1.0625)^{0.1562} = 758.64$, with the difference caused by rounding.

C. The continuously compounded risk-free rate is $r^c = \ln(1.0625) = 0.0606$. The continuously compounded dividend yield is

$$\frac{1}{0.1562}\ln(1.0052) = 0.0332$$

The futures price would then be

$$f_0(0.1562) = 755.42e^{(0.0606 - 0.0332)(0.1562)} = 758.66$$

7.4 Pricing Currency Futures

Given our assumptions about no marking to market, it will be a simple matter to learn how to price currency futures: We price them the same as currency forwards. Recall that in Chapter 2 we described a currency as an asset paying a yield of r^f, which can be viewed as the foreign risk-free rate. Thus, in this sense, a currency futures can also be viewed like a stock index futures, whereby the dividend yield is analogous to the foreign interest rate.

Therefore, an arbitrageur can buy the currency for the spot exchange rate of S_0 and sell a futures expiring at T for $f_0(T)$, holding the position until expiration, collecting the foreign interest, and delivering the currency to receive the original futures price. An important twist, however, is that the arbitrageur must be careful to have the correct number of units of the currency on hand to deliver.

Consider a futures contract on one unit of the currency. If the arbitrageur purchases one unit of the currency up front, the accumulation of interest on the currency will result in having more than one unit at the futures expiration. To adjust for this problem, the arbitrageur should take $S_0/(1 + r^f)^T$ units of his own currency and buy $1/(1 + r^f)^T$ units of the foreign currency.[50] The arbitrageur holds this position and collects interest at the foreign rate. The accumulation of interest is accounted for by multiplying by the interest factor $(1 + r^f)^T$. At expiration, the number of units of the currency will have grown to $[1/(1 + r^f)^T][1 + r^f]^T = 1$. So, the arbitrageur would then have 1 unit of the currency. He delivers that unit and receives the futures price of $f_0(T)$.

To avoid an arbitrage opportunity, the present value of the payoff of $f_0(T)$ must equal the amount initially invested. To find the present value of the payoff, we must discount at the domestic risk-free rate, because that rate reflects the opportunity cost of the arbitrageur's investment of his own money. So, first we equate the present value of the future payoff, discounting at the domestic risk-free rate, to the amount initially invested:

$$\frac{f_0(T)}{(1 + r)^T} = \frac{S_0}{(1 + r^f)^T}$$

Then we solve for the futures price to obtain

$$f_0(T) = \left(\frac{S_0}{(1 + r^f)^T}\right)(1 + r)^T \tag{3-18}$$

This formula is the same one we used for currency forwards.

An alternative variation of this formula would apply when we use continuously compounded interest rates. The adjustment is very slight. In the formula above, dividing S_0 by

[50] In other words, if S_0 buys 1 unit, then $S_0/(1 + r^f)^T$ buys $1/(1 + r^f)^T$ units.

$(1 + r^f)^T$ finds a present value by discounting at the foreign interest rate. Multiplying by $(1 + r)^T$ is finding a future value by compounding at the domestic interest rate. The continuously compounded analogs to those rates are $r^{fc} = \ln(1 + r^f)$ and $r^c = \ln(1 + r)$. Then the formula becomes

$$f_0(T) = (S_0 e^{-r^{fc}T})e^{r^cT} \quad \textbf{(3-19)}$$

We also saw this formula in Chapter 2.

Consider a futures contract expiring in 55 days on the euro. Therefore, T = 55/365 = 0.1507. The spot exchange rate is \$0.8590. The foreign interest rate is 5.25 percent, and the domestic risk-free rate is 6.35 percent. The futures price should, therefore, be

$$f_0(T) = \left(\frac{S_0}{(1 + r^f)^T}\right)(1 + r)^T$$

$$f_0(0.1507) = \left(\frac{0.8590}{(1.0525)^{0.1507}}\right)(1.0635)^{0.1507} = 0.8603$$

If the futures is selling for more than this amount, the arbitrageur can buy the currency and sell the futures. He collects the foreign interest and converts the currency back at a higher rate than 0.8603, resulting in a risk-free return that exceeds the domestic risk-free rate. If the futures is selling for less than this amount, the arbitrageur can borrow the currency and buy the futures. The end result will be to receive money at the start and pay back money at a rate less than the domestic risk-free rate.

If the above problem were structured in terms of continuously compounded rates, the domestic rate would be ln(1.0635) = 0.0616 and the foreign rate would be ln(1.0525) = 0.0512. The futures price would then be

$$f_0(T) = (S_0 e^{-r^{fc}T})e^{r^cT}$$

$$f_0(0.1507) = (0.85890 e^{-0.0512(0.1507)})e^{0.0616(0.1507)} = 0.8603$$

which, of course, is the same price we calculated above.

Exhibit 3-11 summarizes the formulas for pricing currency futures.

EXHIBIT 3-11 Pricing Formulas for Currency Futures Contract

Futures price = (Spot exchange rate discounted by Foreign interest rate) compounded at Domestic interest rate:

Discrete interest:	$f_0(T) = \left(\frac{S_0}{(1 + r^f)^T}\right)(1 + r)^T$
Continuous interest:	$f_0(T) = (S_0 e^{-r^{fc}T})e^{r^cT}$

PRACTICE PROBLEM 6

The spot exchange rate for the Swiss franc is \$0.60. The U.S. interest rate is 6 percent, and the Swiss interest rate is 5 percent. A futures contract expires in 78 days.

A. Find the appropriate futures price.

B. Find the appropriate futures price under the assumption of continuous compounding.

C. Using Part A, execute an arbitrage resulting from a futures price of \$0.62.

SOLUTIONS

T = 78/365 = 0.2137

A. $f_0(0.2137) = \frac{\$0.60}{(1.05)^{0.2137}}(1.06)^{0.2137} = \$0.6012$

B. The continuously compounded equivalent rates are

$r^{fc} = \ln(1.05) = 0.0488$

$r^c = \ln(1.06) = 0.0583$

The futures price is

$$f_0(0.2137) = (\$0.60e^{-0.0488(0.2137)})e^{0.0583(0.2137)}$$
$$= \$0.6012$$

C. At \$0.62, the futures price is too high, so we will need to sell the futures. First, however, we must determine how many units of the currency to buy. It should be

$$\frac{1}{(1.05)^{0.2137}} = 0.9896$$

So we buy this many units, which costs 0.9896(\$0.60) = \$0.5938. We sell the futures at \$0.62. We hold the position until expiration. During that time the accumulation of interest will make the 0.9896 units of the currency grow to 1.0000 unit. We convert the Swiss franc to dollars at the futures rate of \$0.62. The return per dollar invested is

$$\frac{0.62}{0.5938} = 1.0441$$

This is a return of 1.0441 per dollar invested over 78 days. At the risk-free rate of 6 percent, the return over 78 days should be $(1.06)^{0.2137} = 1.0125$. Obviously, the arbitrage transaction is much better.

7.5 Futures Pricing: A Recap

We have now examined the pricing of short-term interest rate futures, intermediate- and long-term interest rate futures, stock index futures, and currency futures. Let us recall the intuition behind pricing a futures contract and see the commonality in each of those special cases. First recall that under the assumption of no marking to market, at expiration the short makes delivery and we assume that the long pays the full futures price at that point. An arbitrageur buys the asset and sells a futures contract, holds the asset for the life of the futures, and delivers it at expiration of the futures, at which time he is paid the futures price. In addition, while holding the asset, the arbitrageur accumulates costs and accrues cash flows, such as interest, dividends, and benefits such as a convenience yield. The value of the position at expiration will be the futures price net of these costs minus benefits and cash flows. The overall value of this transaction at expiration is known when the transaction is initiated; thus, the value at expiration is risk-free. The return from a risk-free transaction should equal the risk-free rate, which is the rate on a zero-coupon bond whose maturity is the futures expiration day. If the return is indeed this risk-free rate, then the futures price must equal the spot price compounded at the risk-free rate plus the compound value of these costs net of benefits and cash flows.

It should also be noted that although we have taken the more natural approach of buying the asset and selling the futures, we could just as easily have sold short the asset and bought the futures. Because short selling is usually a little harder to do as well as to understand, the approach we take is preferable from a pedagogical point of view. It is important, nonetheless, to remember that the ability to sell short the asset or the willingness of parties who own the asset to sell it to offset the buying of the futures is critical to establishing the results we have shown here. Otherwise, the futures pricing formulas would be inequalities—limited on one side but not restricted on the other.

We should remind ourselves that this general form of the futures pricing model also applied in Chapter 2 in our discussion of forward contracts. Futures contracts differ from forward contracts in that the latter are subject to credit risk. Futures contracts are marked to market on a daily basis and guaranteed against losses from default by the futures clearinghouse, which has never defaulted. Although there are certain institutional features that distinguish futures from forwards, we consider those features separately from the material on pricing. Because the general economic and financial concepts are the same, for pricing purposes, we treat futures and forwards as the same.

8 THE ROLE OF FUTURES MARKETS AND EXCHANGES

We conclude this chapter with a brief look at the role that futures markets and exchanges play in global financial systems and in society. Virtually all participants in the financial markets have heard of futures markets, but many do not understand the role that futures markets play. Some participants do not understand how futures markets function in global financial systems and often look at futures with suspicion, if not disdain.

In Chapter 1, we discussed the purposes of derivative markets. We found that derivative markets provide price discovery and risk management, make the markets for the underlying assets more efficient, and permit trading at low transaction costs. These characteristics are also associated with futures markets. In fact, price discovery is often cited by others as the primary advantage of futures markets. Yet, all derivative markets provide these benefits. What characteristics do futures markets have that are not provided by comparable markets as forward markets?

First recall that a major distinction between futures and forwards is that futures are standardized instruments. By having an agreed-upon set of homogeneous contracts, futures markets can provide an orderly, liquid market in which traders can open and close positions without having to worry about holding these positions to expiration. Although not all futures contracts have a high degree of liquidity, an open position can nonetheless be closed on the exchange where the contract was initiated.[51] More importantly, however, futures contracts are guaranteed against credit losses. If a counterparty defaults, the clearinghouse pays and, as we have emphasized, no clearinghouse has ever defaulted. In this manner, a party can engage in a transaction to lock in a future price or rate without having

[51] Recall that there is no liquid market for previously opened forward contracts to be closed, but the holder of a forward contract can re-enter the market and establish a position opposite to the one previously established. If one holds a long forward contract to buy an asset in six months, one can then do a short forward contract to sell the asset in six months, and this transaction offsets the risk of changing market prices. The credit risk on both contracts remains. In some cases, the offsetting contract can be done with the same counterparty as in the original contract, permitting the two parties to arrange a single cash settlement to offset both contracts.

to worry about the credit quality of the counterparty. Forward contracts are subject to default risk, but of course they offer the advantage of customization, the tailoring of a contract's terms to meet the needs of the parties involved.

With an open, standardized, and regulated market for futures contracts, their prices can be disseminated to other investors and the general public. Futures prices are closely watched by a vast number of market participants, many trying to discern an indication of the direction of future spot prices and some simply trying to determine what price they could lock in for future purchase or sale of the underlying asset. Although forward prices provide similar information, forward contracts are private transactions and their prices are not publicly reported. Futures markets thus provide transparency to the financial markets. They reveal the prices at which parties contract for future transactions.

Therefore, futures prices contribute an important element to the body of information on which investors make decisions. In addition, they provide opportunities to transact for future purchase or sale of an underlying asset without having to worry about the credit quality of the counterparty.

In Chapters 2 and 3, we studied forward and futures contracts and showed that they have a lot in common. Both are commitments to buy or sell an asset at a future date at a price agreed on today. No money changes hands at the start of either transaction. We learned how to determine appropriate prices and values for these contracts. In Chapter 6, we shall look at a variety of strategies and applications using forward and futures contracts. For now, however, we take a totally different approach and look at contracts that provide not the obligation but rather the right to buy or sell an asset at a later date at a price agreed on today. To obtain such a right, in contrast to agreeing to an obligation, one must pay money at the start. These instruments, called options, are the subject of Chapter 4.

KEY POINTS

- Futures contracts are standardized instruments that trade on a futures exchange, have a secondary market, and are guaranteed against default by means of a daily settling of gains and losses. Forward contracts are customized instruments that are not guaranteed against default and are created anywhere off of an exchange.
- Modern futures markets primarily originated in Chicago out of a need for grain farmers and buyers to be able to transact for delivery at future dates for grain that would, in the interim, be placed in storage.
- Futures transactions are standardized and conducted in a public market, are homogeneous, have a secondary market giving them an element of liquidity, and have a clearinghouse, which collects margins and settles gains and losses daily to provide a guarantee against default. Futures markets are also regulated at the federal government level.
- Margin in the securities markets is the deposit of money, the margin, and a loan for the remainder of the funds required to purchase a stock or bond. Margin in the futures markets is much smaller and does not involve a loan. Futures margin is more like a performance bond or down payment.
- Futures trading occurs on a futures exchange, which involves trading either in a physical location called a pit or via a computer terminal off of the floor of the futures exchange as part of an electronic trading system. In either case, a party to a futures contract goes long, committing to buy the underlying asset at an agreed-upon price, or short, committing to sell the underlying asset at an agreed-upon price.

- A futures trader who has established a position can re-enter the market and close out the position by doing the opposite transaction (sell if the original position was long or buy if the original position was short). The party has offset the position, no longer has a contract outstanding, and has no further obligation.
- Initial margin is the amount of money in a margin account on the day of a transaction or when a margin call is made. Maintenance margin is the amount of money in a margin account on any day other than when the initial margin applies. Minimum requirements exist for the initial and maintenance margins, with the initial margin requirement normally being less than 10 percent of the futures price and the maintenance margin requirement being smaller than the initial margin requirement. Variation margin is the amount of money that must be deposited into the account to bring the balance up to the required level. The settlement price is an average of the last few trades of the day and is used to determine the gains and losses marked to the parties' accounts.
- The futures clearinghouse engages in a practice called marking to market, also known as the daily settlement, in which gains and losses on a futures position are credited and charged to the trader's margin account on a daily basis. Thus, profits are available for withdrawal and losses must be paid quickly before they build up and pose a risk that the party will be unable to cover large losses.
- The margin balance at the end of the day is determined by taking the previous balance and accounting for any gains or losses from the day's activity, based on the settlement price, as well as any money added or withdrawn.
- Price limits are restrictions on the price of a futures trade and are based on a range relative to the previous day's settlement price. No trade can take place outside of the price limits. A limit move is when the price at which two parties would like to trade is at or beyond the price limit. Limit up is when the market price would be at or above the upper limit. Limit down is when the market price would be at or below the lower limit. Locked limit occurs when a trade cannot take place because the price would be above the limit up or below the limit down prices.
- A futures contract can be terminated by entering into an offsetting position very shortly before the end of the expiration day. If the position is still open when the contract expires, the trader must take delivery (if long) or make delivery (if short), unless the contract requires that an equivalent cash settlement be used in lieu of delivery. In addition, two participants can agree to alternative delivery terms, an arrangement called exchange for physicals.
- Delivery options are features associated with a futures contract that permit the short some flexibility in what to deliver, where to deliver it, and when in the expiration month to make delivery.
- Scalpers are futures traders who take positions for very short periods of time and attempt to profit by buying at the bid price and selling at the ask price. Day traders close out all positions by the end of the day. Position traders leave their positions open overnight and potentially longer.
- Treasury bill futures are contracts in which the underlying is $1,000,000 of a U.S. Treasury bill. Eurodollar futures are contracts in which the underlying is $1,000,000 of a Eurodollar time deposit. Treasury bond futures are contracts in which the underlying is $100,000 of a U.S. Treasury bond with a minimum 15-year maturity. Stock index futures are contracts in which the underlying is a well-known stock index, such as the S&P 500 or FTSE 100. Currency futures are contracts in which the underlying is a foreign currency.

- An expiring futures contract is equivalent to a spot transaction. Consequently, at expiration the futures price must converge to the spot price to avoid an arbitrage opportunity in which one can buy the asset and sell a futures or sell the asset and buy a futures to capture an immediate profit at no risk.
- The value of a futures contract just prior to marking to market is the accumulated price change since the last mark to market. The value of a futures contract just after marking to market is zero. These values reflect the claim a participant has as a result of her position in the contract.
- The price of a futures contract will equal the price of an otherwise equivalent forward contract one day prior to expiration, or if interest rates are known or constant, or if interest rates are uncorrelated with futures prices.
- A futures price is derived by constructing a combination of a long position in the asset and a short position in the futures. This strategy guarantees that the price received from the sale of the asset is known when the transaction is initiated. The futures price is then derived as the unknown value that eliminates the opportunity to earn an arbitrage profit off of the transaction.
- Futures prices are affected by the opportunity cost of funds tied up in the investment in the underlying asset, the costs of storing the underlying asset, any cash flows paid on the underlying asset, such as interest or dividends, and nonmonetary benefits of holding the underlying asset, referred to as the convenience yield.
- Backwardation describes a condition in which the futures price is lower than the spot price. Contango describes a condition in which the futures price is higher than the spot price.
- The futures price will not equal the expected spot price if the risk premium in the spot price is transferred from hedgers to futures traders. If the risk premium is transferred, then the futures price will be biased high or low relative to the expected future spot price. When the futures price is biased low (high), it is called normal backwardation (normal contango).
- T-bill futures prices are determined by going short a futures contract and going long a T-bill that will have the desired maturity at the futures expiration. At expiration, the T-bill is delivered or cash settled to a price locked in when the transaction was initiated through the sale of the futures. The correct futures price is the one that prohibits this combination from earning an arbitrage profit. Under the assumptions we make, the T-bill futures price is the same as the T-bill forward price.
- The implied repo rate is the rate of return implied by a transaction of buying a spot asset and selling a futures contract. If financing can be obtained in the repo market at less than the implied repo rate, the transaction should be undertaken. If financing can be supplied to the repo market at greater than the implied repo rate, the transaction should be reversed.
- Eurodollar futures cannot be priced as easily as T-bill futures, because the expiration price of a Eurodollar futures is based on a value computed as 1 minus a rate, whereas the value of the underlying Eurodollar time deposit is based on 1 divided by a rate. The difference is small but not zero. Hence, Eurodollar futures do not lend themselves to an exact pricing formula based on the notion of a cost of carry of the underlying.
- Treasury bond futures prices are determined by first identifying the cheapest bond to deliver, which is the bond that the short would deliver under current market conditions. Then one must construct a combination of a short futures contract and a long position in that bond. The bond is held, and the coupons are collected and reinvested.

At expiration, the underlying bond is delivered and the futures price times the conversion factor for that bond is received. The correct futures price is the one that prevents this transaction from earning an arbitrage profit.

- Stock index futures prices are determined by constructing a combination of a long portfolio of stocks identical to the underlying index and a short futures contract. The stocks are held and the dividends are collected and reinvested. At expiration, the cash settlement results in the effective sale of the stock at the futures price. The correct futures price is the one that prevents this transaction from earning an arbitrage profit.
- Currency futures prices are determined by buying the underlying currency and selling a futures on the currency. The position is held, and the underlying currency pays interest at the foreign risk-free rate. At expiration, the currency is delivered and the futures price is received. The correct futures price is the one that prevents this transaction from earning an arbitrage profit.
- Futures markets serve our financial systems by making the markets for the underlying assets more efficient, by providing price discovery, by offering opportunities to trade at lower transaction costs, and by providing a means of managing risk. Futures markets also provide a homogeneous, standardized, and tradable instrument through which participants who might not have access to forward markets can make commitments to buy and sell assets at a future date at a locked-in price with no fear of credit risk. Because futures markets are so visible and widely reported on, they are also an excellent source of information, contributing greatly to the transparency of financial markets.

PROBLEMS

1. **A.** In February, Dave Parsons purchased a June futures contract on the Nasdaq 100 Index. He decides to close out his position in April. Describe how would he do so.

   B. Peggy Smith is a futures trader. In early August, she took a short position in an S&P 500 Index futures contract expiring in September. After a week, she decides to close out her position. Describe how would she do so.

2. A gold futures contract requires the long trader to buy 100 troy ounces of gold. The initial margin requirement is $2,000, and the maintenance margin requirement is $1,500.

   A. Matthew Evans goes long one June gold futures contract at the futures price of $320 per troy ounce. When could Evans receive a maintenance margin call?

   B. Chris Tosca sells one August gold futures contract at a futures price of $323 per ounce. When could Tosca receive a maintenance margin call?

3. A copper futures contract requires the long trader to buy 25,000 lbs of copper. A trader buys one November copper futures contract at a price of $0.75/lb. Theoretically, what is the maximum loss this trader could have? Another trader sells one November copper futures contract. Theoretically, what is the maximum loss this trader with a short position could have?

4. Consider a hypothetical futures contract in which the current price is $212. The initial margin requirement is $10, and the maintenance margin requirement is $8. You go long 20 contracts and meet all margin calls but do not withdraw any excess margin.

   A. When could there be a margin call?

   B. Complete the table below and explain any funds deposited. Assume that the contract is purchased at the settlement price of that day so there is no mark-to-market profit or loss on the day of purchase.

Day	Beginning Balance	Funds Deposited	Futures Price	Price Change	Gain/Loss	Ending Balance
0			212			
1			211			
2			214			
3			209			
4			210			
5			204			
6			202			

   C. How much are your total gains or losses by the end of day 6?

5. Sarah Moore has taken a short position in one Chicago Board of Trade Treasury bond futures contract with a face value of $100,000 at the price of 96 6/32. The initial margin requirement is $2,700, and the maintenance margin requirement is $2,000. Moore would meet all margin calls but would not withdraw any excess margin.

   A. Complete the table below and provide an explanation of any funds deposited. Assume that the contract is purchased at the settlement price of that day, so there is no mark-to-market profit or loss on the day of purchase.

Day	Beginning Balance	Funds Deposited	Futures Price	Price Change	Gain/Loss	Ending Balance
0			96-06			
1			96-31			
2			97-22			
3			97-18			
4			97-24			
5			98-04			
6			97-31			

B. How much are Moore's total gains or losses by the end of day 6?

6. A. The IMM index price in yesterday's newspaper for a September Eurodollar futures contract is 95.23. What is the actual price of this contract?

B. The IMM index price in today's newspaper for the contract mentioned above is 95.25. How much is the change in the actual futures price of the contract since the previous day?

7. Jason Hathaway, a speculator, has purchased a March Eurodollar futures contract at a price of 93.35.

A. What is the annualized LIBOR rate priced into this contract?

B. A month later, the interest rate has decreased to 6.5 percent. Would the futures price go up or down?

C. How much is Hathaway's gain or loss in dollar terms?

8. Mary Craft is expecting large-capitalization stocks to rally close to the end of the year. She is pessimistic, however, about the performance of small-capitalization stocks. She decides to go long one December futures contract on the Dow Jones Industrial Average at a price of 9,020 and short one December futures contract on the S&P Midcap 400 Index at a price of 369.40. The multiplier for a futures contract on the Dow is $10, and the multiplier for a futures contract on the S&P Midcap 400 is $500. When Craft closes her position towards the end of the year, the Dow and S&P Midcap 400 futures prices are 9,086 and 370.20, respectively. How much is the net gain or loss to Craft?

9. A. The current price of gold is $300 per ounce. Consider the net cost of carry for gold to be zero. The risk-free interest rate is 6 percent. What should be the price of a gold futures contract that expires in 90 days?

B. Using Part A above, illustrate how an arbitrage transaction could be executed if the futures contract is priced at $306 per ounce.

C. Using Part A above, illustrate how an arbitrage transaction could be executed if the futures contract is priced at $303 per ounce.

10. Consider an asset priced at $90. A futures contract on the asset expires in 75 days. The risk-free interest rate is 7 percent. Answer the following questions, each of which is independent of the others, unless indicated otherwise.

A. Find the appropriate futures price if the underlying asset has no storage costs, cash flows, or convenience yield.

B. Find the appropriate futures price if the underlying asset's storage costs at the futures expiration equal $3.

C. Find the appropriate futures price if the underlying asset has positive cash flows. The future value of these cash flows is $0.50 at the time of futures expiration.

D. Find the appropriate futures price if the underlying asset's storage costs at the futures expiration equal $3.00 and the compound value at the time of the futures expiration of the positive cash flow from the underlying asset is $0.50.

E. Using Part D above, illustrate how an arbitrage transaction could be executed if the futures contract is trading at $95.

F. Using Part A above, determine the value of a long futures contract an instant before marking to market if the previous settlement price was $89.50.

G. What happens to the value of the futures contract in Part F above as soon as it is marked to market?

11. A 45-day T-bill has a discount rate of 5.50 percent. A 135-day T-bill has a discount rate of 5.95 percent.

A. What should be the price of a futures contract that expires in 45 days? Assume $1 par value.

B. Show that the purchase of a 135-day T-bill, with its price in 45 days hedged by the sale of a 45-day futures contract that calls for the delivery of a 90-day T-bill, is equivalent to purchasing a 45-day T-bill and holding it to maturity.

12. The discount rate on a 60-day T-bill is 6.0 percent, and the discount rate on a 150-day T-bill is 6.25 percent.

A. Based on the 60-day and 150-day T-bill discount rates, what should be the price of a 60-day futures contract? Assume $1 par value.

B. If the actual price of a 60-day futures contract is 0.9853, outline the transactions necessary to take advantage of the arbitrage opportunity, and show the outcome.

C. Calculate the implied repo rate and discuss how you interpret it to determine the profitability of the arbitrage strategy outlined in Part B.

13. A futures contract on a T-bill expires in 30 days. The T-bill matures in 120 days. The discount rates on T-bills are as follows:

30-day bill: 5.4 percent

120-day bill: 5.0 percent

A. Find the appropriate futures price by using the prices of the 30- and 120-day T-bills.

B. Find the futures price in terms of the underlying spot price compounded at the appropriate risk-free rate.

C. Convert the futures price to the implied discount rate on the futures.

D. Now assume that the futures is trading in the market at an implied discount rate 20 basis points lower than is appropriate, given the pricing model and the rule of no arbitrage. Demonstrate how an arbitrage transaction could be executed.

E. Now assume that the futures is trading in the market at an implied discount rate 20 basis points higher than is appropriate, given the pricing model and the rule of no arbitrage. Demonstrate how an arbitrage transaction could be executed.

14. A $1 face value bond pays an 8 percent semiannual coupon. The annual yield is 6 percent. The bond has 10 years remaining until maturity, and its price is $1.1488. Consider a futures contract calling for delivery of this bond only. The contract expires in 18 months. The risk-free rate is 5 percent.

A. Compute the appropriate futures price.

B. Assuming that the futures contract is appropriately priced, show the riskless strategy involving the bond and the futures contract that would earn the risk-free rate of return.

15. Consider a six-year $1 par Treasury bond. The bond pays a 6 percent semiannual coupon, and the annual yield is 6 percent. The bond is priced at par. A futures contract expiring in 15 months calls for delivery of this bond only. The risk-free rate is 5 percent.

A. Find the future value in 15 months of the coupons on this bond.

B. Find the appropriate futures price.

C. Now suppose that the above bond is only one of many deliverable bonds. The contract specification calls for the use of a conversion factor to determine the price paid for a given deliverable bond. Suppose the bond described here has a conversion factor of 1.0567. Find the appropriate futures price.

16. A stock index is at 1,521.75. A futures contract on the index expires in 73 days. The risk-free interest rate is 6.10 percent. The value of the dividends reinvested over the life of the futures is 5.36.

A. Find the appropriate futures price.

B. Find the appropriate futures price in terms of the two specifications of the dividend yield.

C. Using your answer in Part B, find the futures price under the assumption of continuous compounding of interest and dividends.

17. A stock index is at 443.35. A futures contract on the index expires in 201 days. The price of the futures contract is 458.50.The risk-free interest rate is 6.50 percent. The value of the dividends reinvested over the life of the futures is 5.0.

A. Show that the futures contract above is mispriced by computing what the price of this futures contract should be.

B. Show how an arbitrageur could take advantage of the mispricing.

18. The spot exchange rate for the British pound is $1.4390. The U.S. interest rate is 6.3 percent, and the British interest rate is 5.8 percent. A futures contract on the exchange rate for the British pound expires in 100 days.

A. Find the appropriate futures price.

B. Find the appropriate futures price under the assumption of continuous compounding.

C. Suppose the actual futures price is $1.4650. Is the future contract mispriced? If yes, how could an arbitrageur take advantage of the mispricing? Use discrete compounding as in Part A.

SOLUTIONS

1. **A.** Parsons would close out his position in April by offsetting his long position with a short position. To do so, he would re-enter the market and offer for sale a June futures contract on the Nasdaq 100 index. When he has a buyer, he has both a long and a short position in the June futures contract on the Nasdaq 100 index. From the point of view of the clearinghouse, he no longer has a position in the contract.
   B. Smith would close out her position in August by offsetting her short position with a long position. To do so, she would re-enter the market and purchase a September futures contract on the S&P 500. She then has both a short and a long position in the September futures contract on the S&P 500. From the point of view of the clearinghouse, she no longer has a position in the contract.
2. The difference between initial and maintenance margin requirements for one gold futures contract is \$2,000 − \$1,500 = \$500. Because one gold futures contract is for 100 troy ounces, the difference between initial and maintenance margin requirements per troy ounce is \$500/100, or \$5.
   A. Because Evans has a long position, he would receive a maintenance margin call if the price were to *fall* below \$320 − \$5, or \$315 per troy ounce.
   B. Because Tosca has a short position, he would receive a maintenance margin call if the price were to *rise* above \$323 + \$5, or \$328 per troy ounce.
3. *Trader with a long position*: This trader loses if the price falls. The maximum loss would be incurred if the futures price falls to zero, and this loss would be \$0.75/lb × 25,000 lbs, or \$18,750. Of course, this scenario is only theoretical, not realistic.

   Trader with a short position: This trader loses if the price increases. Because there is no limit on the price increase, there is no theoretical upper limit on the loss that the trader with a short position could incur.
4. **A.** The difference between the initial margin requirement and the maintenance margin requirement is \$2. Because the initial futures price was \$212, a margin call would be triggered if the price falls below \$210.
   B.

Day	Beginning Balance	Funds Deposited	Futures Price	Price Change	Gain/Loss	Ending Balance
0	0	200	212			200
1	200	0	211	−1	−20	180
2	180	0	214	3	60	240
3	240	0	209	−5	−100	140
4	140	60	210	1	20	220
5	220	0	204	−6	−120	100
6	100	100	202	−2	−40	160

On day 0, you deposit \$200 because the initial margin requirement is \$10 per contract and you go long 20 contracts (\$10 per contract times 20 contracts equals \$200). At the end of day 3, the balance is down to \$140, \$20 below the \$160 maintenance margin requirement (\$8 per contract times 20 contracts). You must deposit enough money to bring the balance up to the initial margin requirement of \$200. So, the next day (day 4), you deposit \$60. The price change on day 5 causes a gain/loss of −\$120, leaving you with a balance of \$100 at the end of day 5. Again, this amount is less than the $160 maintenance margin requirement. You must deposit enough money to bring the balance up to the initial margin requirement of $200. So on day 6, you deposit $100.

C. By the end of day 6, the price is $202, a decrease of $10 from your purchase price of $212. Your loss so far is $10 per contract times 20 contracts, or $200.

You could also look at your loss so far as follows. You initially deposited $200, followed by margin calls of $60 and $100. Thus, you have deposited a total of $360 so far and have not withdrawn any excess margin. The ending balance, however, is only $160. Thus, the total loss incurred by you so far is $360 − $160, or $200.

5. A.

Day	Beginning Balance	Funds Deposited	Futures Price	Price Change	Gain/Loss	Ending Balance
0	0	2,700.00	96-06			2,700.00
1	2,700.00	0	96-31	25/32	−781.25	1,918.75
2	1,918.75	781.25	97-22	23/32	−718.75	1,981.25
3	1,981.25	718.75	97-18	−4/32	125.00	2,825.00
4	2,825.00	0	97-24	6/32	−187.50	2,637.50
5	2,637.50	0	98-04	12/32	−375.00	2,262.50
6	2,262.50	0	97-31	−5/32	156.25	2,418.75

On day 0, Moore deposits $2,700 because the initial margin requirement is $2,700 per contract and she has gone short one contract. At the end of day 1, the price has increased from 96-06 to 96-31—that is, the price has increased from $96,187.50 to $96,968.75. Because Moore has taken a short position, this increase of $781.25 is an adverse price movement for her, and the balance is down by $781.25 to $1,918.75. Because this amount is less than the $2,000 maintenance margin requirement, she must deposit additional funds to bring her account back to the initial margin requirement of $2,700. So, the next day (day 2), she deposits $781.25. Another adverse price movement takes place on day 2 as the price further increases by $718.75 to $97,687.50. Her ending balance is again below the maintenance margin requirement of $2,000, and she must deposit enough money to bring her account back to the initial margin requirement of $2,700. So, the next day (day 3), she deposits $718.75. Subsequently, even though her balance falls below the initial margin requirement, it does not go below the maintenance margin requirement, and she does not need to deposit any more funds.

B. Moore bought the contract at a futures price of 96-06. By the end of day 6, the price is 97-31, an increase of 1 25/32. Therefore, her loss so far is 1.78125 percent of $100,000, which is $1,781.25.

You could also look at her loss so far as follows: She initially deposited $2,700, followed by margin calls of $781.25 and $718.75. Thus, she has deposited a total of $4,200 so far, and has not withdrawn any excess margin. Her ending balance is $2,418.75. Thus, the total loss so far is $4,200 − $2,418.75, or $1,781.25.

6. A. Because the IMM index price is 95.23, the annualized LIBOR rate priced into the contract is 100 − 95.23 = 4.77 percent. With each contract based on $1 million notional principal of 90-day Eurodollars, the actual futures price is $1,000,000[1 − 0.0477(90/360)] = $988,075.

B. Because the IMM index price is 95.25, the annualized LIBOR rate priced into the contract is 100 − 95.25 = 4.75 percent. The actual futures price is \$1,000,000[1 − 0.0475(90/360)] = \$988,125. So, the change in actual futures price is \$988,125 − \$988,075 = \$50.

You could also compute the change in price directly by noting that the IMM index price increased by 2 basis points. Because each basis point move in the rate moves the actual futures price by \$25, the increase in the actual futures price is 2 × \$25, or \$50.

7. A. Because the IMM index price is 93.35, the annualized LIBOR rate priced into the contract is 100 − 93.35 = 6.65 percent.

B. Because the interest rate has decreased, the futures price would have increased.

C. With each contract based on \$1 million notional principal of 90-day Eurodollars, the actual futures price at the time of purchase was \$1,000,000[1 − 0.0665 (90/360)] = \$983,375. The actual futures price a month later is \$1,000,000 [1 − 0.0650(90/360)] = \$983,750. The increase in futures price is \$983,750 − \$983,375 = \$375. Thus, Jason Hathaway's gain is \$375.

You could also compute the change in price directly by noting that the interest rate decreased by 15 basis points (and the IMM index price increased by 15 basis points). Because each basis point move in the rate moves the actual futures price by \$25, the increase in actual futures price is 15 × \$25, or \$375.

8. Her gain caused by the increase in the price of Dow Jones Industrial Average futures is \$10(9,086 − 9,020) = \$660. Because Craft had a short position in S&P Midcap 400 futures, her loss caused by the increase in the price of S&P Midcap 400 futures is \$500(370.20 − 369.40) = \$400. Craft's net gain is \$660 − \$400 = \$260.

9. A. T = 90/365 = 0.2466. The futures price is

$$f_0(T) = S_0(1 + r)^T$$

$$f_0(0.2466) = 300(1.06)^{0.2466} = \$304.34 \text{ per ounce}$$

B. Do the following:

- Enter a short futures position—that is, sell the futures at \$306.
- Buy gold at \$300.
- At expiration, deliver an ounce of gold and receive \$306.

This amount is \$1.66 more than \$304.34, which is the sum of the cost of the asset (\$300) and the loss of interest on this amount at the rate of 6 percent a year (\$4.34). Thus, the overall strategy results in a riskless arbitrage profit of \$1.66 per futures contract. You can also look at this scenario in terms of returns: Investing \$300 and receiving \$306 90 days later is an annual return of 8.36 percent, because $300(1.0836)^{(90/365)} = 306$. This return is clearly greater than the risk-free return of 6 percent.

C. The steps in this case would be the reverse of the steps in Part B above. So, do the following:

- Enter a long futures position; that is, buy the futures at \$303.
- Sell short the gold at \$300.
- At expiration, take the delivery of an ounce of gold and pay \$303.

This amount paid is \$1.34 less than \$304.34, which is the sum of the funds received from the short sale of the asset (\$300) and the interest earned on this at the rate of 6 percent per year (\$4.34). Thus, the overall strategy results in a riskless arbitrage profit of \$1.34 per futures contract. In terms of rates, receiving \$300 up front and paying \$303 90 days later represents an annual rate of 4.12 percent, because $300(1.0412)^{(90/365)} = 303$. This rate is clearly less than the risk-free rate of 6 percent. Thus, the overall transaction is equivalent to borrowing at a rate less than the risk-free rate.

10. A. T = 75/365 = 0.2055. The futures price is $f_0(0.2055) = 90(1.07)^{0.2055} = 91.26$.

B. Storage costs must be covered in the futures price, so we add them:

$$f_0(0.2055) = 91.26 + 3 = 94.26$$

C. A positive cash flow, such as interest or dividends on the underlying, reduces the futures price:

$$f_0(0.2055) = 91.26 - 0.50 = 90.76$$

D. We add the storage costs and subtract the positive cash flow:

$$f_0(0.2055) = 91.26 + 3 - 0.50 = 93.76$$

E. We would do the following:

- Sell the futures at $95.
- Buy the asset at $90.
- Because the asset price compounded at the interest rate is $91.26, the interest forgone is 1.26. So the asset price is effectively $91.26 by the time of the futures expiration.
- We have incurred storage costs of $3 on the asset. We have received $0.50 from the asset. At expiration, we deliver the asset and receive $95. The net investment in the asset is $91.26 + $3.00 − $0.50 = $93.76. If we sell it for $95, we make a net gain of $1.24. Thus, the overall strategy results in a riskless arbitrage profit of $1.24 per futures contract. One can also look at this profit in terms of returns. Investing $90 and receiving a net of $95.00 − $3.00 + $0.50 = $92.50 75 days later is an annual return of 14.26 percent, because $\$90(1.14264)^{(75/365)} = \$92.50$. This return is clearly greater than the risk-free return of 7 percent.

F. The last settlement price was $89.50, and the price in our answer in Part A is $91.26. The value of a long futures contract is the difference between these prices, or $1.76.

G. When the futures contact is marked to market, the holder of the futures contract receives a gain of $1.76, and the value of the futures contract goes back to a value of zero.

11. A. h = 45 and h + m = 135

$$r_0^d(h) = r_0^d(45) = 0.055$$

$$r_0^d(h + m) = r_0^d(135) = 0.0595$$

The prices of these T-bills will, therefore, be

$$B_0(h) = 1 - r_0^d(h)\left(\frac{h}{360}\right)$$

$$B_0(45) = 1 - 0.055\left(\frac{45}{360}\right) = 0.9931$$

$$B_0(h + m) = 1 - r_0^d(h + m)\left(\frac{h + m}{360}\right)$$

$$B_0(135) = 1 - 0.0595\left(\frac{135}{360}\right) = 0.9777$$

So the price of a futures contract expiring in 45 days is

$$f_0(h) = \frac{B_0(h + m)}{B_0(h)}$$

$$f_0(45) = \frac{B_0(135)}{B_0(45)} = \frac{0.9777}{0.9931} = 0.9845$$

The discount rate implied in the futures would be

$$r_0^{df}(h) = [1 - f_0(h)]\left(\frac{360}{m}\right)$$

$$r_0^{df}(45) = [1 - 0.9845]\left(\frac{360}{90}\right) = 0.0620$$

In other words, in the T-bill futures market, the rate would be stated as 6.20 percent, which would imply a futures price of 0.9845. Alternatively, the implied futures discount rate could be obtained from the spot rates as

$$r_0^{df}(h) = \left\{1 - \left[\frac{1 - r_0^d(h + m)\left(\frac{h + m}{360}\right)}{1 - r_0^d\left(\frac{h}{360}\right)}\right]\right\}\left(\frac{360}{m}\right)$$

$$r_0^{df}(45) = \left\{1 - \left[\frac{1 - 0.0595\left(\frac{135}{360}\right)}{1 - 0.055\left(\frac{45}{360}\right)}\right]\right\}\left(\frac{360}{90}\right) = 0.0622$$

with the slight difference caused by rounding.

B. Suppose one purchases the 135-day T-bill for 0.9777 and sells the 45-day futures contract at a price of 0.9845. Then, 45 days later, the T-bill would be a 90-day T-bill and would be delivered to settle the futures contract. Thus, at that time, one would receive the original futures price of 0.9845. One initially paid 0.9777 and 45 days later received 0.9845. The return per dollar invested is

$$\frac{0.9845}{0.9777} = 1.00696$$

If instead one purchases a 45-day T-bill at the price of 0.9931 and holds it for 45 days, the return per dollar invested is

$$\frac{1}{0.9931} = 1.00695$$

Thus, the return per dollar invested is the same in both transactions (with a slight difference caused by rounding), and both transactions are free of risk.

12. A. First compute the prices of the 60-day and 150-day T-bills. With h = 60 and h + m = 150,

$$r_0^d(h) = r_0^d(60) = 0.060$$

$$r_0^d(h + m) = r_0^d(150) = 0.0625$$

The prices of these T-bills will, therefore, be

$$B_0(h) = 1 - r_0^d(h)\left(\frac{h}{360}\right)$$

$$B_0(60) = 1 - 0.06\left(\frac{60}{360}\right) = 0.99$$

$$B_0(h + m) = 1 - r_0^d(h + m)\left(\frac{h + m}{360}\right)$$

$$B_0(150) = 1 - 0.0625\left(\frac{150}{360}\right) = 0.974$$

So the price of a futures expiring in 60 days is

$$f_0(h) = \frac{B_0(h + m)}{B_0(h)}$$

$$f_0(45) = \frac{B_0(150)}{B_0(60)} = \frac{0.974}{0.990} = 0.9838$$

B. As the actual futures price of 0.9853 is more than the implied futures price computed in Part A, you should sell the futures contract. So, do the following.

- Buy the 150-day T-bill at 0.974.
- Sell the 60-day futures contract at 0.9853.

The return per dollar would be

$$\frac{0.9853}{0.9740} = 1.0116$$

Note that this return is risk free. It compares favorably with return per dollar on purchasing a 60-day T-bill and holding it to maturity, which is

$$\frac{1}{0.99} = 1.0101$$

C. The repo rate is the annualization of the return per dollar because of the arbitrage transactions outlined in Part B:

$$1.0116^{365/60} - 1 = 0.0727$$

Thus, the rate of return from a cash-and-carry transaction implied by the futures price relative to the spot price is 7.27 percent. If the financing rate available in the repo market is less than this rate, the arbitrage strategy outlined in Part B is worthwhile, because the cost of funds is less than the return on the funds.

13. A. First, find the prices of 30- and 120-day bills:

$$B_0(30) = 1 - 0.054(30/360) = 0.9955$$

$$B_0(120) = 1 - 0.050(120/360) = 0.9833$$

$$f_0(h) = \frac{B_0(h + m)}{B_0(h)}$$

$$f_0(45) = \frac{B_0(120)}{B_0(30)} = \frac{0.9833}{0.9955} = 0.9877$$

B. We must find the rate at which to compound the spot price of the 120-day T-bill. The spot rate, obtained from the 30-day T-bill, is

$$[1 + r_0(h)]^{h/365} = \frac{1}{0.9955} = 1.0045$$

Based on the rate $r_0(h)$, every dollar invested should grow to a value of 1.0045. Thus, the futures price should be the spot price (price of the 120-day T-bill) compounded by the factor 1.0045:

$$f_0(30) = 0.9833(1.0045) = 0.9877$$

That is, $[1 + r_0(h)]^{h/365} = 1.0045$. Note that we do not actually need $r_0(h)$.

C. Given the futures price of 0.9877, the implied rate is

$$r_0^{df}(h) = (1 - 0.9877)\left(\frac{360}{90}\right) = 0.0492$$

D. If the futures contract is trading for 20 basis points lower, it is trading at a rate of 4.72 percent. So the futures price would be

$$f_0(30) = 1 - 0.0472\left(\frac{90}{360}\right) = 0.9882$$

Do the following:
- Buy the 120-day bill at 0.9833.
- Sell the futures at 0.9882.

This transaction produces a return per dollar invested of

$$\frac{0.9882}{0.9833} = 1.0050$$

which is a risk-free return and compares favorably with a return per dollar invested of 1.0045 (computed in Part B above) for a 30-day T-bill.

E. If the futures contract is trading for 20 basis points higher than is appropriate, it trades at a rate of 5.12 percent. So the futures price would be

$$f_0(30) = 1 - 0.0512\left(\frac{90}{360}\right) = 0.9872$$

Do the following:
- Sell the 120-day bill at 0.9833.
- Go long a 30-day futures contract at 0.9872.

Thus, at the beginning, you received 0.9833. At expiration 30 days later, the 120-day bill you sold short at the beginning is a 90-day bill. You would take care of this by paying 0.9872 and taking the delivery of a 90-day T-bill (because you had bought a 30-day futures contract at the beginning). Effectively, what you have done is borrowed at a rate of

$$\frac{0.9872}{0.9833} = 1.0040$$

which compares favorably with the risk-free rate of 1.0045 computed in Part B above.

You could also look at the above as follows. You go long a 30-day futures contract at 0.9872. You sell the 120-day bill at 0.9833. Invest this amount in a 30-day T-bill. At maturity, you will have 0.9833(1.0045), or 0.9877. This return compares favorably with the 0.9872 that you owe at expiration.

14. A. Because the futures contract expires in 18 months, $T = 1.5$. The risk-free rate, $r_0(T)$, is 0.05. When computing the accumulated value of the coupons on the bond and the interest on them until the futures contract expires, note that the first coupon is paid in exactly six months and reinvested for the one year remaining until expiration. Also, the second coupon is paid in exactly one year and reinvested for the six months remaining until expiration, and the third coupon is paid in exactly one and a half years and not reinvested. So, the accumulated value of the coupons on the bond and the interest on them is

$$0.04(1.05)^{1} + 0.04(1.05)^{0.5} + 0.04 = 0.1230$$

Because the underlying bond is the only deliverable bond in this simplistic problem, the conversion factor is 1.0, so no adjustment is required. Now the futures price is easily obtained as

$$f_0(T) = B_0^c(T + Y)[1 + r_0(T)]^T - FV(CI,0,T)$$
$$f_0(1.5) = 1.1488(1.05)^{1.5} - 0.1230$$
$$= 1.1130$$

B. Buy the five-year bond for \$1.1488 and sell the futures for \$1.1130. Hold the position for one and a half years until the futures expiration. Collect and reinvest the coupons in the meantime. When the futures contract expires, deliver the bond and receive the futures price of \$1.1130. In addition, you will have the coupons and interest on them of \$0.1230 for a total of \$1.1130 + \$0.1230 = \$1.2360. You invested \$1.1488 and end up with \$1.2360 a year and a half later, so the return per dollar invested is \$1.2360/\$1.1488 = 1.0759. Because this amount is paid in 1.5 years, the annual equivalent of this is

$$1.0759^{1/1.5} = 1.05$$

This return is equivalent to the 5 percent risk-free rate.

15. A. Because the futures contract expires in 15 months, T = 1.25. The risk-free rate, $r_0(T)$, is 0.05. To compute the accumulated value of the coupons on the bond and the interest on them until the futures contract expires, we note that the first coupon is paid in exactly six months and reinvested for the nine months (0.75 years) remaining until expiration. Also, the second coupon is paid in exactly one year and reinvested for the three months (0.25 years) remaining until expiration. So, the accumulated value of the coupons on the bond and the interest on them is

$$0.03(1.05)^{0.75} + 0.03(1.05)^{0.25} = 0.0615$$

B. Because the underlying bond is the only deliverable bond in this part of the problem, the conversion factor is 1, and no adjustment is required. So, the futures price is

$$f_0(T) = B_0^c(T + Y)[1 + r_0(T)]^T - FV(CI,0,T)$$
$$f_0(1.25) = 1(1.05)^{1.25} - 0.0615$$
$$= 1.0014$$

C. The futures price now is the price computed in Part B above divided by the conversion factor. Because the conversion factor is 1.0567, the futures price is

$$\frac{1.0014}{1.0567} = 0.9477$$

16. A. T = 73/365 = 0.20. The futures price should be

$$f_0(T) = S_0(1 + r)^T - FV(D,0,T)$$
$$f_0(0.20) = 1{,}521.75(1.0610)^{0.20} - 5.36$$
$$= 1{,}534.52$$

Alternatively, we can find the present value of the dividends:

$$PV(D,0,T) = \frac{FV(D,0,T)}{(1 + r)^T}$$

$$PV(D,0,0.20) = \frac{5.36}{(1.0610)^{0.20}} = 5.30$$

Then the futures price would be

$$f_0(T) = [S_0 - PV(D,0,T)](1 + r)^T$$

$$f_0(0.20) = (1{,}521.75 - 5.30)(1.061)^{0.20}$$

$$= 1{,}534.52$$

B. One specification based on the yield δ is

$$\frac{1}{(1 + \delta)^T} = 1 - \frac{FV(D,0,T)}{S_0(1 + r)^T}$$

$$= 1 - \frac{5.36}{1{,}521.75(1.061)^{0.20}} = 0.9965$$

So, $(1 + \delta)^T$ is 1/0.9965 = 1.0035. Then the futures price is

$$f_0(T) = \left(\frac{S_0}{(1 + \delta)^T}\right)(1 + r)^T$$

$$f_0(0.20) = \left(\frac{1{,}521.75}{1.0035}\right)(1.061)^{0.20}$$

$$= 1{,}534.51$$

The difference comes from rounding.

Under the other specification, the yield would be found as

$$\delta^* = \frac{PV(D,0,T)}{S_0}$$

$$= \frac{5.30}{1{,}521.75} = 0.0035$$

Then the futures price would be

$$f_0(T) = S_0(1 - \delta^*)(1 + r)^T$$

$$f_0(0.20) = 1{,}521.75(1 - 0.0035)(1.061)^{0.20}$$

$$= 1{,}534.49$$

The difference comes from rounding.

C. The continuously compounded risk-free rate is $r^c = \ln(1 + r) = \ln(1.061) = 0.0592$. The continuously compounded dividend yield is $\delta^c = \ln(1 + \delta) = (1/T)\ln[(1 + \delta)^T] = (1/0.20)\ln(1.0035) = 0.0175$. The futures price is

$$f_0(T) = S_0 e^{(r^c - \delta^c)T}$$

$$f_0(0.20) = 1{,}521.75e^{(0.0592 - 0.0175)0.20}$$

$$= 1{,}534.49$$

The difference comes from rounding.

17. A. T = 201/365 = 0.5507. The futures price should be

$$f_0(T) = S_0(1 + r)^T - FV(D,0,T)$$

$$f_0(0.5507) = 443.35(1.0650)^{0.5507} - 5.0 = 454.0$$

Alternatively, we can find the present value of the dividends:

$$PV(D,0,T) = \frac{FV(D,0,T)}{(1 + r)^T}$$

$$PV(D,0,0.5507) = \frac{5.0}{(1.0650)^{0.5507}} = 4.83$$

Then the futures price would be

$$f_0(T) = [S_0 - PV(D,0,T)](1 + r)^T$$

$$f_0(0.5507) = (443.35 - 4.83)(1.065)^{0.5507}$$

$$= 454.0$$

Because the futures contract is selling at 458.50, which is higher than the price computed above, the futures contract is overpriced.

B. The arbitrageur will buy the stocks underlying the index at their current price of \$443.35. Also, the arbitrageur will sell the futures contract at the settlement price of \$458.50. The arbitrageur will collect and reinvest the dividends, which would be worth \$5 at the time of the futures expiration. At the time of expiration, the arbitrageur will get the settlement price of \$458.50. So, the arbitrageur invests \$443.35 at the beginning and receives \$5.00 + \$458.50 = \$463.50 at the expiration 201 days later. The return per dollar invested over the 201-day period is

$$\frac{463.50}{443.35} = 1.0454$$

The annual risk-free rate is 6.5 percent, equivalent to a return per dollar invested of $(1.065)^{0.5507} = 1.0353$ over the 201-day period. Thus, the return to the arbitrageur from the transactions described above exceeds the risk-free return. Alternatively, one could see that to the arbitrageur, the return per dollar invested, over a year, is $1.0454^{365/201} = 1.0832$. This annualized return of 8.32 percent is clearly greater than the annual risk-free rate of 6.5 percent.

18. T = 100/365 = 0.274

A. The futures price is

$$f_0(T) = \left(\frac{S_0}{(1 + r^f)^T}\right)(1 + r)^T$$

$$f_0(0.274) = \left(\frac{1.4390}{1.058^{0.274}}\right)(1.063)^{0.274}$$

$$= 1.4409$$

B. The continuously compounded equivalent rates are

$$r^{fc} = \ln(1.058) = 0.0564$$

$$r^c = \ln(1.063) = 0.0611$$

The futures price is

$$f_0(T) = (S_0 e^{-r^{fc}T})e^{r^cT}$$

$$f_0(T) = (1.4390e^{-0.0564(0.274)})e^{0.0611(0.274)}$$

$$= 1.4409$$

C. The actual futures price of \$1.4650 is higher than the price computed above—the futures contract is overpriced. To take advantage, the arbitrageur needs to buy the foreign currency and sell the futures contract. First, however, we must determine how many units of the currency to buy. Because we need to have 1 unit of currency, including the interest, the number of units to buy is

$$\frac{1}{(1.058)^{0.274}} = 0.9847$$

So we buy 0.9847 units, which costs 0.9847(\$1.4390) = \$1.417. We sell the futures at \$1.4650 and hold until expiration. During that time, the accumulation of interest will make the 0.9847 units of the currency grow to one unit. Using the futures contract, at expiration we convert this unit at the futures rate of \$1.4650. The return per dollar invested is

$$\frac{1.4650}{1.417} = 1.0339$$

or a return of 3.39 percent over 100 days. The U.S. annual risk-free rate is 6.3 percent, which is equivalent to a return per dollar invested of $(1.063)^{0.274} = 1.0169$, over the 100-day period. Thus, the return to the arbitrageur from the transactions described above exceeds the risk-free return. Alternatively, one could see that to the arbitrageur, the return per dollar invested, over a year, is $(1.0339)^{365/100} = 1.1294$. This annualized return of 12.94 percent is more than double the annual risk-free rate of 6.3 percent.