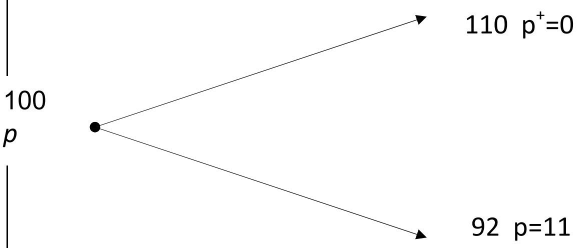
Revision questions study unit 4 to 5 **Total [35]**

1. A stock is currently trading at R100. It is expected to increase by 10% and decrease by 8% over the next six months. The risk-free interest rate is 5.5%. Calculate the price of a European put option expiring in the six months with an exercise price of R103. (5)

Put option:

u = 1.10
d = 0.92



Step1 Find the underlying prices in the binomial tree:

$S_u = 100(1.10) = 110$
$S_d = 100(0.92) = 92$

Step 2 Find option prices at expiration (put):

$P^+ = \max(0; 103 - 110) = 0$
$P^- = \max(0; 103 - 92) = 11$

Step 3 Find the risk-neutral probability (π) (sometimes also referred to as p)

$$\pi = \frac{(1+r)-d}{u-d}$$
$$= \frac{1.055-0.92}{1.10-0.92}$$
$$= \frac{0.1350}{0.18}$$
$$= 0.75$$

1-π = 1 – 0.75
= 0.25

Step 4: Price today

$$p = \frac{\pi p^+ + (1-\pi)p^-}{1+r}$$
$$= \frac{0.75(0)+0.25(11)}{1.055}$$
$$= \frac{2.75}{1.055}$$
$$= 2.6066$$
$$= 2.61$$

2. The following information is available regarding the option contracts on shares of PPC Limited:

Option style	European
Share spot price	R65
Strike price	R70
Time to maturity	Three months
Risk-free interest rate	8% per annum
Put option price	R7.62
Call option price	R?

Calculate the price of the call option by using put-call parity. (3)

$$S + p = c + X(1+r)^{-t}$$

$$c = S + p - X(1+r)^{-t}$$

$$= 65 + 7.62 - 70(1.08)^{-0.25}$$

$$= 72.62 - 68.6661$$

$$= 3.95$$

3. Peter wants to buy a European call option on a non-dividend-paying share to which the following information pertains:

- Share price R 62
- Strike price R 59
- Risk-free interest rate 9% per annum (continuous compounding)
- Volatility 28% per annum
- Time to maturity 3 months

Use the Black-Scholes-Merton model to calculate the price of the call option. (7)

$$d_1 = \frac{ln\left(\frac{62}{59}\right) + \left(0.09 + \frac{0.28^2}{2}\right)0.25}{0.28\sqrt{0.25}}$$

$$= \frac{0.04960 + 0.03230}{0.14}$$

$$= 0.5850$$

$$d_2 = 0.5850 - 0.14$$

$$= 0.4450$$

N(d1) = 0.7224
N(d2) = 0.6736

$$c = SN(d_1) - Xe^{-r^cT}N(d_2)$$

$$= 62 \times 0.7224 - 59\ ^{-0.09\times0.25} \times \quad 0.6736$$

$$= 44.7888 - 38.8582$$

$$= 5.93$$

4. The following information is available regarding option contracts on the shares of Hull Limited:

Option style	European
Share spot price	R70
Strike price	R77
Time to maturity	9 months
Risk-free interest rate	8% per annum
Put option premium	R7.62
Call option premium	R?

4.1 Calculate the price of the call option assuming that put-call parity exists. (3)

$$S + p = c + X(1+r)^{-t}$$

$$c = S + p - X(1+r)^{-t}$$

$$= 70 + 7.62 - 77(1.08)^{-0.75}$$

$$= 77.62 - 72.6813$$

$$= 4.94$$

4.2 Calculate the upper bound for the call option and lower bound for the put option premium respectively. (2)

upper bound call: $c \leq S_0 \leq 70$

lower bound put:

$$p \geq \left[X(1+r)^{-t} - S\right]$$

$$p \geq 77(1.08)^{-0.75} - 70$$

$$\geq 72.6813 - 70$$

$$\geq 2.68$$

5. Consider a two-year interest rate swap with semi-annual payments. Assume a notional principal of $25 million.

5.1 Calculate the semi-annual fixed payment and the annualised fixed rate on the swap if the current term structure of LIBOR interest rates is as follows:

$L_0(180) = 0.0715$; $L_0(360) = 0.0705$
$L_0(540) = 0.0695$; $L_0(720) = 0.0685$ (7)

$$B_0(180) = \frac{1}{1+0.0715(180/360)} = 0.9655$$

$$B_0(360) = \frac{1}{1+0.0705(360/360)} = 0.9341$$

$$B_0(540) = \frac{1}{1+0.0695(540/360)} = 0.9056$$

$$B_0(720) = \frac{1}{1+0.0685(720/360)} = 0.8795$$

$$FS(0,4,180) = \frac{1-0.8795}{0.9655+0.9341+0.9056+0.8795}$$

$$= \frac{0.1205}{3.6847}$$

$$= 0.0327$$

Fixed payment $= 0.0327 \times \$25{,}000{,}000 = \$817{,}500$ or R817 570

Annualized fixed rate $= 3.27\%(360/180) = 6.54\%$

Note: Take 0.0715 x 180/360 =0.0358 then +1 = 1.0358, then only 1/1.0358 = 0.9655

5.2 Calculate the market value of the swap 120 days later from the point of view of the party paying the fixed rate and receiving the floating rate if the term structure 120 days later is as follows:

$L_{120}(60) = 0.0697$; $L_{120}(240) = 0.0653$
$L_{120}(420) = 0.0629$; $L_{120}(600) = 0.0613$ (8)

$$B_{120}(180) = \frac{1}{1+0.0697(60/360)} = 0.9885$$

$$B_{120}(360) = \frac{1}{1+0.0653(240/360)} = 0.9583$$

$$B_{120}(540) = \frac{1}{1+0.0629(420/360)} = 0.9316$$

$$B_{120}(720) = \frac{1}{1+0.0613(600/360)} = 0.9073$$

$$\text{Fixed} = 0.0327(0.9885+0.9583+0.9316+0.9073)+1(0.9073)$$
$$= 1.0311$$

$$1^{st}\text{Floating payment} = \left[1+(0.0715)(180/360)\right] = 1.0358$$

Discounted with the 60 day present value factor of 0.9885:

$$\text{Float} = 1.0358 \times 0.9885 = 1.0238$$

$$V_{pay\_float} = \$25{,}000{,}000(1.0311-1.0238) = \$182{,}500$$
$$V_{pay\_fixed} = \$25{,}000{,}000(1.0238-1.0311) = -\$182{,}500$$

Total [35]