

Revision questions Forward and futures contracts (SU2 and SU3)

- 1 A stock is expected to pay a dividend of \$2 per share in three and six months. The stock price is \$50, and the risk-free rate is 7.5% p.a. (annual compounding). An investor has taken a short position in a six-month forward contract.

1.1 What are the forward price and the initial value of the forward contract? (3)

$$\begin{aligned}PV(D) &= \frac{2}{(1.075)^{3/12}} + \frac{2}{(1.075)^{6/12}} \\ &= 1.9642 + 1.9290 \\ &= \$3.8932 \\ &= \$3.89\end{aligned}$$

$$\begin{aligned}F_T &= [S_0 - PV(D)](1+r)^T \\ &= [50 - 3.89](1.075)^{6/12} \\ &= \$47.8079 \\ &= \$47.81\end{aligned}$$

Initial value of forward contract = zero

1.2 Four months later, the price of the stock is \$48 and the risk-free rate is still 7.5% pa. What are the forward price and value of the short position in the forward contract? (3)

$$\begin{aligned}PV(D) &= \frac{2}{(1.075)^{2/12}} \\ &= \$1.9760\end{aligned}$$

$$\begin{aligned}F_T &= [S_0 - PV(D)](1+r)^T \\ &= [48 - 1.9760](1.075)^{2/12} \\ &= \$46.5822 \\ &= \$46.58\end{aligned}$$

$$\begin{aligned}
V_t &= S_t - PV(D) - \frac{F_T}{(1+r)^{T-t}} \\
&= 48 - 1.976 - \frac{47.81}{1.075^{2/12}} \\
&= 48 - 1.976 - 47.2372 \\
&= -1.2132 \\
&= -1.21
\end{aligned}$$

Negative for short position = gain

2. Marx Corporation plans to borrow \$45 million in 30 days. The following information is available regarding the loan:

Maturity of loan	270 days
Interest rate	LIBOR + 25 basis points

Marx Corporation is concerned that the interest rate will increase and enters into a forward rate agreement (FRA) at 7.25%. Calculate the effective cost of the loan if the 270-day LIBOR in 30 days is 8.50%. (8)

$$\begin{aligned}
FRA_{\text{payoff}} &= NP \left[\frac{\left(U_{\text{rate}} - FRA_{\text{rate}} \right) \left(\frac{U_{\text{days}}}{360} \right)}{1 + U_{\text{rate}} \left(\frac{U_{\text{days}}}{360} \right)} \right] \\
&= 45,000,000 \left[\frac{\left(0.085 - 0.0725 \right) \left(\frac{270}{360} \right)}{1 + 0.085 \left(\frac{270}{360} \right)} \right] \\
&= 45,000,000 \left[\frac{0.0094}{1.0638} \right] \\
&= 45,000,000 \times 0.0088 \\
&= 397,649.82
\end{aligned}$$

This payoff reduces the amount that has to be borrowed. The amount borrowed is \$45,000,000 - \$397,649.82 = \$44,602,350.18

$$\begin{aligned}
\text{Repay interest and principal} &= \$44,602,350.18 \left[1 + (0.085 + 0.0025) \left(\frac{270}{360} \right) \right] \\
&= \$47,529,379.41
\end{aligned}$$

$$\begin{aligned}
\text{Effective rate} &= \left(\frac{47,529,379.41}{45,000,000} - 1 \right) \left(\frac{360}{270} \right) \\
&= 7.49\%
\end{aligned}$$

If not rounded:

$$\begin{aligned} \text{FRA}_{\text{payoff}} &= \text{NP} \left[\frac{\left(U_{\text{rate}} - \text{FRA}_{\text{rate}} \right) \left(\frac{U_{\text{days}}}{360} \right)}{1 + U_{\text{rate}} \left(\frac{U_{\text{days}}}{360} \right)} \right] \\ &= 45,000,000 \left[\frac{(0.085 - 0.0725) \left(\frac{270}{360} \right)}{1 + 0.085 \left(\frac{270}{360} \right)} \right] \\ &= 45,000,000 \left[\frac{0.0094}{1.0638} \right] \\ &= 45,000,000 \times 0.0088 \\ &= 396,592.24 \end{aligned}$$

This payoff reduces the amount that has to be borrowed. The amount borrowed is $\$45,000,000 - \$396,592.24 = \$44,603,407.78$

$$\begin{aligned} \text{Repay interest and principal} &= \$44,603,407.78 \left[1 + (0.085 + 0.0025) \left(\frac{270}{360} \right) \right] \\ &= \$47,530,506.42 \end{aligned}$$

$$\begin{aligned} \text{Effective rate} &= \left(\frac{47,530,506.42}{45,000,000} - 1 \right) \left(\frac{360}{270} \right) \\ &= 7.50\% \end{aligned}$$

3. Explain why the valuation of forward contract to the investor are important. (6)

Value can be defined as the amount of money you have to pay for something or the amount of money you can sell something for.

The importance of valuation of forward contracts are as follows:

- important to business owners in order to know the value of their future commitments (good business sense).
- accounting rules require forward contracts to be included in income statements and balance sheets.
- the value gives good measure on credit exposure.
- the value are used to determine the amount of money one party has to pay another party to terminate a position.

4. Peter Matseke wants to buy a dividend-paying share currently worth R165. He plans to buy the share in 250 days. In order for him to hedge against a possible increase in the price of the share, he wishes to take a long position in the forward contract that expires in 250 days. The risk-free rate is 5%. Over the next 250 days, the stock will pay the following dividends according to the following schedule:

<i>Days to next dividend</i>	<i>Dividend per share</i>
30	R1.75
120	R1.75
210	R1.75

- 4.1 Calculate the forward price of a contract established today and expiring in 250 days. (3)

$$\begin{aligned}
 PV(D) &= \frac{1.75}{(1.05)^{30/365}} + \frac{1.75}{(1.05)^{120/365}} + \frac{1.75}{(1.05)^{210/365}} \\
 &= 1.7430 + 1.7222 + 1.7016 \\
 &= R5.1668 \\
 &= R5.17
 \end{aligned}$$

$$\begin{aligned}
 F_T &= [S_0 - PV(D)](1+r)^T \\
 &= [165 - 5.17](1.05)^{250/365} \\
 &= R165.26
 \end{aligned}$$

- 4.2 It is now 100 days since Matseke entered into the forward contract. The share price is R174. Calculate the value of the forward contract at this point and indicate whether there is a gain or a loss in the transaction? (4)

$$\begin{aligned}
 PV(D) &= \frac{1.75}{(1.05)^{20/365}} + \frac{1.75}{(1.05)^{110/365}} \\
 &= 1.7453 + 1.7245 \\
 &= R3.4698 \\
 &= R3.47
 \end{aligned}$$

$$\begin{aligned}
 V_t &= S_t - PV(D) - \left[\frac{F_T}{(1+r)^{T-t}} \right] \\
 &= 174 - 3.47 - \left[\frac{165.26}{(1.05)^{150/365}} \right] \\
 &= 170.53 - 161.98 \\
 &= 8.55
 \end{aligned}$$

Positive value indicates a **gain** to the long position

- 4.3 At expiration, the price of the share is R160. Calculate the value of the contract at expiration and indicate whether Matseke has made a gain or loss on the transaction? (3)

$$\begin{aligned}
 V_T &= S_T - F_T \\
 &= 160 - 165.26 \\
 &= -5.26
 \end{aligned}$$

The contract expires with a negative value which indicates that Matseke made a **loss** on the transaction.

5. A US investor gathered the following information on interest rates and currencies in the UK and USA. The discrete risk-free rates are 5.25% in the US and 6.25% in the UK. The spot exchange rate is \$1.65 per pound and the market price of a 75-day futures contract on the pound is \$1.6498. Calculate the no-arbitrage futures price on the 75-day contract and determine the appropriate arbitrage strategy. (7)

$$r_d^c = \ln(1.0525) = 0.0512$$

$$r_f^c = \ln(1.0625) = 0.0606$$

$$f_0(T) = 1.6500e^{(0.0512-0.0606)(75/365)} = \$1.6468$$

Futures contract is overpriced

Short the futures contract

Go long in the pound in the spot market

$$(1.0525)^{75/365} \neq (1.0625)^{75/365} \times \left(\frac{1.6498}{1.6500} \right)$$

$$1.0106 < 1.0124$$

Borrow dollars at 5.25% for 75 days

Convert to pounds (buy in spot market) and invest at 6.25%

Enter into futures contract to sell pounds (convert at \$1.6498)

6. A gold futures contract requires the long trader to buy 100 troy ounces of gold. The initial margin requirement is R3 550, and the maintenance margin requirement is R1 500.
- 6.1 Matthew Evans goes long one June gold futures contract at the futures price of R425 per troy ounce. When could Evans receive a maintenance margin call? (show all calculations) (3)

The difference between the initial margin and the maintenance margin for one gold futures contract is as follows: $3\,550 - 1\,500 = 2\,050$
One gold futures contract = 100 troy ounce of gold. Thus: $2\,050/100 = 20.50$ or R20.50. (1 mark)

A long position holder will **receive a margin call when the price FALLS**.

Evans will receive a margin call if the price of gold **decreases** (1 mark) by R20.50.
Thus: $R425 - R20.50 = \mathbf{R404.50}$ per troy ounce. (1 mark)

6.2 Chris Tosca sells one August gold futures contract at a futures price of R397 per ounce.
When could Tosca receive a maintenance margin call? (show all calculations) (2)

A short position holder will receive a margin call if the price **INCREASES**.

Tosca will receive a margin call if the price of gold **increases** (1 mark) by R20.50.
Thus: $R397 + R20.50 = \mathbf{R417.50}$ per troy ounce. (1 mark)