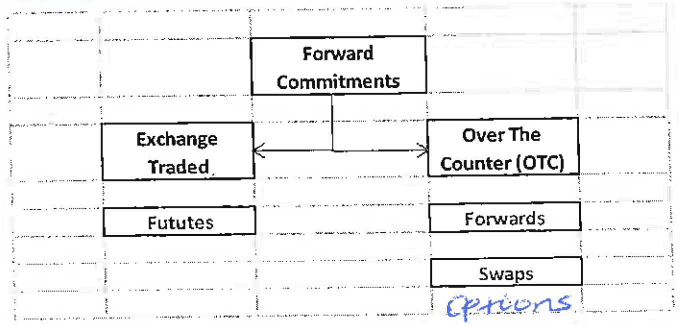
LO 5.22: Determine how credit risk arises in a swap and distinguish between current credit risk and potential credit risk.
LO 5.23: Identify and assess at what point in a swap's life credit risk is the greatest.
LO 5.24: Interpret the swap spread and what it represents.
LO 5.25: Illustrate how swap credit risk is reduced by both netting and marking to market.
LO 5.26: Evaluate the role that swaps play in the financial system.

Study Unit 1: Derivative markets and instruments

LO 1.1: Argue the concept of a derivative
A derivative is a financial instrument whose value depends on the value of some other instrument
A derivative is simply an agreement between a future buyer and a future seller (known as counterparties)
Every derivative specifies a future price at which some item can or must be sold. This item, known as the underlier, can be a physical commodity such as maize or gold, or some financial security such as shares of bonds, or something abstract such as share indexes.
Every derivative also specifies a future delivery date on or before which the transaction must occur
The common elements of all derivatives are:
- The buyer and seller (the counterparties)
- The underlier
- The future price
- The future date (delivery date or contract date)

LO 1.2: Differentiate between exchange traded and over-the-counter (OTC) derivatives.
An exchange traded instrument is traded on a recognised exchange such as CBOT (the Chicago Board of Trade) and in South Africa SAFEX (the South African Futures Exchange) and are in the public domain
An over the counter instruments are traded between individuals and are essentially private agreements
The relevance and importance of this distinction will be discussed more fully later

LO 1.3: Assess a forward commitment and identify the different type of forward commitments.
The word commitment suggests a binding agreement where the counterparties have no choice but to perform their obligations in terms of an agreement.
More formally a forward commitment is an agreement between two parties in which one party, the buyer, agrees to buy from the other party, the seller, an underlying asset at a future date at a price established at the inception of the agreement



Study unit 4: Option markets and contracts

LO 4.1: Identify the basic elements and characteristics of option contracts.
LO 4.2: Assess European option, American option, moneyness, payoff, intrinsic value, and time value.
LO 4.3: Differentiate between exchange-traded options and over-the-counter options.
LO 4.4: Identify the different varieties of options in terms of the types of instruments underlying them.
LO 4.5: Compare and contrast interest rate options with forward rate agreements (FRAs).
LO 4.6: Determine option payoffs, and differentiate between interest rate option payoffs and payoffs of other types of options.
LO 4.7: Contrast interest rate caps and floors.
LO 4.8: Identify the minimum and maximum values of European options and American options.
LO 4.9: Illustrate how the lower bounds of European calls and puts are determined by constructing portfolio combinations that prevent arbitrage and calculate an Option's lower bound.
LO 4.10: Determine the lowest prices of European and American calls and puts based on the rules for lower bounds.
LO 4.11: Illustrate how a portfolio (combination) of options establishes the relationship between options that differ only by exercise price.
LO 4.12: Determine how option prices are affected by differences in the time to expiration.
LO 4.13: Illustrate how put-call parity for European options is established by comparing the payoffs on a fiduciary call and a protective put and use the result to create synthetic instruments. Argue why an investor would want to do so.
LO 4.14: Illustrate how violations of put-call parity for European options can be exploited and how those violations are eliminated.
LO 4.15: Compare American options with European options in terms of the lower bounds on and the possibility of early exercise.
LO 4.16: Assess how cash flows on the underlying asset affect put-call parity and the lower bounds on option prices.
LO 4.17: Identify the directional effect of an interest rate change on an option's price.
LO 4.18: Determine an option price and illustrate how an arbitrage opportunity can be exploited in a one-period binomial model.
LO 4.19: Determine an option price in a two-stage binomial model.
LO 4.20: Calculate prices of options on bonds and interest rate options in one- and two-period binomial models.
LO 4.21: Determine how the binomial model value converges as time periods are added.
LO 4.22: Identify and assess the assumptions underlying the Black-Scholes-Merton model.
LO 4.23: Calculate the value of a European option using the Black-Scholes-Merton model.
LO4.24: Determine how an option price, as represented by the Black-Scholes-Merton model, is affected by each of the input vales (the Greeks).

LO 4.25: Illustrate and interpret the concept of an option's delta and how it is used in dynamic hedging.
LO 4.26: Determine the gamma effect on an option's price and delta.
LO 4.27: Determine how cash flows on the underlying asset affect an option's price.
LO 4.28: Identify and illustrate the two methods of estimating the volatility of the underlying.
LO 4.29: Illustrate how put-call parity for options on forwards (or futures) is established.
LO 4.30: Identify the similarities in American options on forwards and futures, and differentiate them from European options.
LO 4.31: Calculate the value of a European option on forwards (or futures) using the Black model.
LO 4.32: Calculate the value of a European interest rate option using the Black model.
LO 4.33: Evaluate the role of options markets in financial systems and society.

Study unit 5: Swap markets and contracts

LO 5.1: Identify the characteristics of swap contracts.
LO 5.2: Demonstrate how swaps are terminated.
LO 5.3: Identify the types of currency swaps.
LO 5.4: Calculate the payments on a currency swap.
LO 5.5: Classify a plain vanilla interest rate swap.
LO 5.6: Calculate the payments on an interest rate swap.
LO 5.7: Identify the types of equity swaps.
LO 5.8: Calculate the payments on an equity swap.
LO 5.9: Distinguish between the pricing and valuation of swaps.
LO 5.10: Illustrate the equivalence of swaps to combinations of other instruments.
LO 5.11: Determine how interest rate swaps are equivalent to a series of off-market forward rate agreements (FRAs).
LO 5.12: Determine how a plain vanilla swap is equivalent to a combination of an interest rate call and an interest rate put.
LO 5.13: Determine the fixed rate on a plain vanilla interest rate swap and the market value of the swap during its life.
LO 5.14: Determine the fixed rate, if applicable, and the foreign notional principal for a given domestic notional principal on a currency swap, and determine the market values of each of the different types of currency swaps during their lives.
LO 5.15: Determine the fixed rate, if applicable, on an equity swap and the market values of the different types of equity swaps during their lives.
LO 5.16: Identify and interpret the characteristics of swaptions, including the difference between payer and receiver swaptions.
LO 5.17: Determine why swaptions exist and identify their applications.
LO 5.18: Illustrate how the payoffs of an interest rate swaption are like those of an option on a coupon-bearing bond.
LO 5.19: Calculate the value of an interest rate swaption on the expiration day.
LO 5.20: Contrast the different ways in which the market value of a swaption at expiration can be received.
LO 5.21: Evaluate forward swaps and distinguish between forward swaps and swaptions.

LO 1.4: Contrast the basic characteristics of forward contracts, futures contracts and swaps.

The first thing to note is that forward, future and swap contracts are all classified as forward commitments

Definition of a forward contract
A forward contract is an agreement where the buyer agrees to purchase the underlier from the seller at a specified price on a specified future date.
You may view this as a "deferred sale" meaning that all of the terms of the agreement are made at the inception of the contract but the deal will actually only take place at some point in the future. There is no exchange of money at inception. The cash flows in the contract only happens in the future i.e. when the seller delivers the underlying and the buyer pays for the underlying

Example
Mr A is a manufacturer and is concerned that the price of widgets will go up in six months time. He enters into a forward contract with Ms B to buy one widget at a price of R1000 for delivery in six months time

At the delivery date (six months) Scenario 1
Now let us assume that the market price of widgets has risen to R1200. Mr A will take delivery of the widget and pay R1000 (He is therefore better off by R200 i.e. he could take delivery of the widget for R1000 and immediately sell it for R1200)
Ms B on the other hand will be worse off by R200. She has to deliver the widget to Mr A and receive R1000 for the widget which in now worth R1200

At the delivery date (six months) Scenario 2
Now let us assume that the market price of widgets has fallen to R900. Mr A will take delivery of the widget and pay R1000 (He is therefore worse off of by R100 i.e. he could take delivery of the widget for R1000 but can only sell it for R900)
Ms B on the other hand will be better off by R100. She has to deliver the widget to Mr A and receive R1000 for the widget which in now only worth R900

(Note that both Mr A and Ms B are bound to perform in terms of the agreement i.e. they cannot change their minds no matter how depressed they may be! Thus we say that a forward contract is *forward commitment*)

Definition of a futures contract
A futures contract is an agreement where the buyer agrees to purchase the underlier from the seller at a specified price on a specified future date. However a futures contract is a standard agreement which is traded on a recognised exchange This means that the exchange guarantees that both parties will fulfill their obligations (how this happens will be discussed later)

Standard here means that the contracts are for standard quantities and qualities of the underlying at standard delivery dates. Perhaps the most important point to note is the fact the there is no default risk (because the exchange guarantees performance of the counterparties)

Definition of a swap contract
A swap is an agreement where the counterparties agree to exchange future cash flows. Swap contracts can be based on interest rates, indexes or on foreign currencies Note that swap contracts are traded over the counter
A swap contract is in fact a series of forward commitments More of this later

LO 1.5: Assess a contingent claim and identify the different types of contingent claims.
The words "contingent claim" implies that something will happen upon the outcome of some future event. This also suggests that one of the parties will be able to exercise a choice based on the outcome of some future event. We call this choice an option (as opposed to an obligation where performance is required no matter what) In other words the holder of an option can make a choice as to whether he wants the contract to continue or to walk away from the deal

Definition of an option contract
An option gives the holder (i.e.THE BUYER never the seller) the right, but not the obligation to buy or sell something at a specified price, on or before a specified future date.
Note that the majority of option contracts are exchange traded but some of them are traded OTC

LO 1.6: Identify the basic characteristics of options and distinguish between an option to buy and an option to sell.
Returning to the definition of an option: ***An option gives the holder (i.e.THE BUYER never the seller) the right, but not the obligation to buy or sell something at a specified price, on or before a specified future date***
From the above definition we note the following:
- An option always gives the buyer the right to exercise or not exercise the option
- An option that gives the buyer the right to buy is known as a call option
- An option that gives the buyer the right to sell is known as a put option
- An option that can only be exercised on a specified date is known as a European option
- An option that can be exercised any time between inception and maturity is known as an American option
- The specified price is also known as the strike price or exercise price

LO 1.7: Differentiate between the ways of measuring the size of the global derivatives market.
The approximate size of the global derivatives market can be measured in one of two ways:
- Notional principal i.e. the total amount of the underlyings on which the derivatives contracts are based or
- Market value i.e. the economic value of worth of the derivatives

LO 1.8: Evaluate the purposes and criticisms of derivative markets.

Purposes:
- Price discovery
- Facilitating risk management
- Making markets more efficient
- Lowering of transaction costs

The greatest criticism of derivatives is that through leverage (a topic discussed more fully later) unknowledgeable participants can get into serious financial distress. A typical example of this was the demise of Barings Bank and more recently the blaming of the global credit crisis on derivatives. Warren Buffet has been quoted as saying that "derivatives are the weapons of mass destruction" in the financial markets. In my opinion he was referring to the use or misuse of derivatives by individuals and institutions that lack the knowledge to effectively use these instruments.

Study unit 2: Forward markets and contracts

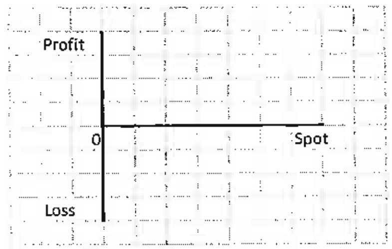
LO 2.1: Distinguish between the positions held by the long and short parties to a forward contract.

Long means to own an asset or your intention is to buy an asset i.e. if I say I am long 2 contracts it means that I own 2 contracts. If I say I'm going to go long 2 contracts it means I'm going to buy 2 contracts

Short means not to own or your intention is to sell. If I say I'm short 3 contracts it means that I do not own them. If I say I'm going to go short 3 contracts it means that I'm going to sell 3 contracts

Now let's consider these positions with the use of a payoff diagram. (You will find these diagrams extremely useful in you study of the theory and a good knowledge and understanding of the diagrams is extremely important)

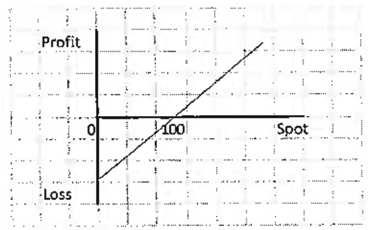
A payoff off diagram looks as follows:



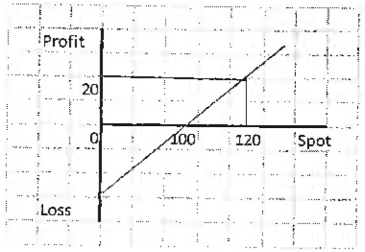
We see from the above diagram that the x axis measures the spot price of the underlying asset. We simply abbreviate the axis title to "spot" instead of writing "spot price of the underlying asset" We note also that the lowest that a spot price can ever go is zero i.e. the underlying spot price can never be a negative number

The y axis measures profit above the zero and loss below the zero

Now consider Mr A who goes long (i.e. buys) a share for R100. The payoff diagram would look like the following:

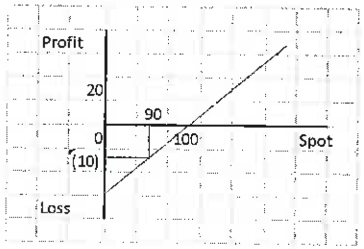


Now what is the diagram telling us if the spot price of the underlying increases to R120 per share. We can see form the diagram that the long position would be making a R20 profit as follows:



This is correct because 120 – 100 = 20 (logically if you bought a share for R100 and sold it for R120 you would make R20 profit)

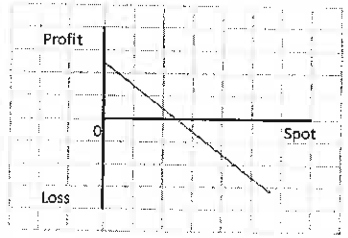
Now what would happen if the price dropped to R90 From the diagram we see that the buyer would be making a R10 loss as follows:



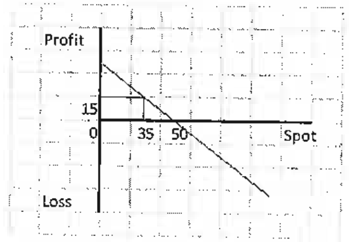
This is correct because 90 – 100 = -10 (logically if you bought a share for R100 and sold it for R90 then you would make a R10 loss)

It is also clear that the motivation for going long (i.e. buying) an asset is that you are hoping that the spot prices would rise above your purchase price and you would thus realise a gain on your investment This simply confirms the oldest trading rule in the world i.e. "buy low and sell high"

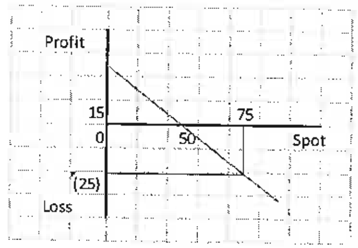
Let us now consider the short position. This may occur where the owner of an asset wishes to sell the asset at some point in the future but is concerned that prices may decline in the future. The way to avoid this risk is to sell the asset now for delivery in the future. For example, Mr X is concerned that the price of his asset will decline in the future. He therefore goes short (sells the asset now) for delivery in the future Think of this as an adaptation of our trading rule that now says " sell high and buy low"



From the above we see that if the spot price decline, say to R35 then the seller would make a profit of R15 This makes sense because he sold the asset for R50 and could replace it at R35 thus yielding a R15 profit. This can be read directly off the diagram below:



However if the spot prices rise to, say R75, then the seller would incur a loss of R25 This makes sense because he sold the asset for R50 and would have to replace it at R75 thus yielding a R25 loss
This can also be seen in the diagram below:



LO 2.2: Identify the procedures for selling a forward contract at expiration.

There are two ways to settle a forward contract at expiration:

- **Delivery** This means that at expiry the buyer will accept delivery of the underlying asset and pay the seller the price Alternatively the seller will make delivery of the underlying asset and accept the price
- **Cash settlement**. This means that the underlying will not be physically delivered. If the asset is worth more (less) than the forward price the short (long) pays the long (short) the cash difference between the market price or rate and the price or rate agreed on in the contract

LO 2.3: Demonstrate how a party to a forward contract can terminate a position prior to expiration and how credit risk is affected by the way in which a position is terminated.

A party to a forward contract can terminate the position by taking an opposite position with the same counterparty or with counterparty. If the party is long she can go short and vice versa.

LO 2.4: Differentiate between a dealer and an end user of a forward contract.

A dealer is an institution that makes a market in derivatives i.e. they will buy and sell instruments. An end user buys or sells derivatives to dealers usually to hedge a particular risk exposure.

LO 2.5: Identify the essential characteristics of equity forward contracts.

Equity forward contracts can be written on individual shares (stocks) or portfolios of shares or on share indexes. We will see later that the pricing and value of equity forward contracts must take into account the possibility that the underlying shares may pay dividends. We will learn how to adjust for dividend payments when calculating price and value later in the course

LO 2.6: Identify the essential characteristics of forward contracts on zero-coupon and coupon bonds.

A forward contract on a bond must mature before the maturity of the underlying bond. The forward contracts can be based on individual zero coupon bonds or individual coupon bonds or on bond portfolios or on bond indices.
Zero coupon bonds pay their return by discounting the face value (usually using a 360 day year)
A forward contract on a bond can be affected by embedded options in the bonds e.g. callable bonds or convertible bonds.

LO 2.7: Identify the characteristics of the Eurodollar time deposit market.

There is a large global market for time deposits in various currencies issued by large creditworthy banks.
This market is primarily centered in London but also exists elsewhere, THOUGH NOT IN THE USA.
The primary time deposit instrument is called the EURODOLLAR, which is a dollar deposited outside the USA
Banks borrow dollars from other banks by issuing Eurodollar time deposits which are essentially short term unsecured loans
In London the rate on such dollar loans is called the LIBOR (London Inter Bank Offer Rate)

Even though it represents a loan outside the USA LIBOR is considered to be the best representative rate on a dollar borrowed by a private i.e. nongovernmental high quality borrower
Trading in Euros and Euro deposits occurs in most world cities and two similar rates on such euro deposits are commonly quoted.
One called EUROLIBOR is complied in London by the British Bankers Association and the other is called Euribor and is compiled in Frankfurt and published by the European Central Bank.
Euribor is the most widely used.

LO 2.8: Interpret LIBOR and Euribor.

LIBOR is the rate at which London banks are willing to lend to other London banks
Euribor is the rate at which banks loan to other banks in Frankfort in which the currency is euro

LO 2.9: Identify the essential characteristics of forward rate agreements (FRAs).

A Forward Rate Agreement is a forward contract in which the long party agrees to pay a fixed interest rate at a future date and receive an interest payment at a rate to be determined at expiration.
FRA's are described by a special notation e.g. a 6 x 9 FRA expires in six months and the underlying is a Eurodollar deposit that begins in six months and ends three months later or nine months from now.
A 6 X 12 FRA expires in six months and the underlying is a Eurodollar deposit that begins in six months and ends 12 months later or 12 months from now

LO 2.10: Calculate the payment at expiration of an FRA and evaluate each of the component terms.

EXAMPLE
A FRA is based on the 90 day LIBOR and will expire in 30 days time. (This is referred to as a 1x4 FRA since it expires in one month's time and the period that its interest rate relates to is the three months after expiry i.e. from the end of month 1 to the end of month 4. The FRA is quoted at 6%. An investor buys the FRA based on a notional principal of $1m

Scenario 1
If in 30 days time, the 90 day LIBOR is trading at 4% calculate the payments made under the FRA
Scenario 2
If in 30 days time, the 90 day LIBOR is trading at 7% calculate the payments made under the FRA

Scenario 1 Answer
The payment made is based on the differences between the FRA rate and the underlying rate for 90 day LIBOR on the day the FRA expires

$$1000000\,x\,(4-6)\,x\,\frac{90}{360}=\$5{,}000$$

The underlying LIBOR is actually lower than the rate agreed under the FRA, meaning that the investor who has bought the FRA will have to make a payment to the FRA dealer.
The payment is made when the FRA expires, at the beginning of the 90 day period to which the LIBOR relates (in this case at the end of the first month). However, LIBOR interest is usually paid at the end of the period to which it relates. In order to allow for this, the payment value of the payment 90 days earlier is calculated based on the final underlying LIBOR

Payment made by the investor is $-5{,}000\,x\,\frac{1}{1.01111}=-4950$

The 1.01 is calculated as $1+90\,day\,LIBOR=1+0.01$ $\left[4\%\,x\,\frac{90}{360}=1\%=0.01\right]$

Scenario 2 Answer

$$1000000\,x\,(7\%-6\%)\,x\,\frac{90}{360}=2500$$

Payment made by bank to investor $2500\,x\,\frac{1}{1.0175}=2457$

From the above we observe the following rules for FRA's:

- If at expiry the market spot rate is above the FRA rate then the dealer pays the purchaser (an amount equilivant to the present value of the difference between the spot price and the FRA rate times the nominal amount over the contract period discounted at the prevailing spot rate)
- If at expiry the market spot rate is below the FRA rate the purchaser pays the dealer (an amount equilivant to the present value of the difference between the spot price and the FRA rate times the nominal amount over the contract period discounted at the prevailing spot rate)

We will investigate FRA's in more detail in **LO 2.8** below

LO 2.11: Identify the essential characteristics of currency forward contracts.

A currency forward contract is a contract in which the long agrees to exchange one currency for another currency at a future date The contract can be settled by actual delivery or the counterparties can choose to settle in cash on the expiration date

LO 2.12: Determine the price of a forward contract.

LO2.12.1: The Cost of Carry Model

The price of a forward contract is a spot price of an underlying adjusted for time and other costs. Remember that you are certain that the future transaction will occur (i.e. a forward contract is a forward commitment) Because we know for certain that the future transaction will occur we adjust the spot price of the underlying for interest, storage costs and other costs that stem from the deferral of the transaction (In LO 1.4 we said that a forward contract is just a "deferred sale") At the future date the long party will buy and the short party will sell
The delivery (or contract price) is the fixed price at which the long party commits to buy the underlying and the short party commits to sell the underlying at the contract delivery date (i.e. expiration of the forward)
We will now discuss how we determine the delivery price (contract price) of the forward contract.
The delivery price (contract price) is the price such that the value of the contract at inception is zero. This means that at inception neither the long nor the short has an advantage.
[In this course you can safely assume that the calculation of a forward price is the same as that of a futures price. Also remember that all the maths in derivatives assumes that the calculations are done from the LONG position or longs point of view. We can do that because derivatives are a zero sum game meaning that the gain for the long party is exactly equal to the loss for the short party and vice versa. So if you are doing a calculation from the short party's point of view you perform the maths as usual but simply change the sign of the answer – more of this later]

When we study forwards (and any other derivatives for that matter) it is extremely important that we distinguish the difference between "delivery price", "value" and "forward price"

i) **Delivery price** – is the guaranteed price that the long will buy and the short will sell at the future delivery date. This price is determined at inception of the contract and is "cast in stone" i.e.it does not and cannot change over the life of the forward contract.
ii) **The value** - is a measure of how much better or worse off the parties are for having entered into the forward contract.
iii) **A forward price** - is the delivery price of a theoretical new contract (with the same terms and conditions) and whose value is zero

Simply put a forward price becomes a delivery price (or contract price) when a forward contract is entered into i.e. the long and short agree to do a deal. The reason why this is important will become clear when we do the calculations
The value of an existing forward contract is then simply the present value of the difference between the current spot price of the underlying and the delivery price

LO 2.13: Determine the price and value of a forward contract at initiation, during the life of the contract and at expiration.

Before we begin let us establish the notation that we will use in this course

The diagram above sets out a typical time line starting now i.e. point 0 going through to time T i.e. some point in the future. Time t is some random point between now and point T in the future.

We are going to find the following on various forward contracts:
- The price and value at inception of the contract
- The price and value at some random point t
- The price ad value at termination i.e. point T

Let us start by developing a generic model on a forward contract on an underlying asset that produces no income.

.Notation

F_0 = Forward price at inception(Delivery price or contract price)
S_0 = Spot price at inception
r = Risk free rate
F_t = Forward price at some random point t
S_t = Spot price at some random price t
F_T =Forward price at expiration
S_T = Spot price at expiration

Generic Pricing Model – where the underlying is a non income producing asset

Example 1:
Mr Lennon purchases a one year forward contract on an asset that produces no income, has a spot price of R50 and the risk free rate is 10% p.a.

i) Find the price and the value of the contract at inception?
ii) It is now 5 months later and the spot price of the underlying is R60 and the risk free rate is still 10% p.a. What is the price and value of the forward contract at this time?

iii) Want is the price and value of the contract at expiration is the spot price of the underlying is R72?

At Inception

$S_0 = 50$
r=10% or 0.10
T=1

Price
$F_0 = S_0(1+r)^T = 50(1.10)^1 = 55$

Value
Remember that the value of a contract is zero at inception
The following calculation serves as proof

$V_0 = S_0 - \frac{F_0}{(1+r)^T} = 50 - \frac{55}{(1.1)^1} = 50 - 50 = 0$

i) **Five months later**

$S_t = 60$
r=10% or 0.10
$T=7 = \frac{7}{12} = 0.5833$

Price
$F_t = S_t(1+r)^{T-t} = 60(1.10)^{0.5833} = 60(1.0572) = 63.5301$

Value
$V_t = S_t - \frac{F_0}{(1+r)^{T-t}} = 60 - \frac{55}{(1.1)^{0.5833}} = 60 - \frac{55}{1.0583} = 60 - 51.9701 = 8.0299$

ii) **At termination**

$S_T = 72$
$r = 10\%$ or 0.10
$T = 0$

Price

$F_T = S_T(1+r)^0 = 72(1.10)^0 = 72$

Value

$V_T = S_T - \frac{F_0}{(1+r)^{T-t}} = 72 - \frac{55}{(1.1)^0} = 72 - 55 = 17$

OR

$V_T = S_T - F_0 = 72 - 55 = 17$

EXAMPLE 2:
Lucy Sky believes that the price of widgets will decline within six months. (Widgets are non income producing assets). The current price of widgets is R220 and the risk free rate is 6%. She decides to speculate and goes short a forward contract on the widgets. Answer the following questions

i) What is the contract price at inception?
ii) Two months later the spot price of widgets has risen to R240. What is the forward price of widgets and what is the value of the contract if the interest rate is still 6%?
iii) It is now the expiry date and the spot price of widgets is R196. What is the forward price of the contract and what is Lucy's gain or loss?

i) **At Inception**

$S_0 = 220$
$r = 0.06$ (6%)
$T = \frac{6}{12} = 0.5$

Price

$F_0 = S_0(1+r)^T = 220(1.06)^{0.5} = 220(1.0296) = 226.5039$

Value

Remember that the value of a contract is zero at inception
The following calculation serves as proof

$V_0 = S_0 - \frac{F_0}{(1+r)^T} = 220 - \frac{226.5039}{(1.06)^{0.5}} = 220 - \frac{226.5039}{1.0296} = 0$

ii) **Two months later**

$S_t = 240$
$r = 0.06$ (6%)
$T = \frac{4}{12} = 0.3333$

Price

$F_t = S_t(1+r)^{T-t} = 240(1.06)^{0.3333} = 240(1.0196) = 244.7040$

Value

$V_t = S_t - \frac{F_0}{(1+r)^{T-t}} = 240 - \frac{226.5039}{(1.06)^{0.333}} = 240 - \frac{226.5039}{1.0196} = 240 - 222.1498 = 17.8502$

Remember that the maths is from the long position.
Lucy is short and is therefore making a loss

iii) **At termination**

$S_T = 196$
$r = 0.06$ (6%)
$T = 0$

Price

$F_T = S_T(1+r)^0 = 196(1.06)^0 = 196$

Value

$V_T = S_T - \frac{F_0}{(1+r)^{T-t}} = 196 - \frac{226.5029}{(1.06)^0} = 196 - 226.5029 = -30.5029$

OR

$V_T = S_T - F_0 = 196 - 226.5029 = -30.5029$

Remember the maths is from the long point of view
Therefore Lucy made a profit of 30.5029

LO 2.14: Assess why valuation of a forward contract is important.

Valuation of forward contracts is important because:
- It is good to know the value of future commitments
- Accounting rules require that forward contracts be accounted for in the financial statements at value
- Value gives a good idea of credit exposure

- It gives a good idea of termination value should one party wish to terminate the position

LO 2.15: Contrast an off-market forward contract with the standard type of forward contract.

We have already seen that forward contracts at inception have a value of zero. However there is an exception and this is known as an off market forward contract. This an off market forward contract has a non zero value at inception which is negotiated between the contracting parties. The contact may have a positive of negative value at inception. A positive value is paid by the long to the short and vice versa

LO 2.16: Calculate the price and value of an equity forward contract, given the different possible patterns of dividend payments.

Firstly note the following about equities:

- Not all equities pay dividends i.e. some equities are non income producing assets (In fact companies are under no legal obligation to pay dividends)
- Forward contracts can be taken out on individual equities, portfolios of equities or equity indexes
- If an equity produces income it is in the form of a dividend and the amount is in dollars (rands) However income on equity portfolios and indexes are quoted as yields

We now deal with each of the above mentioned situations with the aid of examples

LO 2.16.1: Price and value of a forward contract on an equity that pays no income (dividend)

At Inception
Price
$F_0 = S_0(1+r)^T$ Remember that when the deal is struck then this price becomes the delivery price (or contract price)
Value
Value of a forward contract at inception is zero

At some random point between inception and expiration
Price
$F_t = S_t(1+r)^{T-t}$
Value
$V_t = S_t - \frac{F_0}{(1+r)^{T-t}}$

At expiration
Price
$F_T = S_T$

Value
$V_T = S_T - F_0$

Example:
Mr Pepper purchases a forward contract on a non dividend paying share when the share price is R245, the risk free rate is 8% and the term of the contract is six months
Calculate the following:

i) Price and value at inception
ii) It is now three months later and the spot price of the share is R270.What would the forward price be on a new contract and what is the value of the contract?
iii) What is the price and value of the contract at expiration if the spot price of the share is R265

Answer
i) Price and value at inception

Price
$F_0 = S_0(1+r)^T = 245(1.08)^{0.5} = 245(1.0392) = 254.6115$
Value
The value of the forward contract at inception is zero

ii) Three months later

Price
$F_t = S_t(1+r)^{T-t} = 270(1.08)^{\frac{3}{12}} = 270(1.08)^{0.25} = 270(1.0194) = 275.2452$

Value
$V_t = S_t - \frac{F_0}{(1+r)^{T-t}} = 270 - \frac{254.6115}{(1.08)^{\frac{3}{12}}} = 270 - \frac{254.6115}{1.0194} = 270 - 249.7660 = 20.2340$

iii) At expiration

Price
$F_T = S_T = 265$

Value
$V_T = S_T - F_0 = 265 - 254.6115 = 10.3885$

LO 2.16.2: Price and value of a forward contract on an equity that pays a dividend (a known income)

At Inception
Price
$F_0 = [S_0 - PV(Div)](1+r)^T$ Remember that when the deal is struck then this price becomes the delivery price (or contract price)
Value
Value of a forward contract at inception is zero

At some random point between inception and expiration
Price
$F_t = [S_t - PV(Div)](1+r)^{T-t}$
Or
$F_t = S_t(1+r)^{T-t} - FV(Div)$
Value
$V_t = [S_t - PV(Div)] - \frac{F_0}{(1+r)^{T-t}}$
Or
$V_t = S_t - \frac{F_0}{(1+r)^{T-t}} - FV(Div)$

At expiration
Price
$F_T = S_T$
Value
$V_T = S_T - F_0$

Example

Mrs Rigby enters into a one year forward contract on a share priced at R200 that will pay a dividend of R2 in 3 months and again in 9 months. The current risk free rate is 10%
Calculate:
i) Price and value of the forward at inception
ii) It is now 6 months later the spot price of the share is R210 and the risk free rate is still 10%. What are the price and the value of the forward contract?
iii) What is the price and value of the forward contract if at expiration the spot price of the share is R230

At inception
Price
$F_0 = [S_0 - PV(div)](1+r)^T$

$PV(Div) = \frac{2}{(1.1)^{\frac{3}{12}}} + \frac{2}{(1.1)^{\frac{9}{12}}} = \frac{2}{(1.1)^{0.2500}} + \frac{2}{(1.1)^{0.7500}} = \frac{2}{1.0241} + \frac{2}{1.0741} = 1.9529 + 1.8620 = 3.8149$

$F_0 = [200 - 3.8149](1.1)^1 = 196.1851(1.1)^1 = 215.8036$

OR

$F_0 = S_0(1+r)^t - FV(Div)$

$FV(Div) = 2(1.1)^{\frac{9}{12}} + 2(1.1)^{\frac{3}{12}} = 2(1.0741) + 2(1.0241) = 2.1482 + 2.0482 = 4.1964$

$F_0 = 200(1.1)^1 - 4.1964 = 220 - 4.1964 = 215.8036$

Value
The value of a forward contract at inception is zero

Six months later
Price
$F_t = [S_t - PV(Div)](1+r)^{T-t}$

$PV(Div) = \frac{2}{(1.1)^{\frac{3}{12}}} = \frac{2}{(1.1)^{0.2500}} = \frac{2}{1.0241} = 1.9529$

$F_t = [210 - 1.9529](1.1)^{\frac{12-6}{12}} = [208.0471](1.1)^{0.5000} = 208.0471(1.0488) = 218.1998$

OR

$F_t = S_t(1+r)^T - FV(Div)$

$FV(Div) = 2(1.1)^{\frac{3}{12}} = 2(1.1)^{0.2500} = 2(1.0241) = 2.0482$

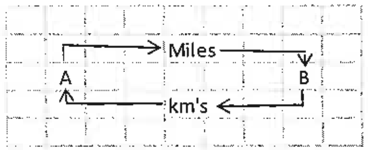
$F_t = 210(1.1)^{\frac{6}{12}} - 2.0482 = 220.2499 - 2.0482 = 218.2017$ Difference due to rounding

At Expiration
Price
$F_t = S_T = 230$
Value
$V_T = S_T - F_0 = 230 - 215.8036 = 14.1964$

LO 2.16.3: Price and value of a forward contract on an equity portfolio whose income is quoted as a continuously compounded rate

First let's discuss continuously compounded rates

If we have two points in time and space, say point A and Point B we can measure the distance between Point A and Point B in terms of Miles or in terms of km's. Although the units of measure are different the physical distance between the two points is the same



We can say the same time about measuring interest rates. We can measure interest in terms of discrete rates or in terms of continuous rates

Example 1

Invest 100 at 12%pa for one year. What is the future value of the investment?

$$FV = PV(1+r)^T = 100(1.12)^1 = 112$$

Now we can do the same calculation using a continuous interest rate. First we have to convert the discrete rate into a continuous rate as follows:

$$r^c = m\text{Ln}\left(1+\frac{r^d}{m}\right)$$

Where:

r^c = Continuous rate

m = number of compounding periods per year

r^d = Discreet rate

Ln = Natural Log

In our example

$$r^c = m\text{Ln}\left(1+\frac{r^d}{m}\right) = 1\,\text{Ln}\left(1+\frac{0.12}{1}\right) = 0.1133$$

$$FV = PVe^{r^cT} = 100\,(2.7183)^{0.1133} = 100\,(1.12) = 112$$

Note that the value of "e" in any formula = 2.7183

Formulas

At Inception

Price

$F_0 = [S_0e^{-\delta^cT}]e^{r^cT}$

The term in the square brackets is the share price discounted at the dividend yield rate which is equilivant to the share price minus the present value of the dividends. This value is then compounded at the risk free rate over the life of the contract just as we have done previously.

Value

The value of a forward contract at inception is zero

At some point between inception and expiration

Price

$F_t = [S_te^{-\delta^c(T-t)}]e^{r^c(T-t)}$

Value

$V_t = S_t{}^{-\delta(T-t)} - F_0e^{-r^c(T-t)}$

At Expiration

Price

$F_T = S_T$

Value

$V_T = S_T - F_0$

Example

A portfolio manager expects to purchase a portfolio of shares in 90 days. In order to hedge against a potential price increase over the next 90 days she decides to take a long position on a 90 day forward contract on the S&P 500 share index. The index is currently at 1245. The continuously compounded yield is 1.75 percent The discreet risk free rate is 4.25%

i) Calculate the no-arbitrage forward price on this contract at inception and the value at inception
ii) It is now 28 days since the manager entered into the contract. The index value is 1325. Calculate the value of the contract 28 days into the contract.
iii) At expiration, the index is 1335. Calculate the value of the forward contract

Answer

First calculate the continuous rate for the risk free rate as follows:

$$r^c = m\text{Ln}\left(1+\frac{r^d}{m}\right) = \text{Ln}(1.0425) = 0.0416$$

At Inception
Price

$F_0 = (S_0 e^{-\delta T})e^{r^c T} = (1245)(2.7183)^{-0.0175(\frac{90}{365})}(2.7183)^{0.0416(\frac{90}{365})}$

$F_0 = (1245)(2.7183)^{-0.0175(0.2466)}(2.7183)^{0.0416(0.2466)}$

$F_0 = 1245(2.7183)^{-0.0043}2.7183^{0.0103} = 1245(0.9966)(1.0104) = 1253.6710$

Value
The value of the contract at inception is zero

28 Days Later
Value

$V_t = S_t^{-\delta(T-t)} - F_0 e^{-r^c(T-t)}$

LO 2.17: Calculate the price and value of a forward contract on a fixed-income security.

The price and value of fixed income securities is not very different from equity forward contracts

To price a fixed interest forward contract we take the bonds spot price, subtract the present value of the coupons over the life of the contract, and compound this amount at the risk free rate to the expiration of the contract. As we see this is exactly the same as pricing an equity contract except that the income is in the form of a dividend.

To value a fixed income forward contract we take the bonds spot price, subtract the present value of the coupons and subtract the present value of the delivery price that will be paid at the contract's expiration.

Again note the following about bonds:
- Not all bonds pay coupons i.e. these zero coupon bonds
- Forward contracts can be taken out on individual bonds, portfolios of bonds or bond indexes
- If a bond produces income it is in the form of coupons and the amount is in dollars (rands) However income on bond portfolios and indexes are quoted as yields

LO 2.17.1: Calculate the price and value of a forward contract on a zero coupon bond. (i.e. a non income producing asset)

At Inception
Price
$F_0 = S_0(1+r)^T$ Remember that when the deal is struck then this price becomes the delivery price (or contract price)
Value
Value of a forward contract at inception is zero

At some random point between inception and expiration
Price
$F_t = S_t(1+r)^{T-t}$
Value

$V_t = S_t - \frac{F_0}{(1+r)^{T-t}}$

At expiration
Price
$F_T = S_T$
Value
$V_T = S_T - F_0$

LO 2.17.2: Calculate the price and value of a forward contract on an income producing bond

At Inception
Price
$F_0 = [S_0 - PV(Cpn)](1+r)^T$ Remember that when the deal is struck then this price becomes the delivery price (or contract price)
Value
Value of a forward contract at inception is zero

At some random point between inception and expiration
Price
$F_t = [S_t - PV(Cpn)](1+r)^{T-t}$
Or
$F_t = S_t(1+r)^{T-t} - FV(Cpn)$
Value

$V_t = [S_t - PV(Cpn)] - \frac{F_0}{(1+r)^{T-t}}$

Or

$V_t = S_t - \frac{F_0}{(1+r)^{T-t}} - FV(Cpn)$

At expiration
Price
$F_T = S_T$
Value
$V_T = S_T - F_0$

Example

A 6% Treasury bond is trading at $1,044 per $1,000 of face value. It will make a coupon payment 98 days from now. The yield curve is flat at 5% over the next 150 days. Calculate the forward price per $1,000 of face value for a 120 day forward contract.

Answer

$$\text{Coupon} = 1000 \times 0.06 = 60/2 = 30$$

$$PV(Cpn) = \frac{30}{(1.05)^{\frac{98}{365}}} = \frac{30}{(1.05)^{0.2685}} = \frac{30}{1.0132} = 29.6092$$

$$F_t = (S_t - PV(Cpn))[1+r]^{T-t} = (1044 - 29.6092)(1.05)^{\frac{120}{365}} = 1014.3908(1.05)^{0.3288} = 1030.7951$$

LO 2.17.3: Calculate the price and value of a forward contract on a portfolio of bonds and a bond index

Formulas
At Inception

Price

$F_0 = [S_0 e^{-\delta^c T}]e^{r^c T}$

The term in the square brackets is the bond price discounted at the coupon yield rate which is equilivant to the bond price minus the present value of the coupons. This value is then compounded at the risk free rate over the life of the contract just as we have done previously.

Value

The value of a forward contract at inception is zero

At some point between inception and expiration

Price

$F_t = [S_t e^{-\delta^c (T-t)}]e^{r^c (T-t)}$

Value

$V_t = S_t^{-\delta(T-t)} - F_0 e^{-r^c (T-t)}$

At Expiration

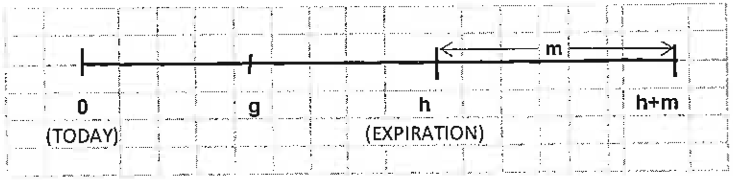
Price

$F_T = S_T$

Value

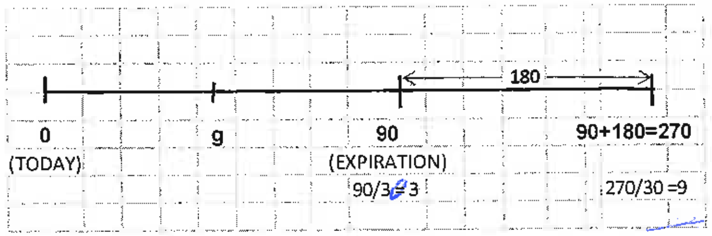
$V_T = S_T - F_0$

LO 2.18: Calculate the price and value of an FRA.



0 = Contract inception
g = Some random point between inception (o) and FRA expiration (h)
h = FRA Expiration date
m= The term of the underlying
h+m = the time from FRA expiration to until the maturity date of the underlying instrument on which the FRA is based

Now let's consider the nature of FRA's.The best way to consider FRA's is to use and example. Assume that you are the treasurer of a company that wants to borrow $2 million in 90 days time for a period of 180 days. You would be concerned that in 90 days time when you borrow the money the interest rate would have risen thus making your borrowing more expensive. You therefore decide to enter into a FRA in order to lock in the interest rate in 90 days time which rate would apply for the 180 day period. We can consider our scenario on a time line as follows:



The FRA that you will enter into is known as a 3X9 FRA

This FRA would lock in the rate today starting in 90 days time for a period of 180 days

At expiry of the FRA (i.e. 90 days time):

- If the ruling underlying rate i.e. spot rate is above the FRA rate then the broker pays you
- If the ruling underlying rate i.e. spot rate is below the FRA rate then you pay the broker

Now let us consider an example

Example 1

You wish to hedge against an increase in future borrowing costs due to a possible rise in short term interest rates. You decide to hedge against this risk by entering into a long 6X12 FRA. The current term structure for LIBOR is as follows:

Term	Interest Rate
30 day	5.10%
90 day	5.25%
180 day	5.70%
360 day	5.95%

a) Indicate when this 6X12 FRA expires and identify which term of LIBOR this FRA is based on
b) Calculate the rate you would receive on a 6X12 FRA (i.e. the delivery rate of the FRA rate at inception)
c) Suppose that it is now 45 days since you entered into the FRA. The interest rates have risen and the LIBOR term structure is as follows:

Term	Interest Rate
135 day	5.90%
315 day	6.15%

Calculate the market value of this FRA based on a notional principal of $10 million
d) At expiration, the 180 day LIBOR rate is 6.25%. Calculate the payoff on the FRA. Do you receive payment or make payment to the broker?

Answers

Answer a)

A 6X12 FRA expires in 180 days (time h) and is based on a 180 day LIBOR (m=180)

Answer b)

$h = 180$

$m = 180$

$h + m = 360$

$L_0(h+m) = L_0(360) = 0.0595$

$L_0(h) = L_0(180) = 0.057$

$$FRA_0 = \left[\frac{1 + L_0(h+m)\left(\frac{h+m}{360}\right)}{1 + L_0(h)\left(\frac{h}{360}\right)} - 1\right]\left(\frac{360}{m}\right)$$

$$FRA_0 = \left[\frac{1 + 0.0595\left(\frac{360}{360}\right)}{1 + 0.057\left(\frac{180}{360}\right)} - 1\right]\left(\frac{360}{180}\right)$$

$$FRA_0 = \left[\frac{1 + 0.0595}{1 + 0.0285} - 1\right](2)$$

$$FRA_0 = \left[\frac{1.0595}{1.0285} - 1\right](2)$$

$FRA_0 = 0.0603 = 6.03\%$

Answer c)

After 45 days

h=180

m=180

g=45

h − g=180 − 45=135

L + m − g= 315

$L_{45}(h-m)=L_{45}(135)=L_{45}(0.0590)$

$L_{45}(h+m-g)=L_{45}(315)=0.0615$

$$V_g=\frac{1}{1+L_g(h-g)\left(\frac{h-g}{360}\right)}-\frac{1+FRA_0\left(\frac{m}{360}\right)}{1+L_g(h+m-g)\left(\frac{h+m-g}{360}\right)}$$

$$V_{45}=\frac{1}{1+0.0590\left(\frac{135}{360}\right)}-\frac{1+0.0603\left(\frac{180}{360}\right)}{1+0.0615\left(\frac{315}{360}\right)}$$

$$V_{45}=\frac{1}{1.0221}-\frac{1.0302}{1.0538}$$

$V_{45}=0.9784-0.9776$

$V_{45}=0.0008$

For a \$10m notional

10,000,000 x 0.0008 = \$8,000

Answer d)

Pay off

h=180

m=180

$L_{180}(m)=L_{180}(180)=0.0625$

$$FRA_h=\frac{[L_h(m)-FRA_0]\left(\frac{m}{360}\right)}{1+L_h\left(\frac{m}{360}\right)}$$

$$FRA_{180}=\frac{[0.0625-0.0603]\left(\frac{180}{360}\right)}{1+0.0625\left(\frac{180}{360}\right)}$$

$$FRA_{180}=\frac{(0.0022)(0.5)}{1.0313}$$

$$FRA_{180}=\frac{0.0011}{1.0313}$$

$FRA_{180}=0.001067$

On a notional principal of \$10m

10,000,000 x 0.001067 =10,670

Now because the spot rate at expiration (6.25) is greater than the FRA rate (6.03%) the dealer pays you

LO 2.19: Calculate the price and value of a forward contract on a currency.

In this section we assume that foreign exchange rates are quoted on the basis of a direct quote which means how many units of domestic currency per unit of foreign currency
Clearly an indirect quote is how many units of foreign currency per unit of domestic currency

The price (i.e. exchange rate) of a forward contract on a currency is the spot rate discounted at the foreign interest rate over the life of the contract and then compounded at the domestic interest rate to the expiration of the contract

Price of a forward exchange contract

Where interest is quoted on a discreet basis

$$F_0 = \left[\frac{S_0}{(1+r^f)^T}\right](1+r^d)^T$$

Where

r^f = foreign interest rate

r^d = domestic interest rate

Where Interest is quoted on a continuous basis

$$F_0 = \left(S_0 e^{-r^f T}\right) e^{r^d T}$$

Where

r^f = forein interest rate on a continuous basis

r^d = domestic interest rate on a continuous basis

Value of a forward exchange contract

Where interest is quoted on a discreet basis

$$V_t = \left[\frac{S_t}{(1+r^f)^{T-t}}\right] - \frac{F_0}{(1+r^d)^{T-t}}$$

Where interest is quoted on a continuous basis

$$V_t = \left[S_0 e^{-r^f (T-t)}\right] - F_0 e^{r^d (T-t)}$$

Example 1

The US risk free rate is 6% and the Swiss risk free rate is 4%. The spot exchange rate between the US and Switzerland is $0.6667

a) Calculate the continuously compounded rates for the US and Swiss interest rates.
b) Calculate the forward contracts price at inception
c) Calculate the value of the forward contract 25 days into the contract if the spot rate is $0.65

Answer a)

US Interest rate

$$r^c = m\text{Ln}\left(1+\frac{r}{m}\right)$$

$$r^c = 1\,\text{Ln}\left(1+\frac{0.06}{1}\right)$$

$$r^c = 0..0583$$

Swiss Interest rate

$$r^c = m\text{Ln}\left(1+\frac{r}{m}\right)$$

$$r^c = 1\text{Ln}\left(1+\frac{0.04}{1}\right)$$

$$r^c = 0.0392$$

Answer b)

Using discreet rates

$$F_0 = \left(\frac{S_0}{(1+r^f)^T}\right)(1+r)^T$$

$$F_0 = \left(\frac{0.6667}{(1.04)^{\frac{90}{365}}}\right)(1.06)^{\frac{90}{365}}$$

$$F_0 = \left(\frac{0.6667}{1.0097}\right)(1.0145)$$

$$F_0 = 0.6698$$

Using continuous rates

$$F_0 = (S_0 e^{-r^f T})(e^{r T})$$

$$F_0 = (0.6667)(2.7183)^{-0.0392\left(\frac{90}{365}\right)} \text{x}(2.7183)^{0.0583\left(\frac{90}{365}\right)}$$

$$F_0 = 0.6667)(2.7183)^{-0.0392(0.2466)}(2{,}7183)^{0.0584(0.2466)}$$

$$F_0 = (0.6667)(2.7183)^{-0.0097}(2.7183)^{0.0144}$$

$$F_0 = (0.6667)(0.9903)(1.0145)$$

$$F_0 = 0.6698$$

Answer c)

$$V_t = \left[S_0 e^{-r^f(T-t)}\right] - F_0 e^{r^d(T-t)}$$

$$V_t = \left[0.65(2{,}7183)^{-0.0392\left(\frac{65}{365}\right)}\right] - 0{,}6698(2.7183)^{-0{,}0583\left(\frac{65}{365}\right)}$$

$$V_t = \left[0.65(2.7183)^{-0.0392(0.1781)}\right] - 0.6698(2.7183)^{-0.0583(0.1781)}$$

$$V_t = \left[0.65(2{,}7813)^{-0.0070}\right] - 0.6698(2.7183)^{-0.0104}$$

$$V_t = (0.65)(0.9930) - 0.6698(1.0105)$$

$$V_t = 0.6455 - 0.6629$$

$$V_t = -\$0.0174$$

LO 2.20: Determine how credit risk arises in a forward contract and how market value is a measure of the credit risk to a party in a forward contract

Credit risk in a forward contract arises when one of the parties owes a large amount to the other. The market value of the contract is a measure of the net amount the one party owes the other. Remember that derivatives are a zero sum game. Market values can change resulting in the credit risk reducing or in fact moving to the other party

Study unit 3: Futures markets and contracts

LO 3.1: Identify the institutional features that distinguish futures contracts from forward contracts

FORWARD CONTRACTS	FUTURES CONTRACTS
Contracts are traded on commodities, interest bearing securities, shares, and currencies	Contracts exist against the same underlying assets as forward contracts, but there are also contracts on notional assets such as share and bond indices
Exist worldwide in all open economies	Exist in most well developed financial centers
Deals are concluded through any acceptable communication method such as the telephone, Reuters dealing system and the internet	Deals can only be concluded on a formal exchange either telephonically, through open outcry on an exchange floor, or through an electronic network-trading platform
Participants are usually banks, large companies or co-operatives. Brokers are active in facilitating trade between counter-parties	Participants are similar to those in the forward market, although the general public has access to the futures market. Participants must either be members of the exchange or transact as clients of member firms
Default risk exists between the parties to the contracts	The clearing-house of the exchange guarantees the contracts
There is very little secondary market trade in these contracts	These contracts trade actively in the secondary market
The size , delivery terms and expiry of contracts, are tailored to suit the counter-parties	Size, delivery terms and expiry of contracts are standardised by the exchange
Usually terminate with physical delivery of the underlying asset	Most contracts are cash settled with very few terminating in physical delivery of the underlying asset
Traded for any maturity, but most trades occur for periods up to one year	Trade for maturity's of up to one year with few exceptions
Contracts are very rarely subjected to margining	All contracts are margined
The markets are generally self-regulated	The markets are strictly regulated

LO 3.2: Determine the origins of modern futures markets.

Japanese Rice markets in 18th century
Mid 1800's Chicago (CBOT)
Clearing house in 1920's

LO 3.3: Identify the primary characteristics of futures contracts.

These contracts have generally accepted terms and the exchange standardises the instruments. They are liquid with a well developed secondary market (can easily be bought and sold) with very little credit risk

LO 3.4: Differentiate between margin in the securities markets and margin in the futures markets

In the stock market margin means that a loan is taken using the portfolio as collateral

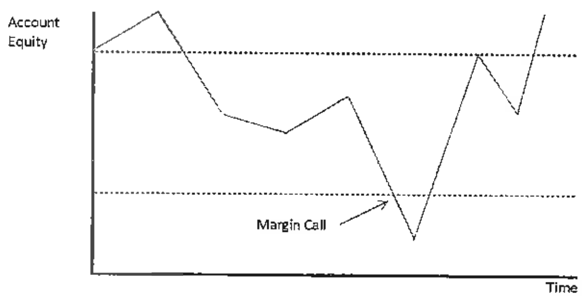
In the futures market the word margin is used to describe the amount of money that must be put into an account by a party opening up a futures position

LO 3.5: Demonstrate how a futures trade takes place and how a futures position may be closed out prior to expiration.

LO 3.6: Interpret initial margin, maintenance margin, variation margin, and settlement price.

There are three types of margins. The first deposit is called the **initial margin**. The initial margin must be posted before any trading takes place. The initial margin is fairly low and equals about one day's maximum price fluctuation. The margin requirement is low because at the end of every day there is a daily settlement process called "marking to market". In the marking to market, any losses for the day are removed from the traders account. If the margin balance in the traders account falls below a certain level (called the **maintenance margin**) the trader will get a **margin call** and will have to deposit more money (called the **variation margin**) into the account to bring the account back up to the initial margin

The initial margin is payable when the futures contract is initially entered into. This margin account then earns interest at a money market related interest rate and the balance is returned to the investor when the contract is eventually closed out



LO 3.7: Formulate the process of marking to market.

As per LO 3.6 above

LO 3.8: Calculate the margin balance given the previous day's balance and the new futures price.

EXAMPLE

Suppose you enter into a three month futures contract to buy 100 tons of maize for R940.00 per ton. You are required to deposit R9 000 in a margin account and you are informed that the maintenance margin is R5 000

(i) What change in the futures price will lead to a margin call

The price must drop to R900.00 a ton (4000/100 = 40 then 940.00 -40.00 =900)

(ii) What happens if the margin call is not met?

The exchange will close out the position

LO 3.9: Interpret price limits, limit move, limit up, limit down and locked limit.

Some futures exchanges impose limits on the price changes that can occur one day to the next and these are called price limits. These limits are usually set as an absolute change over the previous day

Thus the next day's settlement price cannot go beyond the price limit and thus no transaction can take place beyond these limits

If the price at which a transaction would be made exceeds the limits, then the price freezes at one of the limits, which is called a limit move

If the price is stuck at the upper limit it is called limit up.

If the price is stuck at the lower limit it is called limit down.

If a transaction cannot take place because the price would be beyond the limits it is called locked limits

Not all contracts have limits

Exchange can mark to market more than once per day to eliminate chance of default in markets experiencing large movements

LO 3.10: Demonstrate how a futures contract can be terminated by a closeout at expiration, a delivery, an equivalent cash settlement, or an exchange for physical.

LO 3.11: Identify delivery options in futures contracts.

LO 3.12: Distinguish between scalpers, day traders, and position traders.

Scalper – buys at bid and sell at ask price – holds positions for very short periods

Day trader – holds positions but closes out positions at end of day

Position Trader – Holds open positions over night

Day traders and position traders are quite distinct from scalpers in that they attempt to profit from the anticipated directions of the market, scalpers are trying to simply to buy at bid and sell at ask

LO 3.13: Identify the primary characteristics of the following types of futures contracts: Treasury bill, Eurodollar, Treasury bond, stock index, and currency futures.

Eurodollar futures:
Are based on 90 day LIBOR which is an interest add on instrument. The price is quoted as 100 – annualised LIBOR in percent
These contracts settle in cash and the minimum [rice change is on tick which is a price change of 0.0001 = 0.1% or $25 per $1 million contract

Treasury Bond (T bond) futures
Are traded for T Bonds with a maturity of 15 years or more. The contract is deliverable with a face value of $100,000

(The short in the contract has the option to deliver any of several bonds which satisfy the delivery terms of the contract. This option has value to the short and is known as the delivery option. Each bond is given a conversion factor which is used to adjust the longs payment at delivery so that the more valuable bonds receive a higher payment. These factors are multipliers for the futures price at settlement. The long pays the furures at expiry multiplied by the conversion factor)

Stock Index Futures
Are based on the level on an equity index such as Dow Jones Industrial Average

Currency futures
Are set in units of foreign currency and the price is stated in US dollars per unit of forign currency

LO 3.14: Argue why the futures price must converge to the spot price at expiration.

The futures price must converge to the spot price to avoid an arbitrage opportunity in which one can buy the asset and sell a futures or sell an asset and buy a futures to capture an immediate profit at no risk

LO 3.15: Determine the value of a futures contract.

Because of the mark to market process the value of a futures contract just prior to the marking to market process is the accumulated price change since the last mark to market. The value of a futures contract immediately after the mark to market process is zero

LO 3.16: Contrast forward and futures prices.

LO 3.17: Demonstrate how an arbitrage transaction is constructed to derive the futures price.

LO 3.18: Identify the different types of monetary and nonmonetary benefits and costs associated with holding the underlying asset, and determine how they affect the futures price.

LO 3.19: Contrast backwardation and contango.

Contango – The situation where the futures price exceeds the spot price
Backwardation – The situation where the futures price is less than the spot price

LO 3.20: Argue whether futures prices equal expected spot prices.

The most likely likely situation is that futures prices are not predicyors of future spot prices

LO 3.21: Demonstrate how to price Treasury bill futures.

LO 3.22: Assess the concept of an implied repo rate.

LO 3.23: Identify and illustrate the difficulties in determining the price of Eurodollar futures.

LO 3.24: Identify and illustrate how to price Treasury bond futures.

LO 3.25: Identify and illustrate how to price stock index futures.

LO 3.26: Identify and illustrate how to price currency futures.

LO 3.27: Evaluate the role of futures markets and exchanges in financial systems and in society.
The most appropriate use of futures contracts is hedging of risk. Futures cantracts are also used for speculative purposes

Study unit 4: Option markets and contracts

LO 4.1: Identify the basic elements and characteristics of option contracts.

Definition of an option:
An option gives the **holder the right**, but not the obligation to buy or sell a specified quantity and quality of a certain asset within a specified period or on a specific date, at a price agreed (the exercise or strike price) when the contract was entered into
For the holder to obtain this right she pays a premium to the seller

An option that can be exercised within a specified period is known as an **American option**

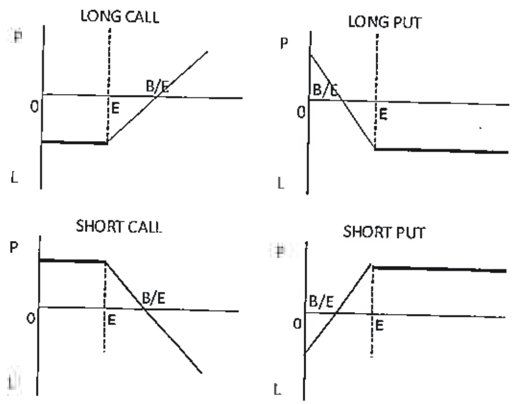
An option that can only be exercised at expiry is a **European option**

An option that gives the holder the **right to buy** the underlying is a **call option**

An option that gives the holder the **right to sell** the underlying is a **put option**

Exercise price (Strike price, striking price or strike) is the price fixed in the options contract at which the **holder** can buy or sell the underlying
The use of this right to buy or sell is referred to as **exercise** or **exercising the option**.
An option has an expiration date thus introducing the notion of an options time to expiration

In observing option prices we observe the following
- Call options have a lower premium the higher the exercise price
- Put options have a lower premium the lower the exercise price
- Both call and put options are cheaper the shorter the time to expiration

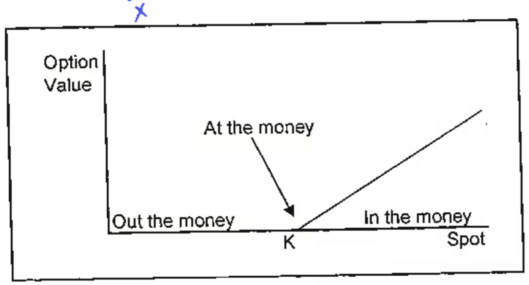


LO 4.2: Assess European option, American option, moneyness, payoff, intrinsic value, and time value.

Options are referred to as "in the money", "at the money" and "out the money"

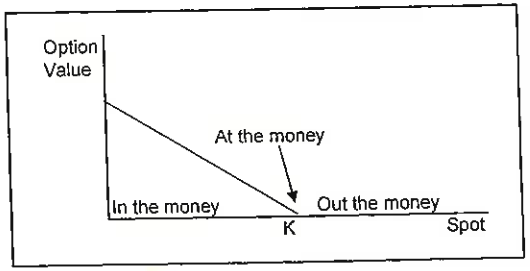
Call options

If S is the stock price (spot) and K is the strike price then:
If S > K the option is in the money
If S = K the option is at the money
If S < K the option is out the money



Put Options

If S is the stock price (spot) and K is the strike price then:
If S < K the option is in the money
If S = K the option is at the money
If S > K the option is out the money



Intrinsic value

It is possible to assess an options value by assessing its intrinsic value and its time value

Intrinsic value is the difference between the options strike price and the spot price of the underlying, or zero

Intrinsic value of a call option = $MAX(0, S_0 - X)$

Intrinsic value of a put option = $MAX(0, X - S_0)$

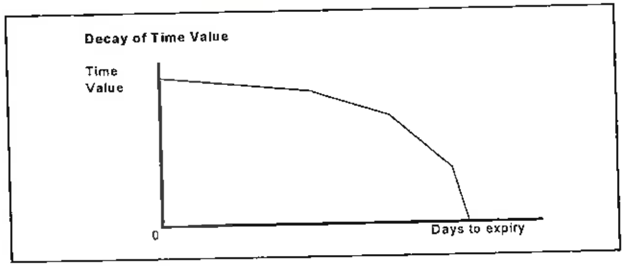
[The above expression is read as follows:
For the intrinsic value of the call "The maximum of 0 or Spot minus Strike"
And for the put "The maximum of zero or Strike minus Spot" This means that the intrinsic value cannot be a negative number]

Time value

Time value reflects the amount of premium in excess of the intrinsic value that you would be prepared to pay in the hope that the option will be worth exercising before it expires.

Even out-the-money options command a price because they may move into the money at some point during the life of the option

Time value does not erode evenly over the life of the option, and the time decay at



the beginning of a long term option is minimal, but increases faster as the option approaches expiry.

The reason for this is that as the option approaches expiry, the likelihood of material price changes diminishes greatly

Example

- You purchase a call option on XYZ share with a strike price of R3.00
- The current market price of the share is R3.20
- You pay a premium of 50c

The following can be deduced

- The call option strike price is below the market price and the option is therefore in the money
- The difference between the strike price and the market price is the intrinsic value i.e. 20c
- The balance i.e. 30c [i.e. 50c – 20c = 30c] is referred to as time value

The factors affecting the time value of an option are:

a) Volatility
b) Time to expiry of the option
c) Strike price of the option in relation to the spot price of the under-lying
d) The risk free interest rate
e) Income derived from the under-lying asset

LO 4.3: Differentiate between exchange-traded options and over-the-counter options.

on option the rest lies on the holder Redo!

LO 4.4: Identify the different varieties of options in terms of the types of instruments underlying them.

Almost anything with a random outcome can have an option on it. In this course we focus on financial options. Financial options are options in which the underlying is a financial asset, an interest rate or a currency

LO 4.5: Compare and contrast interest rate options with forward rate agreements (FRAs).

A FRA is a forward contract in which the underlying is an interest rate. An interest rate option is an option in which the underlying is also an interest rate. Instead of an exercise price it has an exercise rate (strike rate) which is expressed on an order of magnitude of an interest rate. At expiry the option payoff is based on the difference between the underlying rate in the market and the exercise rate. A FRA is a **commitment (obligation)** to make one interest payment and receive another at a future date, an interest rate option is the **right** to make one interest payment and receive another. And just as there are call and put options there is also an interest rate call and an interest rate put.

An interest rate call is an option in which the holder has the right to make a known interest payment and receive an unknown interest payment. The underlying is the unknown interest rate payment
If the unknown underlying interest rate turns out to be higher than the exercise rate at expiration, the option is in the money and is exercised, otherwise the option simply expires.

An interest rate put is an option in which the holder has the right to make an unknown interest payment and receive a known interest payment
If the unknown underlying rate turns out to be lower than the exercise rate at expiration, the option is in the money and is exercised or the option simply expires.
All interest rate option contracts have a specified size, which as in FRA's is called the notional principal

LO 4.6: Determine option payoffs, and differentiate between interest rate option payoffs and payoffs of other types of options.

< >

LO 4.7: Contrast interest rate caps and floors.

LO 4.8: Identify the minimum and maximum values of European options and American options.

An option's value at **expiration** is called its **payoff**

Notation to be used

S_0, S_T = price of the underlying at time 0 (today) and time T (expiration)
X = strike or exercise price
r = risk free rate
T = time to expiration. = number of days to expiration divided by 365
c_0, c_T = price of European call today and at expiration
C_0, C_T = price of American call today and at expiration
p_0, p_T = price of European put today and at expiration
P_0, P_T = price of American put today and at expiration

At expiration a **call option** is worth either zero or the difference between the underlying and the exercise price, whichever is greater

$c_T = Max(0, S_T - X)$
$C_T = Max(0, S_T - X)$

The above expression means the greater of zero or $S_T - X$

At expiration a **put option** is worth either zero or the difference between the exercise price and the underlying price, whichever is greater

$p_T = Max(0, X - S_T)$
$P_T = Max(0, X - S_T)$

The $Max(0, S_T - X)$ for calls or $Max(0, X - S_T)$ for puts is also called the **options intrinsic value** or **exercise value.** Prior to expiry the option will normally sell for more than the intrinsic value. The difference between the market price and the **intrinsic value** is called its **time value** (or **speculative value or its insurance value.)** At expiry the time value is zero (See LO 4.2 above)

LO 4.9: Illustrate how the lower bounds of European calls and puts are determined by constructing portfolio combinations that prevent arbitrage and calculate an Option's lower bound.

In LO 4.8 above we saw that the options value at **expiration** is known as its payoff. Now we will determine the options value at some point prior to expiry

Minimum and Maximum Values

The minimum value of an option

The minimum value of any option is zero

The maximum value of an option.

The maximum value of a call is the current value of the underlying

$c_0 \le S_0, C_0 \le S_0$

A call is a means of buying the underlying. It would not make sense to pay more for the right to buy the underlying than the value of the underlying itself

For put options it makes a difference as to whether the put is European or American

The best outcome for a put holder is that the underlying goes to zero. Then the holder could sell a worthless asset for X.
For an American put option the holder could sell it immediately and capture a value of X.
For a European put option the holder would have to wait until expiration, consequently we must discount X from the expiration day to the present.

Thus the maximum value of a European put is the present value of the exercise price. The maximum value of an American put is the exercise price

$p_0 \le \frac{X}{(1+r)^T}$, $P_0 \le X$

In Summary:

Option	Minimum Value	Maximum Value
European Call	$c_0 \ge 0$	$c_0 \le S_0$
American Call	$C_0 \ge 0$	$C_0 \le S_0$
European Put	$p_0 \ge 0$	$p_0 \le \frac{X}{(1+r)^T}$
American Put	$P_0 \ge 0$	$P_0 \le X$

LO 4.10: Determine the lowest prices of European and American calls and puts based on the rules for lower bounds.

LOWER BOUNDS

We can establish a lower bound on the option price
For **American options**, which are exercisable immediately, we can state that the lower bound of an American option price is its current intrinsic value

$C_0 \ge Max(0, S_0 - X)$
$P_0 \ge Max(0, X - S_0)$

For European call options the lower bound is either zero or the underlying price minus the present value of the exercise price, whichever is the greatest

$$c_0 \ge Max\left(0, \frac{S_0 - X}{(1+r)^T}\right)$$

For European put options the lower bound is the greater of either zero or the present value of the exercise price minus the underlying price

$$p_0 \ge Max\left(0, \frac{X}{(1+r)^T} - S_0\right)$$

Example

Consider call and put options expiring in 90 days, in which the underlying is at 62 and the risk free rate is 5%. The underlying makes no cash payments during the life of the options

a. Find the lower bounds for European calls and puts with exercise prices of 60
b. Find the lower bounds for American calls and puts with exercise prices of 60

Answer

a) European Options

Lower bound of European Call Option

$$c = Max\left[0, S_0 - \frac{X}{(1+r)^T}\right]$$

$$c = MAX\left[0{,}62 - \frac{60}{(1.05)^{\frac{90}{365}}}\right]$$

$$c = MAX\left[0{,}62 - \frac{60}{(1.05)^{0.2466}}\right]$$

$$c = MAX\left[0{,}62 - \frac{60}{1.0121}\right]$$

$$c = MAX[0{,}62 - 58.7659]$$

$$c = MAX[0{,}3.2341]$$

Lower bound of a Euopean Put Option

$$p = MAX\left[0, \frac{X}{(1+r)^T} - S_0\right]$$

$$p = MAX\left[0, \frac{60}{(1.05)^{\frac{90}{365}}} - 62\right]$$

$$p = MAX\left[0, \frac{60}{1.0121} - 62\right]$$

$$p = MAX[0{,}59.2827 - 62]$$

$$p = MAX[0, -2.7173\}$$

$$p = 0$$

b) American Options

Please note that it is never optimal to exercise an American call option on a non income producing asset early i.e. prior to maturity

Lower bound of an American Call Option

$$C_0 = Max\left[0, S_0 - \frac{X}{(1+r)^T}\right]$$

$$C_0 = MAX\left[0{,}62 - \frac{60}{(1.05)^{\frac{90}{365}}}\right]$$

$$C_0 = MAX\left[0{,}62 - \frac{60}{(1.05)^{0.2466}}\right]$$

$$C_0 = MAX\left[0{,}62 - \frac{60}{1.0121}\right]$$

$$C_0 = MAX[0{,}62 - 58.7659]$$

$$C_0 = MAX[0{,}3.2341]$$

Lower Bound of an American Put Option

$$P = MAX[0, X - S_0]$$

$$P = MAX[0{,}60 - 62]$$

$$P = MAX[0, -2.]$$

$$P = 0$$

LO 4.11: Illustrate how a portfolio (combination) of options establishes the relationship between options that differ only by exercise price.

LO 4.12: Determine how option prices are affected by differences in the time to expiration.

Intuitively we would expect a longer term option to be worth more than a shorter dated option because there is a greater probability of the option being in the money We can however state the following rules when considering the impact of time to maturity on the options price:

For European call options
c_0(Longer term option) $\geq c_0$(Shorter term option)

For American call options
C_0(Longer term option) $\geq C_0$(Shorter term option)

For European put options
The longer term European put option may be greater or less than the shorter term European put option

For American put options

P_0(Longer term option) $\geq P_0$(Shorter term option)

LO 4.13: Illustrate how put-call parity for European options is established by comparing the payoffs on a fiduciary call and a protective put and use the result to create synthetic instruments. Argue why an investor would want to do so.

Fiduciary Call

A fiduciary call is a strategy consisting of a European call and a risk free bond that has a maturity date equal to the calls naturity date and a face value equal to the strike price of the call.

If at expiry the spot price is less than the strike price then the option is worth zero and the bond is worth its face value. Combining the positions we have a net amount of X [(0)+X=X]

If at expiry the spot price is greater than the strike price then the option is worth the spot price minus the strike price and the bond is worth its face value. Combining these positions we see that the combination is worth a net amount of S [(S-X)+X=S]

Protective Put

A protective put is a strategy consisting of a European put and the underlying asset.

If at expiry the spot price is less than the strike price then the put is worth X-S and the underlying is worth S Combining these positions we see that the combinantion is worth a net amount of X [(X-S)+S=X

If at expiry the spot price is above the strike price the the option is worth zero and the underlying is worth spot Combining the positions we see that the combination is worth S [(0)+S=S]

From the above we see that:

$$c_0 + \frac{X}{(1+r)^T} = \text{either X or Spot}$$

$$p_0 + S_0 = \text{either X or spot}$$

Therefore

$$c_0 + \frac{X}{(1+r)^T} = p_0 + S_0$$

This is known as the put call parity theorum

Synthetics

By rearranging the variables in the above equation we are able to create synthetic positions as follows

Synthetic call

$$c_0 = p_0 + S_0 - \frac{X}{(1+r)^T}$$

Synthetic put

$$p_0 = c_0 + \frac{X}{(1+r)^T} + S_0$$

Synthetic Spot

$$S_0 = p_0 - c_0 - \frac{X}{(1+r)^T}$$

Synthetic Bond(cash)

$$\frac{X}{(1+r)^T} = p_0 + S_0 - c_0$$

LO 4.14: Illustrate how violations of put-call parity for European options can be exploited and how those violations are eliminated.

Note that the put call parity therom is an equation which tells us that the equation must always balance. If the equation does not balance then there is an arbitrage opportunity. The best way to demonstrate this is to consider the following example:

European put and call options with an exercise price of 45 expires in 115 days. The underlying is priced at 48 and makes no cash payments during the life of the options. The risk free rate is 4.5%. The put is selling 3.75 and the call is selling for 8.00

a. Identify the mispricing by comparing the price of the actual call with the price of the synthetic call
b. Based on your answer in a. demonstrate how an arbitrage transaction is executed

Firstly we test to see if the put call parity holds

$$c_0 + \frac{X}{(1+r)^T} = p_0 + S_0$$

$$8 + \frac{45}{(1.045)^{\frac{115}{365}}} = 3.75 + 48$$

45

$$8 + \frac{45}{(1.045)^{0.3151}} = 51.75$$

$$8 + \frac{45}{1.0141} = 51.75$$

$$8 + 44.3743 = 51.75$$

$$52.3743 \neq 51.75$$

Now we will apply the oldest trading rule in the world that tells us to "buy low and sell high"

From the above result we see that the left hand side of the equation is 52.3743 and the right hand side is 51.75
Therefore we will sell the left hand side and buy the right side as follows:
Sell the call and receive 8
Issue a bond and receive 44.3743
This gives us proceeds of 52.3743
Now we purchase the put for 3.75
Purchase the spot at 48

This strategy will result in a riskless profit of 52.3743-51.75=0.6243

Now lets consider the same problem but this lets assume the the price of the call is 5 and all the other information remains the same

European put and call options with an exercise price of 45 expires in 115 days. The underlying is priced at 48 and makes no cash payments during the life of the options. The risk free rate is 4.5%. The put is selling 3.75 and the call is selling for 5.00

a. Identify the mispricing by comparing the price of the actual call with the price of the synthetic call
b. Based on your answer in a. demonstrate how an arbitrage transaction is executed

$$c_0 + \frac{X}{(1+r)^T} = p_0 + S_0$$

$$5 + \frac{45}{(1.045)^{\frac{115}{365}}} = 3.75 + 48$$

45

$$5 + \frac{45}{(1.045)^{0.3151}} = 51.75$$

$$5 + \frac{45}{1.0141} = 51.75$$

$$5 + 44.3743 = 51.75$$

$$49.3743 \neq 51.75$$

Now we see that the left hand side is less than the right side and applying our trading rule sell high buy low we would enter into the following trades:
Sell put and receive 3.75
Sell spot and receive 48
Buy call for 5
This would leave a cash surplus of 46.75
We would invest this amount at the risk free rate of 4.5% for 115 days thus receiving

$$46.75(1.045)^{\frac{115}{365}} = 46.75(1.0140) = 47.4045$$

Now lets consider what would happen if in 115 days time the underlying spot price was greater than the strike price, say 49

This means that we would exercise the call and purchase the underlying for 45 thus leaving us with a profit of 47.4045 – 45 = 2.4045

If the spot price in 115 days was less than 45, say 43 then the following would happen
The put option is in the money from the buyers point of view which means that we would lose 2 in terms of the put (45-43 -2) Remember that a profit for the long is a loss for the short
We would buy the spot to replace the asset we sold at strategy inception for 43
This would leave us with an overall riskless profit of 47.4045 – 43 – 2 =2.4045

LO 4.15: Compare American options with European options in terms of the lower bounds and the possibility of early exercise.

See LO 4.10 and LO 4.12 above

LO 4.16: Assess how cash flows on the underlying asset affect put-call parity and the lower bounds on option prices.

As we saw previously in the course we simply remove the present value of the future income from the spot price of the underlying

Both the lower bounds on puts and calls and the put call parity must be modified to account for cash flows on the underlying asset

We can restate the lower bounds for European options as:

$$c_0 \geq Max\left[0, (S_0 - PV(CF,0,T) - \frac{X}{(1+r)^T}\right]$$

$$p_0 \geq Max\left[0, \frac{X}{(1+r)^T} - [S_0 - PV(cf,0,T)]\right]$$

The put call parity is restated as follows:

$$c_0 + \frac{X}{(1+r)^T} = p_0 + [S_0 - PV(CF,0,T)]$$

LO 4.17: Identify the directional effect of an interest rate change on an option's price.

When interest rates are higher, call options prices are higher and put option prices are lower

Higher volatility increases call and put option prices because it increases possible upside values and increases possible downside values of the underlying

LO 4.18: Determine an option price and illustrate how an arbitrage opportunity can be exploited in a one-period binomial model.

Binomial suggests two alternatives viz. the underlying price can either increase or decrease and in this case over one period (i.e. this is a one period binomial model)

Notation

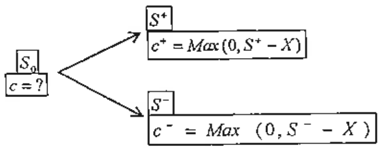
S_0 =Current spot price

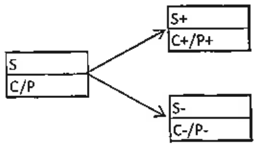
S^+ =One period later price moves up

S^- =One period later price moves down

X=Strike price

r=Risk free rate





$u=\frac{S^+}{S}$

$d=\frac{S^-}{S}$

$d<(1+r)<u$

We start by constructing an arbitrage portfolio by shorting one call option and purchasing an unspecified number of the underlying. Let that number be n. We can calculate the value of n (which is sometimes referred to as the hedge ratio)

The portfolio's current value is H where:

$H=nS-c$

One period later the portfolio will be either H^+ or H^-

$H^+=cS^+-c^+$

$H^-=cS^--c^-$

We can choose the value of n. Let's do so by setting $H^+ = H^-$

Thus

$H^+=H^-$ which means that:

$nS^+-c^+=nS^--c^-$

Therefore

$n=\frac{c^+-c^-}{S^+-S^-}$

The portfolio is thus hedged and should grow at the risk free rate of return. Thus

$H^+=H(1+r)$ or

$H^-=H(1+r)$

We know that $H^+=nS^+-c^+, H^-=nS^--c^-$, and $H=nS-c$.

We also know the values of n, S^+, S^-, c^+, c^- and r.

By Substitution we obtain the value of a call option as follows:

Value of a European Call Option

$$c=\frac{\pi c^+ +(1-\pi)c^-}{(1+r)}$$

Where

$$\pi=\frac{(1+r)-d}{u-d}$$

Using the same logic we can obtain the Value of a European Put Option

$$p=\frac{\pi p^+ +(1-\pi)p^-}{(1+r)}$$

Where

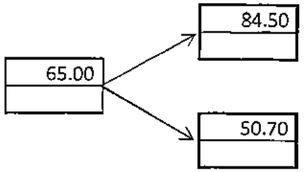
$$\pi=\frac{(1+r)-d}{u-d}$$

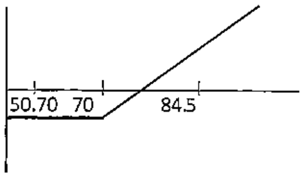
Example

Consider a one period binomial model in which the underlying is at 65 and can go up by 30% or down by 22%. The risk free rate is 8%

a. Determine the price of a European call option with an exercise price of 70
b. Assume that the call is selling for 9 in the market. Demonstrate how to execute an arbitrage transaction and calculate the rate of return. Use 10,000 call options in your calculations.

a. Value the call





u=1.30

d=(1−0.22)=0.78

$S^+ = Su = 65(1.30) = 84.50$

$S^- = Sd = 65(0.78) = 50.70$

Therefore

$c^+ = Max(0, 84.50 - 70) = 14.50$

$c^- = Max(0, 50.70 - 70) = 0$

The risk neutral probability is

$$\pi = \frac{1.08 - 0.78}{1.30 - 0.78} = 0.5769$$

$(1-\pi) = 1 - 0.5769 = 0.4231$

The callprice today is

$$c = \frac{\pi c^+ + (1-\pi)c^-}{(1+r)} = c = \frac{0.5769(14.50) + 0.4231(0)}{1.08} = 7.75$$

b Arbitrage opportunity

We need the value of n for calls

$$n = \frac{c^+ - c^-}{S^+ - S^-} = \frac{14.50 - 0}{84.50 - 50.70} = 0.4290$$

The call is overpriced (9 vs 7.75) so we buy low and sell high by selling 10,000 call options and purchasing 4,290 units of the underlying

Cash flows are as follows:

Sell 10,000 calls at 9	+ 90,000

Buy 4290 units of underlying at 65	-278,850
Net cash flow	-188,850

The value of this combination at expiration will be as follows

If $S_T = 85.50$

$4,290(84,50) - (10,000)(14.50) = 217,505$

If $S_T = 50.70$

$4,290(50.70) - (10,000)(0) = 217,503$

The difference is due to a rounding error

The rate of return is

$$\frac{217,505}{188,850} - 1 = 0.1517$$

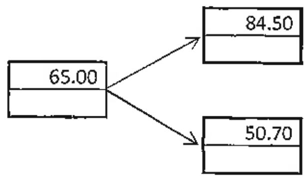
We therefore receive a risk free rate of return almost twice the risk free rate. We could borrow the initial outlay of $188,850 at the risk free rate and capture a risk free profit without any net investment of money

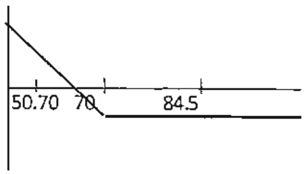
Valuation of a European put option

Example

Consider a one period binomial model in which the underlying is at 65 and can go up by 30% or down by 22%. The risk free rate is 8%

a. Determine the price of a European put option with an exercise price of 70





$u=1.30$

$d=(1-0.22)=0.78$

$S^{+} = Su=65(1.30)=84.50$

$S^{-} = Sd=65(0.78)=50.70$

Therefore

$p^{+} = Max(0,70-84.50)=0$

$p^{-} = Max(0,70-50.70)=19.30$

The risk neutral probability is

$$\pi = \frac{1.08-0.78}{1.30-0.78} = 0.5769$$

$(1-\pi)=1-0.5769=0.4231$

The callprice today is

$$p=\frac{\pi p^{+} + (1-\pi)p^{-}}{(1+r)} = c = \frac{0.5769(0)+0.4231(19.30)}{1.08} = 7.56$$

LO 4.19: Determine an option price in a two-stage binomial model.

Example

Consider a two period binomial model in which the underlying is at 30 and can go up by 14% or down by 11% each period. The risk free rate is 3 % per period.

a. Find the value of a European call option expiring in two periods with an exercise price of 30
b. Find the number of units of the underlying that would be required at each point in the binomial tree to construct a risk free hedge using 10,000 calls

a. Value the call

First find the underlying prices in the binomial tree. We have u=1.14 and d=(1-0.11) = 0.89

$S^{+} = Su = 30(1.14) = 34.20$

$S^{-} = Sd = 30(0.89) = 26.70$

$S^{++} = Su^2 = 30(1.14)^2 = 38.99$

$S^{+-} = Sud = 30(1.14)(0.89) = 30.44$

$S^{--} = Sd^2 = 30(0.89)^2 = 23.76$

Then find the option prices at expiration

$c^{++} = Max(0, 38.99-30) = 8.99$

$c^{+-} = Max(0, 30.44-30) = 0.44$

$c^{--} = Max(0, 23.76-30) = 0$

We need the value of π

$$\pi = \frac{1.03-0.89}{1.14-0.89} = 0.56$$

$(1-\pi)=(1-0.56)=0.44$

$$c^{+} = \frac{\pi c^{++} + (1-\pi)c^{+-}}{(1+r)} = \frac{0.56(8.99)+0.44(0.44)}{1.03} = 5.08$$

$$c^{-} = \frac{\pi c^{+-} + (1-\pi)c^{--}}{(1+r)} = \frac{0.56(0.44)+0.44(0)}{1.03} = 0.24$$

The price today is

$$c = \frac{\pi c^{+} + (1-\pi)c^{-}}{(1+r)} = \frac{0.56(5.08)+0.44(0.24)}{1.03} = 2.86$$

b. Arbitrage opportunity

The number of units of the underlying at each point in the tree is found by first computing the value of n

$$n = \frac{c^{+}-c^{-}}{Su-Sd} = \frac{5.08-0.24}{34.20-26.70} = 0.6453$$

$$n^{+} = \frac{c^{++}-c^{+-}}{Suu-Sud} = \frac{8.99-0.44}{38.99-30.44} = 1.00$$

$$n^{-} = \frac{c^{+-}-c^{--}}{Sud-Sdd} = \frac{0.44-0}{30.44-23.76} = 0.0659$$

The number of units of the underlying required for 10,000 calls would thus be 6,453 today, 10,000 at time 1 if the underlying is at 34.20 and 659 at time 1 if the underlying is at 26.70

LO 4.20: Calculate prices of options on bonds and interest rate options in one- and two-period binomial models.

LO 4.21: Determine how the binomial model value converges as time periods are added.

The more stages we put into the model the more accurate the model outcomes become Ideally we would like an infinite number of stages (i.e. continuous)

LO 4.22: Identify and assess the assumptions underlying the Black-Scholes-Merton model.

The following are the assumptions underlying the BSM model:

- Share price behaviour corresponds to the log-normal model with the mean and standard deviation constant
- There are no transaction costs or taxes. All securities are perfectly divisible
- There are no dividends on the stock during the life of the option
- There are no risk-less arbitrage opportunities
- Security trading is continuous
- Investors can borrow or lend at the same risk free rate of interest
- The short term risk rate of interest is constant and known

LO 4.23: Calculate the value of a European option using the Black-Scholes-Merton model.

$$c = S_0N(d_1) - Xe^{-r^cT}N(d_2)$$

$$p = Xe^{-r^cT}[1-N(d_2)] - S_0[1-N(d_1)]$$

where

$$d_1 = \frac{\ln\left(\frac{S_0}{X}\right) + \left(r^c + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

σ = the annualised standard deviation of the continuously compounded return on the share

r^c = the continuously compounded risk free rate of return

Exercise
Use the Black Scholes Merton model to calculate the prices of European call and pit options on an asset priced at 68.5. The exercise price is 65 the continuously compounded risk free rate is 4%, the options expire in 110 days and the volatility is 0.38. There are no cash flows on the underlying

Solution
The time to expiration will be T=110/365 = 0.3014

$$d_1 = \frac{\ln\left(\frac{68.5}{65}\right) + \left(0.04 + \frac{0.38^2}{2}(0.3014)\right)}{0.38\sqrt{0.3014}} = 0.4135$$

$$d_2 = 0.4135 - 0.38\sqrt{0.3104} = 0.2049$$

Looking up in the normal probability table we have

N(0.41)=0.6591
N(0.20)=0.5793

Plugging into the option price formula

$$c = 68.5(0.6591) - 65e^{-0.04(0.3014)}(0.5793) = 7.95$$

$$p = 65e^{-0.04(0.3014)}(1-0.5793) - 68.5(1-0.6591) = 3.67$$

LO4.24: Determine how an option price, as represented by the Black-Scholes-Merton model, is affected by each of the input values (the Greeks).

LO 4.25: Illustrate and interpret the concept of an option's delta and how it is used in dynamic hedging.

LO 4.26: Determine the gamma effect on an option's price and delta.

LO 4.27: Determine how cash flows on the underlying asset affect an option's price.

We simply use $S_0 - PV(CF)$ in the Black Scholes Merton model instead of S_0
We also used continuous compounding to express the cash flows – for shares we used a continuously compounded dividend rate and for currencies we use a continuously compounded interest rate.

Example
Use the BSM model adjusted for cash flows on the underlying to calculate the price of a call option in which the underlying is priced at 225, the exercise price is 200, the continuously compounded risk free rte is 5.25% the time to expiration is three years

and the volatility is 0.15. The effect of cash flows on the underlying is indicated below for two alternative approaches :

a. The present value of the cash flows over the life of the option is 19.72
b. The continuously compounded dividend yield is 2.7%

Solutions

a. Adjust the price of the underlying to $225 - 19.72 = 205.28$

$$d_1 = \frac{\ln\left(\frac{205.25}{200}\right) + \left[0.0525 + \frac{0.15^2}{2}\right]3.0}{0.15\sqrt{3.0}} = 0.8364$$

$d_2 = 0.8364 - 0.15\sqrt{0.15} = 0.5766$

$N(0.84) = 0.7995$

$N(0.58) = 0.7190$

$c = 205.28(0.7995) - 200e^{-0.0525(3.0)}(0.7190) = 41.28$

b. Adjust the price of the underlying to $S_0 = 225e^{-0.027(3.0)} = 207.49$

$$d_1 = \frac{\ln\left(\frac{207.49}{200}\right) + \left[0.0525 + \frac{0.15^2}{2}\right]3.0}{0.15\sqrt{3.0}} = 0.8776$$

$d_2 = 0.8776 - 0.15\sqrt{3.0} = 0.6178$

$N(0.88) = 0.8106$

$N(0.62) = 0.7324$

$c = 207.49(0.8106) - 200e^{-0.0525(3.0)}(0,7324) = 43.06$

LO 4.28: Identify and illustrate the two methods of estimating the volatility of the underlying.

Volatility can be estimated by calculating the standard deviation of the continuously compounded returns from a sample of recent data for the underlying. This is called the historical volatility.
An alternative measure, called the implied volatility, can be obtained by setting the Black Scholes Merton model price equal to the market price and inferring the volatility. The implied volatility is a measure of the volatility the market is uding to price the option

LO 4.29: Illustrate how put-call parity for options on forwards (or futures) is established.

The payoffs of a call on a forward contract and an appropriately chosen zero coupon bond are equivalent to the payoffs of a put on the forward contract and the forward contract, Thus, their current values must be the same. For this equality to occur, the call price plus the bond price must equal the put price, The appropriate zero coupon bond is one with a face value equal to the exercise price minus the forward price. This relationship is called the put call forward (or futures) parity

Portfolio Combination for Equivalent Packages of Puts, Calls and Forward Contracts (Put-Call Parity for Forward Contracts)

Transaction	Current Value	Value at expiration if $S_T \le X$	Value at expiration if $S_T > X$
Call and Bond			
Buy call	c_0	0	$S_T - X$
Buy bond	$\left[\frac{X - F(0,T)}{(1+r)^T}\right]$	$X - F(0,T)$	$X - F(0,T)$
Total	$c_0 + \frac{X - F(0,T)}{(1+r)^T}$	$X - F(0,T)$	$S_T - F(0,T)$
Put and Forward			
Buy put	p_0	$X - S_T$	0
Buy Forward Contract	0	$S_T - F(0,T)$	$S_T - F(0,T)$
Total	p_0	$X - F(0,T)$	$S_T - F(0,T)$

Forward Contract and Synthetic Forward Contract

Transaction	Current Value	Value at expiration $S_T \le X$	Value at expiration $S_T > X$
Forward Contract			
Long Forward Contract	0	$S_T - F(0,T)$	$S_T - F(0,T)$
Synthetic Forward Contract			
Buy call	c_0	0	$S_T - X$
Sell put	$-p_0$	$-(X - S_T)$	0
Buy (or sell) bond	$\frac{X - F(0,T)}{(1+r)^T}$	$X - F(0,T)$	$X - F(0,T)$
Total	$c_0 - p_0 + \frac{X - F(0,T)}{(1+r)^T}$	$S_T - F(0,T)$	$S_T - F(0,T)$

Example

Determine if a forward contract is correctly priced by using put call forward parity. The option exercise price is 90, the risk free rate is 5%, the options and the forward contract expire in two years, the call price is 15.25, the put price is 3.00 and the forward price is 101.43

Solution

$$p_0 = c_0 + \frac{X - F(0,T)}{(1+r)^T}$$

$$p_0 = 15.25 + \frac{90 - 101.43}{1.05^{2.0}}$$

$$p_0 = 4.88$$

The put is selling for 3.00 and is therefore underpriced. Buy low sell high

Buy put = - 3.00 [cash outflow]

Sell call = +15.25 [cash inflow]

Buy bond = $\frac{90 - 101.43}{(105)^{2.0}} = -10.37$ [cash outflow]

Thus the transaction brings 1.88 up front [15.25 – 3 – 10.37=1.88]

LO 4.30: Identify the similarities in American options on forwards and futures, and differentiate them from European options.

There is no justification for exercising American options on forward contracts early, so they are equivalent to European options on forwards. American options on futures, both calls and puts, can sometimes be exercised early, so they are different from European options on futures and carry a higher price.

LO 4.31: Calculate the value of a European option on forwards (or futures) using the Black model.

$$c = e^{-r^cT}[f_0(T)N(d_1) - XN(d_2)]$$

$$p = e^{-r^cT}(X[1 - N(d_2)] - f_0(T)[1 - N(d_1)])$$

where

$$d_1 = \frac{\ln\frac{f_0(T)}{X} + \left(\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

$f_0(T)$ = Futures price

Example

The price of a forward contract is 139.19. A European option on the forward contract expires in 215 days. The exercise price is 125. The continuously compounded risk free rate is 4.25%. The volatility is 0.15

a. Use the Black model to determine the price of the call option
b. Determine the price of the underlying from the above information and use the Black Scholes Merton model to show that the price of an option on the underlying is the same as the price of the option on the forward

Solution

a. The time to expiry is T = 215/365 = 0.5890

$$d_1 = \frac{\ln\frac{139.19}{125} + \left(\frac{0.15^2}{2}\right)(0.5890}{0.15\sqrt{0.5890}} = 0.9916$$

$d_2 = 0.9916 - 0.15\sqrt{0.5890} = 0.8765$

$N(0.99) = 0.8389$

$N(0.88) = 0.8106$

$c = e^{-0.0425(0.5890)}[139.10(0.8389) - 125(0.8106)] = 15.06$

b. The forward price on the underlying (with no cash flows) is

$F(0,T) = S_0 e^{r^c T}$

By adjusting the formula we get the spot price

$S_0 = F(0,T)e^{-r^c T} = 139.19e^{-0.0425(0.5890)} = 135.75$

Now using the BSM model we get

$$d_1 = \frac{\ln\frac{135.75}{125} + \left(0.0425 + \frac{0.15^2}{2}\right)(0.5890)}{0.15\sqrt{0.5890}} = 0.9916$$

$d_2 = 0.9916 - 0.15\sqrt{0.5890} = 0.8765$

These are as per above thus $N(d_1)$ and $N(d_2)$ are the same

$c = 135.75(0.8389) - 125e^{-0.425(0.5890)}(0.8106) = 15.06$

This price is the same as part a.

LO 4.32: Calculate the value of a European interest rate option using the Black model.

The Black model can be used to price European options on interest rates by entering the forward interest rate into the model for the forward or futures price and the exercise rate for the exercise price

Example
Use the Black model to price an interest rate put that expires in 280 days. The forward rate is currently 6.8%, the 280 day continuously compounded risk free rate is 6.25%, the exercise rate is 7% and the volatility is 0.02. The option is based on a 180 day underlying rate, and the notional principal is $10 million

Solution

The time to expiry is T=280/365=0.7671

$$d_1 = \frac{\ln\frac{0.068}{0.07} + \left(\frac{0.02^2}{2}\right)(0.7671}{0.02\sqrt{0.7671}} = 1.6461$$

$d_2 = 1.6461 - 0.02\sqrt{0.7671} = 1.6636$

$N(-1.65) = 1 - N(1.65) = 1 - 0.9505 = 0.0495$

$N(-1.66) = 1 - N(1.66) = 1 - 0.9515 = 0.0485$

$p_0 = e^{-0.0625(0.7671)}[0.07(1 - 0.0485) - 0.0681(1 - 0.0495)] = 0.00187873$

This formula assumes the option payoff is made at expiry. For an interest rate option this assumption is false. This is a 180 day rate, so the payoff is made 180 days later. Therefore, we discount the payoff over 180 days using the forward rate

$e^{-0.068(\frac{180}{365}}(0.00187873) = 0.00181677$

Interest rate option prices must reflect the fact that the rate used in the formula is quoted as an annual rate. S_0 we must multiply by 180/360 because the transaction is based on a 180 day rate.

$0.00181677\left(\frac{180}{360}\right) = 0.00090839$

Then we multiply the notional principal

10,000,000 x (0.0090839) = $9,084

LO 4.33: Evaluate the role of options markets in financial systems and society.

Study unit 5: Swap markets and contracts

LO 5.1: Identify the characteristics of swap contracts.

A Swap contract is an agreement to exchange future cash. Swaps are Over the Counter instruments.
Swaps are used to exchange cash flows on many different things but typically are based on cash flows stemming from interest payments. In most interest rate swaps one leg of the agreement is based on a variable rate and the other leg on a fixed rate (these swaps are known as plain vanilla swaps)

Swaps are finite instruments meaning that they will expire and payments are made on scheduled dates during the life of the swap. Swaps have a zero value at inception. When payments in the swap are in the same currency then the payments are netted. E.g. party A owes R10 million to party B and party B owes R 8 million to party A. Instead of A paying B R10 million and B paying A 8 million they simply net the payments with the result that party A would simply pay R2 million to B (i.e. 10 million – 8 million) Netting is never applied to foreign currency swaps
Swaps are subject to credit risk by either party

LO 5.2: Demonstrate how swaps are terminated.

Swaps can be terminated:

By one party paying the market value of the swap to the other
By entering into a swap in which the variable payments offset
By selling the swap to another party
By exercising a swaption to enter into an offsetting swap

LO 5.3: Identify the types of currency swaps.

In this course we will deal with Interest Rate Swaps, Currency Swaps and Equity Swaps

In a currency swap each party makes payments to the other in different currencies
Currency swaps can be:

- Fixed to fixed
- Fixed to variable
- Variable to fixed
- Variable to variable

Notional amounts are normally exchanged but this is not mandatory

LO 5.4: Calculate the payments on a currency swap.

The payments on a currency swap are calculated by multiplying the notional principal by the fixed or floating rate times the day count (i.e. a 360 or 365 day year and a 30 day month or actual day month).This is done in each currency with respective parties making separate payments. Payments are not netted

Example
Consider a currency swap in which the domestic party pays a fixed rate in foreign currency, the British pound, and the counterparty pays a fixed rate in US dollars. The notional principals are $50 million and GBP30 million. The fixed rates are 5.6% in dollars and 6.25% in GBP. Both sets of payments are made on the basis of 30 days per month and 365 days per year and the payments are made semi-annually

a. Determine the initial exchange of cash that occurs at the start of the swap
b. Determine the semi-annual payments
c. Determine the final exchange of cash that occurs at the end of the swap
d. Give an example of a situation in which this swap might be appropriate

At inception
Domestic (US) company pays counterparty (UK) $50 million Note that the Domestic company is paying the counterparty in their own currecy and that the counterparty pays the domestic company in their own currency. This happens at inception and the transaction is reversed at contract terminatio (see"At Termination" below).
Counterparty (UK) pays domestic (US) party GBP 30 million.

Semi-annual payments
Domestic party (US) pays the counterparty GBP 30 million (0.625)(180/365) =GBP924,658
Counterparty (UK) pays the domestic party (US) $50 million (0.056)(180/365) = $1,380,822

At Termination
Domestic (US) pays the counterparty (UK) GBP 30 million
Counterparty (UK) pays the domestic (US) $50 million

This swap would be appropriate for a US company that issues a dollar denominated bond but would prefer to borrow GBP or where a company has issued a dual currency bond and wishes to hedge out the currency risk

LO 5.5: Classify a "plain vanilla" interest rate swap.

In a plain vanilla swap , one party makes payments at a fixed rate and the other makes payments at a floating rate, with no exchange of principal.(There is no exchange of principal because both legs of the contract are done in the same currency)
Swaps are often done by a party borrowing variable at a rate tied to a floating rate like LIBOR; that party uses a pay fixed receive variable swap to offset the risk of its exposure to LIBOR and effectively converts its variable rate loan to a fixed rate loan
The other party has an existing obligation to a fixed rate loan and wishes to convert this to a variable rate loan. This can be achieved by entering into receive fixed pay floating irterest rate swap. Note that interest rate swaps can be used to convert fixed to variable rate loans as well as change the nature of the cash flows associated with income from assets.

LO 5.6: Calculate the payments on an interest rate swap.

The payments on an interest rate swap are calculated by multiplying the notional principal by the fixed or floating rate times the day count adjustment. The respective amounts are netted so that the party owing the greater amount makes a net payment to the other

Example
Determine the upcoming payments in a plain vanilla interest rate swap in which the notional principal is $70 million. The end user makes semi-annual fixed payments at the rate of 7% and the dealer makes semi-annual variable payments at Euribor which was 6.25% on the last settlement period. The variable payments are made on the basis of 180 days in the settlement period and 360 days in a year. The fixed payments are made on the basis of 180 days in the settlement period and 365 days in a year. Payments are netted, so determine which party pays which and what amount

Solution
Fixed : $70 million (0.07)(180/365) = $2,416,438
Variable: $70 million (0.0625)(180/360) = $2,187,500

The net payment to be made by the party paying fixed is
2,416,438 – 1,187,500 = $ 228,938

LO 5.7: Identify the types of equity swaps.

Equity options involve one party paying a fixed rate, a variable rate or the return on an equity, while the other party pays an equity return.
This means that an equity swap is a swap in which at least one of the parties pays the return on a share or on a share index

LO 5.8: Calculate the payments on an equity swap.

The equity payment (or payments if both sides of the swap are related to an equity return) is calculated by multiplying the return on the stock over the settlement period by the notional principal.
If there is a fixed or variable payment then it is calculated as for interest rate swaps
Payments in single currencies are netted

Example
A mutual fund has arranged an equity swap with a dealer. The swaps notional principal is $100 million and payments are made semi-annually. The mutual fund agrees to pay the dealer the return on a small cap stock index, and the dealer agrees to pay the mutual fund based on one of the two specifications given below. The small cap index starts off at 1,805.20 and six months later is at 1,796.15

A The dealer pays a fixed rate of 6.75% to the mutual fund with payments made on the basis of 182 days in the period and 365 days in a year. Determine the first payment for both parties, under the assumption of netting, determine the net payment and which party makes it
B The dealer pays the return on a large cap index. The index starts off at 1155.14 and six months later its at 1148.91. Determine the first payment for both parties and, under the assumption of netting, determine the net payment and which party makes it

Solution
A The fixed payment is $100 million (0.0675)(182/365) = $3,365,753

The equity payment is $\left(\frac{1796.15}{1805.20}-1\right)\$100{,}000{,}000 = -\$501{,}329$
Because the fund pays the equity return and the equity return is negative, the dealer must pay the equity return. The dealer also pays the fixed return so the dealer makes both payments which will add up to $3,365,753+501,329 = 3,867,082. The net payment is paid by the dealer to the fund

B The large cap equity payment is
$\left(\frac{1148.91}{1155.14}-1\right)100{,}000{,}000 = -\$539{,}329$
The fund owes -501,329, so the dealer owes the fund 501,329. The dealer owes -$539,329 so the fund owes the dealer 539,329. Therefore the fund pays the dealer the net of 539,329 – 501,329 = $38,000

LO 5.9: Distinguish between the pricing and valuation of swaps.

Let us use a plain vanilla swap as an example. One party agrees to pay floating and receive fixed. At inception of the swap, the fixed rate (i.e. the swap rate or price) is selected such that the present value of the floating rate payments is equal to the present value of the fixed rate payments. This means that the swap has a zero value at inception. This is known as the swap rate and is equilivant to the "price" of the swap.

Now as rates change over the life of the swap, the value of the swap to one party will be positive (an asset) and negative to the other party (a liability)

Stated in a different way:
Swap pricing means to determine the fixed rate and any relevant terms, such as the foreign notional on a currency swap, at the start of the swap.
Valuation means to determine the market value of the swap, which is the present value of one stream of payments less the present value of the other stream of payments.
The market value of a swap is zero at the start but will change to positive for one party and negative for the other during the life of the swap, as market conditions change and time passes

Interest rate swaps

$$B_{60}(90) = \frac{1}{1+0.0425\left(\frac{30}{360}\right)} = 0.9965$$

$$B_{60}(180) = \frac{1}{1+0.0432\left(\frac{120}{360}\right)} = 0.9858$$

$$B_{60}(270) = \frac{1}{1+0.0437\left(\frac{210}{360}\right)} = 0.9751$$

$$B_{60}(360) = \frac{1}{1+0.0444\left(\frac{360}{360}\right)} = 0.9643$$

Now we must value the swap from one of the counterparties perspectives.

Let us value the swap from the party paying fixed and receiving floating's point of view

First we value the fixed leg. We do this by finding the present value of the remaining fixed payments of 0.0092 as follows

0.0092 (0.9965+0.9858+0.9751+0.9643)+1(0.9643) = 1.0004

Now we must find the present value of the variable payments:

On day 0 the 90 day LIBOR is 3.45%. Therefore the first floating payment will be: 0.0345(90/360) = 0.0086.[This is what would be received in 90 days i.e. the first coupon payment date] This payment needs to be discounted back 30 days.[because we have moved sixty days into the contract and there are 30 days left to the coupon date]
What about the remaining payments on the variable rate bond? Remember that the remaining variable rate payments plus the hypothetical notional payment will be equal to 1.
We will therefore discount the payment plus the principal 30 days as follows:
(1+0.0086)(0.9965) = 1.0051

The present value of the fixed leg is 1.0004 and the present value of the variable leg is 1.0051. The value of the swap is therefore:
1.0051-1.0004 = 0.0046 per $1 notional principal

Example 2
Consider a one year interest rate swap with semi-annual payments.

A. Determine the fixed rate on the swap and express it in annualised terms.
The term structure of LIBOR spot rates is given as follows:

Days	Rate
180	7.2%
360	8.0%

B. Ninety days later, the term structure is as follows:

Days	Rate
90	7.1%
270	7.4%

Determine the market value of the swap from the perspective of the party paying the variable rate and receiving fixed rate. Assume a notional principal of $15 million.

Solution

A. First calculate the present value factors for 180 and 360 days

$$B_0(180) = \frac{1}{1+0.072\left(\frac{180}{360}\right)} = 0.9753$$

$$B_0(360) = \frac{1}{1+0.08\left(\frac{360}{360}\right)} = 0.9259$$

The fixed rate is therefore:

$$\frac{1-0.9259}{0.9653-0.9259} = 0.0392$$

The fixed payment would be 0.0392 per $1 notional principal.

The annualised rate would be: $0.0392\left(\frac{360}{180}\right) = 0.0784$

B. Calculate the new present value factors for 90 and 270 days

$$B_{90}(180) = \frac{1}{1+0.071\left(\frac{90}{360}\right)} = 0.9826$$

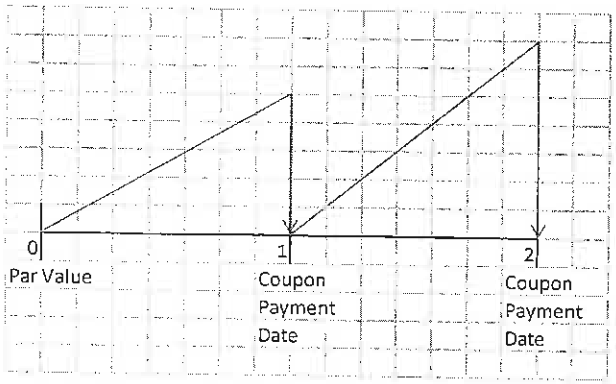
$$B_{90}(360) = \frac{1}{1+0.074\left(\frac{270}{360}\right)} = 0.9474$$

The present value of the remaining fixed payments plus hypothetical $1 notional principal is 0.0392(0.9826+0.9474) + 1(.9474)=1.0231

Calculating and intrepreting the fixed rate on a plain vanilla interest rate swap and the market value during its life.

We can price a plain vanilla (fixed for floating) rate swap by making the assumption that the swap is equilivant to issuing a fixed rate bond and buying an otherwise identical variable rate bond. The fixed rate (the swap rate or price) is set so that the values of the floating rate bond and the fixed rate bond are the same at inception i.e.the value of the swap at inception is zero.

The value of a variable rate bond at inception of the bond is its face value. This value changes over the life of the bond since this rate is reset to the market rate at each payment date and the value returns to par on each of those dates



From the above diagramme we see that the variable rates value at inception is zero. The very next day the bond starts accumulating interest until the coupon payment date is reached (point 1) This coupon is then paid to the bond holder and the bonds value resets to par and the rate for the bonds coupon is reset for the next period. The bond then starts accumulating interest after the last payment date until the next coupon payment date (point 2) At this point the accumulated interest is paid to the bond holder and the bonds value at the coupon payment date resets to par and the bonds coupon rate is reset for the next period and so forth
Thus the variable rate bond will have a value of its par at inception and at each of its interest payment (reset) dates.

Now lets price and value an interest rate swap by using an example:

Example 1
Consider a one year swap with quarterly payments on day 90,180,270 and 360. The underlying is 90 day LIBOR. The annualised LIBOR spot rates today are:

$L_0(90) = 0.0345$

$L_0(180) = 0.0358$

$L_0(270) = 0.0370$

$L_0(360) = 0.0375$

The present value factors are obtained as follows:

First we calculate the discount rates from the givern rates as follows:

$$B_0(90) = \frac{1}{1+0.0345\left(\frac{90}{360}\right)} = 0.9914$$

$$B_0(180) = \frac{1}{1+0.0358\left(\frac{180}{360}\right)} = 0.9824$$

$$B_0(270) = \frac{1}{1+0.0370\left(\frac{270}{360}\right)} = 0.9730$$

$$B_0(360) = \frac{1}{1+0.0375\left(\frac{360}{360}\right)} = 0.9639$$

The fixed payment is found as follows:

$$F_0 = \frac{1-0.9639}{0.9914+0.9824+0.9730+0.9639} = 0.0092$$

Now we calculate the value as follows:

Suppose we entered into the swap and it 60 days later. The new term structure is a follows

$L_{60}(30) = 0.0425$

$L_{60}(120) = 0.0432$

$L_{60}(210) = 0.0437$

$L_{60}(300) = 0.0444$

First we calculate the discount rates as follows:

The 180 day floating rate at the start was 7.2%, so the first variable rate payment would be 0.072(180/360 = 0.036)
The present value of the floating payments plus $1 hypothetical notional principal will be 1.036(0.9826)=1.0180

The value of a pay floating receive fixed is -1.0231+1,0180 = 0.0051.

For a notional principal of $15 million, the market value is:
$15,000,000(0.0051) = $76,500

LO 5.10: Illustrate the equivalence of swaps to combinations of other instruments.

A fixed rate payer swap is equilivant to a series of option positions that pays when floating rates increase and requires a payment to be made when rates fall. This can be accomplished by a series of long interest calls and short interest rate puts with expiration dates the same as the swap payment dates.

A series of short calls and long puts at the same strike price (equal to the fixed rate on the swap) would replicate the payoffs on a receiver swap.

Remember that it takes two options to create an equilivant forward agreement (one to replicate the gains and one to replicate the losses) and that an interest rate swap can be viewed as a series of FRAs

LO 5.11: Determine how interest rate swaps are equivalent to a series of off-market forward rate agreements (FRAs).

LO 5.12: Determine how a plain vanilla swap is equivalent to a combination of an interest rate call and an interest rate put.

Interest rate swaps are like being long (short) interest rate calls and short (long) interest rate puts.

LO 5.13: Determine the fixed rate on a plain vanilla interest rate swap and the market value of the swap during its life.

(See LO 5.9 above for added insight)

The fixed payer on an interest rate swap could gain the same exposure by issuing a fixed rate bond and investing the proceeds in a variable rate bond with the same maturity date and payment dates. On each payment date the fixed payer would pay the fixed interest on the bond and receive the floating rate interest.

We can price an IRS by assuming that the swap is equilivant to issuing a fixed rate bond and investing in a variable rate bond.

The fixed rate must be set such that the values of the floating rate bond and the fixed rate bond are equal at inception. The swap will therefore have a zero value at inception.

At inception a floating rate bond has a value equal to its par value. This value will change over the bonds life because of changing interest rates, however value will return to par on eaxh interest payment date.

Let us investigate a four period bond with a par value of $100. Both the fixed and variable rate bonds must have the same par values to replicate a swap.

Now remember that at inception that variable rate bond has a par value equal to $100.
Now our fixed rate bond must have periodic payments,PMT, such that:

$$\$100 = \frac{PMT}{(1+r_1)} + \frac{PMT}{(1+r_2)} + \frac{PMT}{(1+r_3)} + \frac{PMT}{(1+r_4)} + \frac{100}{(1+r_4)}$$

Let $Z_n = \frac{1}{!+r_n}$ = price of n period zero coupon bond per $ of principal

Now solving for PMT we get:

PMT (which equals the IRS rate at inception i.e. $F_0) = \frac{1-Z_4}{Z_1+Z_2+Z_3+Z_4}$

Example
Consider a one year interest rate swap with semi annual payments

Determine the fixed rate on the swap and express it in annualised terms given that LIBOR spot rates are as follows:

Days	Rate
180	7.2%
360	8.0%

Ninety days later, the term structure is as follows:

Days	Rate
90	7.1%
270	7.4%

Determine the market value of the swap from the perspective of the party paying the floating rate and receiving the fixed rate. Assume a notional principal of $15 million.

Determine the IRS rate at inception

First calculate the relevant discount rates:

$$B_0(180) = \frac{1}{1+0.072\left(\frac{180}{360}\right)} = 0.9653$$

$$B_0(360) = \frac{1}{1+0.08\left(\frac{360}{360}\right)} = 0.9259$$

$$F_0 = \frac{1-0.9259}{(0.9653+0.9259)} = 0.9826$$

The fixed payment would be 0.9826 per $1 per notional principal

The annualised rate would be:

$$0.9826\left(\frac{360}{180}\right) = 0.0784$$

Ninety days later
Now calculate the new discount rates as follows:

$$B_{90}(90) = \frac{1}{1+0.071\left(\frac{90}{360}\right)} = 0.9826$$

$$B_{90}(270) = \frac{1}{1+0.074\left(\frac{270}{360}\right)} = 0.9474$$

The present value of the remaining fixed payments plus hypothetical $1 principal is:

$$0.0392(0.9826+0.9474)+1(0.9474)=1.0231$$

Calculate the present value of the variable rate

The 180 day rate at inception was 7.2% so the floating rate payment to be received 180 days from them was $0.072\left(\frac{180}{360}\right) = 0.036$

The present value of the $1 dollar notional plus interest will be;

$$1.036(0.9826) = 1.080$$

The market value of the pay floating receive fixed swap is

-1.080 (negative for outflow i.e. pay) + 1.0231 (positive for inflow)=0.0051

For a notional principal of $15 million the market value is
$15,000,000 x 0.0051 = $76,500

LO 5.14: Determine the fixed rate, if applicable, and the foreign notional principal for a given domestic notional principal on a currency swap, and

determine the market values of each of the different types of currency swaps during their lives.

With currency swaps there are two yield curves and two swap rates i.e. one for each currency.

The principal amounts must be adjusted for the exchange rate. For example, given a USD/GBP exchange rate of $2/GBP1 and a notional principal of $40 million then the notional amount in GBP would by GBP 20 million

This means that the domestic party (assume US) would pay $40 million to the UK counterparty and the UK counterparty would pay the US counterparty and amount of GBP 20 million at contract inception. This transaction would be reversed at contract termination

Example
Consider a two year currency swap with semi-annual payments. The domestic currency iss UD and the floating currency is GBP. The current excahnge rate is $1.41 per GBP

A. Calculate the annualised fixed rates for dollars and pounds given the following term structures:

USD term	USD rate	GBP term	GBP rate
180	5.85%	180	4.93%
360	6.05%	360	5.05%
540	6.24%	540	5.19%
720	6.65%	720	5.51%

B. Moving forward 120 days. The new exchange rate is $1.35 and the term structures are as follows:

USD term	USD rate	GBP term	GBP rate
60	6.13%	60	5.17%
240	6.29%	240	5.32%
420	6.53%	420	5.68%
600	6.97%	600	5.83%

Calculate the market value of the fixed leg and the variable leg for both the USD's and GBP's

Solution

Part A.
First calculate the discount factors
Dollars