

Tutorial Letter 201/1/2018

Psychological Research

PYC3704

Semester 1

Department of Psychology

- How to cope with formulas
- Feedback for assignment 01

BARCODE

Dear Student

This tutorial letter contains:

- General information
- How to cope with formulas
- Answers and feedback for Assignment 01

1. Some general comments

This Tutorial Letter contains feedback on the first assignment of this semester for PYC3704. The second assignment contains some calculations that you will have to perform. If you struggle to cope with the formulas, we suggest you work carefully through the information given in Section 2 below. You should also work your way through the examples in Appendix E of the Guide if you feel that you need to brush up on your basic numeracy skills.

The team of lecturers who are responsible for PYC3704 for the rest of the semester are listed below.

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Note: If you have e-mailed an academic enquiry to a lecturer, and have not received a reply within a week, please re-submit your queries to Ms Nel, who will forward it to an available lecturer.

If you are interested in joining tutorial classes for this module, you should contact your closest regional Unisa Learning Centre to find out whether the classes are available in your region. You can also contact Mr Khuze Skosana (skosagm@unisa.ac.za or 012 429-2093) who is the liaison in the Psychology Department who deals with the tutor groups.

2. How to cope with formulas



This is a relatively elementary introduction for those of you who find the formulas intimidating. Other students can skip this section.

First, *don't panic*. These formulas may seem scary, but they can usually be solved with a bit of careful work. It takes concentration and attention to detail and becomes easier with practise.

A 'formula' is more properly referred to as an *equation*. This term implies that what is on the left hand side of the formula must be *set equal* to what is written on the right hand side. It represents a concise description of a calculation which you need to perform, displaying the variables you need to consider and the relationships among them. Think of it as a system which tells you how to calculate something in a very precise way. You have to learn how to use it, and this is something you learn by doing, not just by looking at it.

Each symbol in such an equation represents a variable, and you need to find the current value of that variable based on the data which you have. First you substitute the symbol with that number. Then you have to perform the calculation, which is an arithmetic procedure, making use of operations like adding, subtracting, division, multiplication, calculating square roots and so on (see Appendix E in the PYC3704 Guide).

Let us first consider a simple example. Look for at the formula for the mean of a sample of data (given in Appendix C in the Guide):

$$\bar{x} = \frac{1}{n} \sum x$$

The symbol \bar{x} is a conventional way to indicate a *mean* (some books use the symbol M for this). Each of the symbols on the right hand side of the equation is to be replaced by a number. $\sum x$ is a conventional way of writing 'the sum of all the x -values' (see Appendix E in the Guide). To do the calculation you would need to have a set of data representing various measurements of the variable indicated by ' x '. So what this formula tells you is that the mean of the x -values is calculated by adding them up and dividing the result with n (which is equivalent to multiplying the result with $1/n$).

Let us test this on a small set of data (which can be regarded as different measurements of a variable ' x ').

x	14	12	11	22	15	10	15	17	13	15
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Here the sum of x would be: $\Sigma x = 14 + 12 + 11 + 22 + 15 + 10 + 15 + 17 + 13 + 15 = 144$.

Because there are 10 values for x , we know that $n = 10$. So if we substitute this in the formula:

$$\bar{x} = \frac{1}{n} \sum x = \frac{1}{10} \times 144 = 14.4$$

For a more advanced example, let us calculate the value of the sample standard deviation, usually indicated by s . The formula for s (given in Appendix C in the Guide) is as follows:

$$s = \sqrt{\frac{\sum(x - \bar{x})^2}{n - 1}}$$

This formula describes the calculation process which you need to perform to get the standard deviation of the sample of data above. It tells you that you have to subtract the mean (\bar{x}) from each value of x , square the result, and add these together. This total you divide by $n-1$. The square root ($\sqrt{\quad}$) of this result then gives you the value of s . If you do not take the square root here, you will in fact end up with s^2 , which is the sample *variance*.

A good way to calculate a somewhat complicated sum like $\sum(x - \bar{x})^2$ by hand is to first arrange the data in a table, like below:

Measurement no.	x	x - \bar{x}	(x - \bar{x}) ²
1	14	-0.4	0.16
2	12	-2.4	5.76
3	11	-3.4	11.56
4	22	7.6	57.76
5	15	0.6	0.36
6	10	-4.4	19.36
7	15	0.6	0.36
8	17	2.6	6.76
9	13	-1.4	1.96
10	15	0.6	0.36
Sum Σ	144	0.0	104.4

The second column presents the values of x, and the third column shows the values of each x with the mean (which we calculated above as $\bar{x} = 144.4$) subtracted from that value of x. The fourth column gives the *squares* of each of these new values (that is, the column 3 values multiplied with themselves). We are interested in the sum of these (usually referred to as the *sum of squares*), which adds to 104.4, indicated in the bottom row of the table. This result therefore indicates that $\Sigma(x - \bar{x})^2 = 104.4$.

When you use tables like this, make sure you understand how to use this notation. For example, you have to know that if you were working with two variables x and y, Σxy and $\Sigma x \Sigma y$ will *not* produce the same result (see Appendix E in the PYC3704 Guide, especially p. 176).

To determine the standard deviation, we should substitute the sum of squares we calculated above in the formula for s, as follows:

$$s = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n-1}} = \sqrt{\frac{104.4}{10-1}} = \sqrt{\frac{104.4}{9}} = \sqrt{11.6} = 3.406 \text{ (rounded off).}$$

Note how it takes many words to explain this process, and that it can be difficult to express it in a very exact way in words. The formula is a very concise and exact expression of the process, which is why we need to use it.

3. Feedback for Assignment 01 of the first semester of 2018

For your convenience, each question is given followed by the appropriate answer and an explanation of the correct response. Work your way systematically through these, comparing it with your own answer. Even if you chose the correct alternative, you may find that the explanations we give are useful. Try to understand the explanations. Many of the items require insight, not just factual knowledge. You will not pass this course if you try and memorise the questions and answers!

Question 1

In psychological research, a construct may be considered as a(n) - - - - .

1. measurement based on the careful observation of aspects of humans or human behaviour
2. observation of an aspect of humans or human behaviour which was operationalized in some way
3. hypothetical aspect of humans or human behaviour which we wish to investigate
4. explanation of empirical observations based on the measurement of certain variables

→ **Answer:** The correct answer is given in Option 3.

A construct is the aspect of human behaviour or experience that the researcher is interested in. It is often something that cannot be directly observed, but represents a generalisation or abstraction, such as 'intelligence' or 'anxiety' or an 'attitude' (towards something). It is 'hypothetical' in the sense that we suppose it must exist but is not directly observed.

Option 1 refers to a variable (the measurement of the construct), which can be thought of as the result of making an observation. Option 2 is not quite true because operationalization is specifically the process of finding some suitable way to observe the construct and to measure it. So the 'construct' is more the idea which leads to the operationalization to make observation and measurement possible than something which follows it. Option 4 provides a definition of a theory: an explanation of why things are observed to be as they are.

Question 2

Empirical knowledge is knowledge that is based on - - - - .

1. careful reasoning
2. appropriate theories
3. the observation of events
4. published research

→ **Answer:** Option 3 is correct.

The word 'empirical' refers specifically to information (data) which was gathered by observation (see PYC3704 Study Guide p. 2). This would include (for example) data that was gathered by measurements such as by the use of a questionnaire or a psychometric test, or textual data gathered from interviewing people. But reasoning (Option 1) or reading published research regarding the issue under investigation (Option 4) are not regarded as 'observations' as such. Theories are explanations of why the observations are as they are. They are not in themselves observed but are inferred from evaluating relevant hypotheses, which is why Option 2 is not correct.

Question 3

In research, a measurement that stays the same and remains a certain size from case to case is called a - - - - .

1. standard error
2. independent variable
3. constant
4. dependent variable

→ **Answer:** Option 3 is correct.

Numbers that can only take on a certain size are referred to as 'constants', for example, the number of days in a week. This can be contrasted with variable, which are numbers which change from one instance to the next, for example a person's age. (See the PYC3704 Guide, p. 7).

Question 4

In scientific research, the word theory refers to a(n) - - - - .

1. reasonable guess or creative insight which seems to explain a phenomenon
2. method to make the constructs which are involved in a phenomenon visible through a process of operationalisation
3. investigation or procedure which is performed to determine the relationships among variables
4. explanation of why the observations that were made are as they are, or are related in the way that they are related

→ **Answer:** Option 4 is correct.

As explained on page 4 in the Guide for PYC3704, a *theory* is a framework for facts: it is the explanation of why the facts (i.e. observations, measurements) are as they are, or why they are related in the way in which they are related, based on empirical investigations. Option 1 is a description of a hypothesis, but this is often how the word 'theory' is used in informal conversation. Option 2 refers to the process of measurement and option 3 is a description of a process such as an experiment by which a theory can be evaluated but not to a theory as such.

Question 5

In the process of psychological research, researchers try to - - - - .

1. prove that a theory is true
2. account for theoretical assumptions
3. obtain empirical support for a theory
4. change hypotheses into theories

→ **Answer:** Option 3 is correct.

Theories cannot really ever be 'proven' in an absolute sense (there is always room for doubt). In research, psychologists are instead trying to find empirical (observational) support for a theory. The research is not really intended to 'account' for the theory (Option 2), but is rather conducted to show that the theory is supported by the evidence. Hypotheses are developed based on observations (e.g. measurements) and on reasoning about the implications of the theory, and occasionally on inspired guesswork, which implies that a theory follows from the validation of a number of interrelated hypotheses. It is however not specific hypotheses that becomes a theory, as implied in Option 4, but rather a number of hypotheses which, if supported by research, leads to the theory being accepted (which is what is implied in Option 3).

Question 6

An introductory class in statistics has a mixture of students studying for a BA degree and for a BSc degree. There are 9 males and 15 females who are BSc students, as well as 8 males and 12 females who are studying for a BA. If one student is selected at random from this class, what is the probability of it being a BA student?

1. 8/24
2. 20/24
3. 20/44
4. 14/44

→ **Answer:** Option 3 is correct.

There are $9 + 15 + 8 + 12 = 44$ students in total. There are 20 students studying for a BA degree, the sum of 8 males and 12 females. So using the relative frequency method (Section 2.1.1.2 in the PYC3704 Guide), this gives a probability of a random student studying for a BA degree as $20/44$.

Question 7

If a coin is flipped three times, the sample space of possible outcomes (presuming H stands for 'Heads' and T stands for 'Tails') is:

1. HHH; TTT; THT; HTH; HHT; TTH; HTH
2. HTT; THT; HTH; HHH; TTH; TTT
3. HHH; HHT; HTH; HTT; THH; THT; TTH; TTT
4. HHH; HTT; HTH; TTT; HTT; THH; HHT; THT

→ **Answer:** Option 3 is correct.

This question requires that you determine the sample space for three flips of a coin, which in turn means that you must determine all the possible outcomes if a coin is flipped three separate times. Option 3 gives the correct answer, because this is the only option that lists all the possible outcomes. Option 1 does not list THH, Option 2 does not list HHT and THH, and Option 4 does not list TTH.

Question 8

Two pupils, one girl and one boy, will be chosen to go on a field trip from a class of 12 girls and 15 boys. What is the probability that Mary and her brother John (who happen to be in the same class) will both be selected if boys and girls are selected separately at random?

1. 0.0056
2. 0.15
3. 0.0741
4. 0.037

→ **Answer:** Option 1 is correct.

You have to use the basic formula for probability:

$P(A) = (\text{No. of favourable events}) / (\text{No. of possible outcomes})$ separately for boys and girls.

In the case of the girls, we know that there are 12 possible outcomes (any one of the 12 girls can be selected) and we also know that there is only one 'favourable' event (Mary being selected). Therefore, we have:

$$p(\text{Mary}) = 1/12 = 0.0833 \text{ (rounded off)}$$

Similarly, for the boys, there are 15 possible outcomes, so:

$$p(\text{John}) = 1/15 = 0.0667$$

To combine the probabilities for Mary and John, we should use the multiplicative rule (see p. 35 of the PYC3704 Guide):

$$p(\text{Mary AND John}) = p(\text{Mary}) \times p(\text{John}) = 0.0833 \times 0.0667 = 0.0056.$$

Note that since the scenario states 'boys and girls are selected separately at random', this is not a *conditional* probability (the choice of Mary from the group of girls have no effect on the choice of John from the boys) so the formula for conditional probabilities (p. 36 in the Guide) does not apply.

Question 9

A teacher is teaching a class about probabilities. She shows the learners a bowl which contains **3 Red** marbles, **5 Blue** marbles and **7 Yellow** marbles. One of the learners, Vusi, is asked by the teacher to select one marble out of the bowl at random and to hand it to her. She shows the class that Vusi chose a **Blue** marble and she demonstrates to the class how to calculate the probability of this outcome. While holding on to the first marble, she asks Vusi to select another marble in the same way. She now asks the class to calculate the probability that **both** the first **and** the second marble chosen by Vusi would happen to be **Blue**.

Select the *best* estimate out of the options given below.

1. 0.1333
2. 0.1111
3. 0.6190
4. 0.0952

→ **Answer:** Option **4** is the best choice.

There are altogether $3+5+7 = 15$ marbles in the bowl.

Since there are **5 Blue** marbles, it is easy in the first case to apply the classical formula (given on p. 29 of the PYC3704 Guide):

$$p(\text{First marble is BLUE}) = \frac{\text{Number of favourable outcomes}}{\text{Number of possible outcomes}} = \frac{5}{15} = 0.3333$$

However, the outcome in the case of the second marble to be selected by Vusi is not independent of this. When the second marble is to be selected, the class knows that the first marble was blue, and the probability of this outcome can be calculated as above. This is actually an example of a *conditional probability* (see p. 36 in the PYC3704 Guide). Conditional probabilities can be used when additional information that was not previously known is available to improve the estimate.

We know that there are **4 Blue** marbles left out of **14** marbles ($15 - 1$) in total (note that the scenario describes the teacher as "holding on to the first marble"). We can think of this as the probability that 'the Second marble is Blue' *given that* 'the First marble was Blue':

$P(\text{'Second marble is Blue' } | \text{'First marble is Blue'}) = 4/14 = 0.2857$. (Note the use of the vertical bar symbol to signify 'given that').

The overall probability that the *First marble was Blue AND the Second marble is Blue* can then be calculated as follows, using the multiplicative rule for combining conditional probabilities:

$$\begin{aligned} &P(\text{First marble was Blue AND Second marble is Blue}) \\ &= P(\text{First marble is Blue}) \times P(\text{Second marble is Blue } | \text{ First marble was Blue}) \\ &= 0.3333 \times 0.2857 = 0.0952. \end{aligned}$$

Questions 10 to 12 are based on the following research scenario:

A psychologist wants to study how aspects of motivation can influence people's productivity in their work. She reads an article that claims that an important aspect of motivation is locus of control, which distinguishes people who are driven by their own personal ambition from those who act by conforming to a social group. To test this idea, she draws a sample of 100 workers from a number of companies in the information technology sector. She divides the workers into two groups, those with an internal locus of control and those with an external locus of control, based on an appropriate psychometric test. She then uses job evaluation forms to assess the actual work performance of the workers in the two groups.

Question 10

Which of the following gives the best expression of the hypothesis that the researcher wishes to test?

1. Motivation affects the productivity of workers.
2. Internal locus of control is related to high productivity.
3. The job performance of workers is influenced by locus of control.
4. Motivation of workers is influenced by locus of control.

→ **Answer:** Option 3 is correct.

Option 3 is the clearest expression of an appropriate research hypothesis. Option 1 is too vague: 'motivation' is probably too complex to be captured in a single construct. Option 2 is too specific: it is not clear from the information given in the scenario whether internal or external locus of control can be expected to lead to higher productivity. 'Locus of control' is regarded as an aspect of motivation (according to the scenario), not as a separate construct (or variable) that can be compared to it, so Option 4 is also wrong.

Question 11

Given that research is an investigation of a relationship between (two or more) constructs, which of the following constructs have to be compared to do this research?

- (a) job performance
 - (b) motivation
 - (c) locus of control
 - (d) job evaluation forms
 - (e) people who work in the information technology sector
1. (a) and (c)
 2. (a) (b) and (e)
 3. (a) (c) and (e)
 4. (c) and (d)

→ **Answer:** The correct answer is given in Option 1.

'Locus of control' is the specific aspect of 'motivation' that is being studied, which excludes Option 2. In this particular context, 'people who work in the information technology sector' is part of the definition of the population that the researcher chooses to study, not a construct that is being compared to another, so Option 3 is incorrect. Option 4 is incorrect because 'job evaluation forms' are the instruments used to measure the construct 'job performance', and not a construct as such.

Question 12

The dependent variable in the study is - - - - and the independent variable is - - - - .

1. job performance; locus of control
2. locus of control; motivation
3. job performance; productivity
4. locus of control; job performance

→ **Answer:** Option 1 is correct.

The dependent variable is the one that is predicted or explained, and the independent variable is manipulated to see how it affects the dependent variable. In this study, the researcher tries to predict job performance with the aid of locus of control: so the researcher is trying to see if job performance depends, to a significant degree, on locus of control. This rule out Option 4, where it is implied that locus of control depends on job performance. Option 2 is wrong because locus of control is the aspect of motivation that is being studied (it is not a separate variable). In a similar way, job performance is described in the scenario as equivalent to productivity, which excludes Option 3.

Question 13

A Type I Error occurs if - - - - .

1. the null hypothesis is rejected even though it is true
2. the null hypothesis is accepted even though it is false
3. both the null hypothesis and the alternative hypothesis are rejected
4. the null hypothesis is rejected but the effect size is found not to be significant

→ **Answer:** Option 1 is correct.

The risk of rejecting the null hypothesis when this hypothesis is in fact true, and should therefore not be rejected, is referred to as an error of Type I. Note that the whole point of statistics is to affirm or reject hypotheses based on a sample of data and never on all the data (the entire population), so it is always possible to make an error. (See p. 85 in Guide to PYC3704).

Question 14

The z score corresponding to a raw score represents the number of - - - - that the raw score differs from the mean of the raw score distribution.

1. points
2. standard deviations
3. percentiles
4. variances

→ **Answer:** Option 2 is correct.

The z-scores are equivalent to standard deviations on the standard normal distribution, and each unit on the z-scale represent one standard deviation of the raw score. (See section 2.3.3 in the Guide for PYC3704). This implies that $z=1$ is equivalent to one standard deviation of x removed from the raw score mean (μ for a population or \bar{x} for a sample) and $z=2$ is equivalent to two times the standard deviation of x removed from the mean, and so on.

Question 15

Which best describes the *frequency distribution* of the ages of students attending a particular class?

1. A graph of the ages of the students showing the number of students in each of a number of categories, arranged from young to old.
2. The total of all ages, divided by the number of students.
3. The standard deviation of ages, indicating the width of the age distribution.
4. A graph of the ages of the students showing the way that the data is distributed around the mean age of the students relative to the standard deviation.

→ **Answer:** Option 1 is correct.

The expression 'frequency' implies the count of observations in each of a number of categories. A *frequency distribution* of ages will represent the number of students falling in each of a number of age categories, which can be represented graphically in a histogram. Option 2 gives the mean age and Option 3 refers to the standard deviation, and these are not suitable ways of representing a frequency distribution. The frequency distribution will actually give an approximation of the general distribution of the raw scores around the mean as implied in Option 4, but this is relevant only if the raw scores can be assumed to be normally distributed, which is not necessarily true.

Question 16

The lower we set the level of significance, the greater the probability of - - - - .

1. rejecting the null hypothesis
2. a type II error
3. a type I error
4. accepting the alternative hypothesis

→ **Answer:** Option 2 is correct.

The p-value must be less than or equal to the level of significance (α) before the null hypothesis can be rejected. We would choose a low level of significance to reduce the probability of making an error by rejecting the null hypothesis, which is referred to as an error of Type I. So making the level of significance smaller also reduces the probability of this error, which makes Option 3 invalid. A lower level of significance generally reduces the probability of rejecting the null hypothesis (which by implication is the same as accepting the alternative hypothesis), so Option 1 and Option 4 are both invalid.

Question 17

The *sampling distribution of means* refers to the - - - - .

1. normal distribution of a raw score distributed around the mean
2. distribution of the different possible values of the sample means together with their respective probabilities of occurrence
3. distribution of the values of the items in the population
4. distribution of the means of all possible samples of a particular size randomly selected from the same population

→ **Answer:** Option 4 is correct.

Read the discussion of the distribution of sample means in the Guide (pp. 60 - 61). Option 1 is false because it refers to the raw data, not the distribution of means, and Option 2 is incorrect because we typically do not know what the probability of occurrence of a particular sample mean is. Option 3 is too vague to be useful.

Question 18

During the interpretation of psychological measurements the normal distribution is often - - - - .

1. adapted to fit the observed frequency distribution of scores
2. used as a theoretical model for interpreting the observed distribution of scores
3. used to calculate the relative frequency of observed scores
4. used to derive the mean and standard deviation of a sample

→ **Answer:** Option 2 is correct.

Data from the real world are unlikely to follow an exact normal distribution. However, we often find that the normal distribution gives an acceptable approximation for the mean values of any particular distribution of scores as long as the sample is not too small (over 30 or so). This is a consequence of the central limit theorem (see pp. 60 - 62 in the PYC3704 Guide). So if the measurement level is acceptable and the sample size adequate, the normal distribution can be used as a theoretical model for interpreting the observed distribution of scores.

Option 1 does not make much sense, although one could put it the other way round, to say that an observed frequency distribution of scores is often adapted to fit the normal distribution. Option 3 is wrong because the relative frequency of observed scores cannot be *calculated* from the normal distribution. The mean and standard deviation of a sample is calculated from the measurements, not from the normal distribution, so Option 4 is also invalid.

Question 19

The examination marks of statistics course yielded a normal distribution with a mean of 50 and a standard deviation of 10. Determine the probability that the score would be 60 or above, if you were to select the score of one student at random. Into which of the four intervals listed below would the probability fall?

1. 0 to 0.25
2. 0.26 to 0.50
3. 0.51 to 0.75
4. 0.76 to 1.00

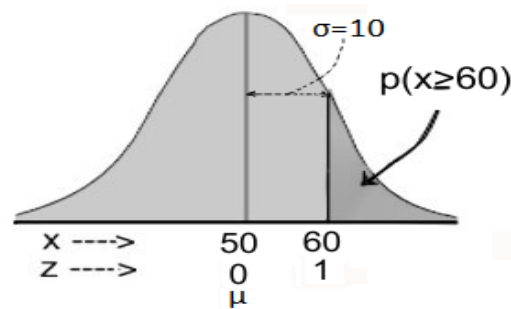
→ **Answer:** Option 1 is correct.

To determine the probability, the given value of x must first be transformed into its equivalent in the z distribution. The z -score is calculated by using the formula for the z -transformation (from p. 55 in the Guide):

$$z = \frac{(x - \mu)}{\sigma} = \frac{(60 - 50)}{10} = \frac{10}{10} = 1.$$

So the z -value associated with 60 is equal to 1. The probability of a score of $x \geq 60$ is therefore equivalent to the probability of a z -value of $z \geq 1$.

If you get a question like this you should draw a graph to help you work out what area under the normal curve you are looking for. This can be illustrated by looking at the graph below:



The probability can be determined from the table of the areas under the standard normal distribution (given in Appendix D of the PYC3704 Guide). You have to look in the table for the probability associated with $z = 1$ under the column marked 'Smaller portion,' because we are interested in the area under the normal curve from the point where $z = 1$ and above (the darker area in the graph); that is, the area in the tail of the normal distribution graph on the far right hand side. (Note that if the question related to the probability of $x < 60$, we would have looked at the 'Larger portion' given in the table, which is the light grey area in our graph).

You should find that $p(z \geq 1) = 0.1587$, which can be rounded off to 0.16. This is between 0 and 0.25, as specified in option 1. (This result can also be calculated from Figure 2.7 on page 53 in the Guide.)

Use the scenario below to answer Questions 20 to 25:

A researcher suspects that the addition of certain food supplements to the diet of elderly people will reduce the decline in cognitive functioning that comes about because of aging. She decides to test this using a neuropsychological test that measures the speed with which objects are identified (the Neuropsychological Perceptual Speed or NPS test). The tests measures the speed with which a person reacts to stimuli in milliseconds. It is known that the distribution of scores on this test is approximately normal and that a mean of $\mu=80$ and $\sigma=20$ was found in the population of persons older than 65.

To investigate her hypothesis, she obtains a random sample of $n=100$ persons older than 65. Each member of this sample is given a daily dose of supplements over a period of six months. At the end of this time, each person is tested on the NPS test and a mean of $\bar{x} = 76$ is found. The researcher plans to test the hypothesis at $\alpha = 0.05$.

Question 20

The appropriate research hypothesis suggested by the scenario above is as follows:

1. Cognitive functioning declines with age.
2. The cognitive functioning of elderly persons is related to their perceptual speed.
3. Cognitive functioning will be better for elderly persons who take the dietary supplement than for those who do not.
4. The perceptual speed of elderly persons who take the dietary supplement will differ from those who do not.

→ **Answer:** Option 3 is the most appropriate choice.

This is the most appropriate way to describe the actual hypothesis that is being tested here: whether taking the dietary supplement reduces cognitive functioning in comparison to not taking it. 'Perceptual speed' is used as a way to indicate 'cognitive functioning'; it is an *operationalization* of cognitive functioning, to enable the researcher to measure it using a relevant test. Option 2 describes this assumption that is made, but this is not the goal of this research. This also implies that Option 4 is not quite correct, as the central research issue is cognitive functioning, not perceptual speed as such. Also the formulation is wrong, since the predicted outcome is that the perceptual or reaction speed should get better, and therefore not only a difference but a *directional* difference is implied. Option 1 does not address the research question at all.

Question 21

The appropriate alternative hypothesis to be tested is - - - - .

1. $H_1: \mu < 80$
2. $H_1: \mu > 76$
3. $H_1: \bar{x} < 80$
4. $H_1: \mu \neq 80$

→ **Answer:** The answer is given in Option 1.

If the hypothesis is valid, one would expect perceptual speed to increase, which means reaction time becomes smaller (since reaction time is what the test measures, according to the scenario). So, if it is effective, you would expect the sample of elderly people who take the dietary supplement to have a significantly *smaller* result than the population mean of $\mu=80$.

Option 3 uses the sample mean symbol, which is wrong, since hypotheses should always be formulated in terms of the *population parameters*, not *sample statistics* (we use the sample to make a decision or inference that refers to the entire population; see Sections 1.4.3 and 3.1.2 in the PYC3704 Guide). Option 4 implies two-tailed testing, yet according to the scenario the researcher wants to know whether cognitive functioning gets *better*, whether it gets worse is not relevant given the way the hypothesis was set up (see Question 20 above).

Option 2 refers to the sample mean that was observed ($\bar{x} = 76$ in the scenario). This result would not be known at the time when the hypothesis was formulated, but is the result of the measurement during the research process. It is this value that should be compared to μ in order to test the hypothesis. In other words, the statistical test is to be performed in order to see whether the measured sample mean ($\bar{x} = 76$) is sufficiently smaller than the population mean of $\mu = 80$ for this to be regarded as statistically significant. There is another problem with Option 2, in that the 'greater than' sign ($>$) would imply that elderly people who use the supplement do *worse* than the population on the test.

Question 22

The mean of the sampling distribution of the mean is - - - - .

1. 80
2. 76
3. 20
4. unknown

→ **Answer:** Option 1 is correct.

According to the central limit theorem, the mean of the sampling distribution of the mean approaches the population mean if the sample is big enough (See Section 2.4.2 on p. 60 of the PYC3704 Guide). For this reason Option 4 is also not correct. Option 2 refers to the sample mean, and the whole point of the test is to see whether this differs significantly from the population mean or not. The central limit theorem gives us a justification to expect that the sample mean will not differ significantly from the population mean *if the null hypothesis is true*. Option 3 refers to the population standard deviation, which is not what is being tested here.

Question 23

The standard error is - - - - .

1. 20
2. 2
3. 0.05
4. unknown

→ **Answer:** Option 2 is correct.

According to the central limit theorem, the standard error is the standard deviation of the sampling distribution of the mean, and it gives an indication of how big the error is when we use the population mean to indicate the mean of the sampling distribution of the mean (referred to in Question 22 above). The formula to calculate this when the population standard deviation (σ) and sample size (n) is known is given on p. 61 of the PYC3704 Guide.

We know from the scenario that the population standard deviation $\sigma = 20$ and that the sample size is $n = 100$. Substituting these into the formula for the standard error gives the following result:

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{20}{\sqrt{100}} = \frac{20}{10} = 2$$

Question 24

With the information as given in the scenario, what would be the most appropriate statistical test to test the hypothesis?

1. A one sample t-test
2. A two sample t-test
3. A test of correlation r for relationship between variables
4. A one sample z-test

→ **Answer:** The best choice is Option 4.

The scenario and the hypothesis to be tested (identified in Question 21) indicate that the mean of a single sample is to be compared with a known population mean. Because the population standard deviation is known (it is specified in the scenario above as $\sigma = 20$) the most appropriate test here would be the single sample z-test ($z_{\bar{x}}$). (See pp. 100 – 102 of the PYC3704 Guide). Using a single-sample t-test (Option 1) would not be totally wrong, but it is not the best test to use of the options given. When the population standard deviation is known, the z-test is in fact more *powerful* than a t-test; i.e. it is more sensitive to statistical effects (see the reference to the 'power' of a test on p. 86 of the PYC3704 Guide).

Question 25

The test statistic is calculated and, based on this, a computer program is used to determine that the two-tailed p-value = 0.055. What conclusion can be drawn?

1. The null hypothesis can be rejected, so the supplement improves cognitive functioning.
2. The null hypothesis cannot be rejected, so the supplement does not improve cognitive functioning.
3. The alternative hypothesis can be rejected, so the supplement does not improve cognitive functioning.
4. The null hypothesis can be rejected, so the supplement does not improve cognitive functioning.

→ **Answer:** Option 1 is correct.

The computer program supplied a two-tailed p-value, while the scenario requires a one-tailed test (consider the hypothesis, as indicated in Question 21). Therefore, the one-sided result should be divided by 2:

$$\text{One-tailed p-value} = \text{two-tailed p-value}/2 = 0.055/2 = 0.0275 \text{ (see p. 81 in the Guide).}$$

Since this value is lower than the chosen level of significance (given in the scenario as $\alpha = 0.05$), the null hypothesis can be rejected. Consequently, we can conclude that the food supplement improves cognitive functioning. Note that while the rejection of the null hypothesis in Option 4 is correct, the wrong conclusion is drawn.

Use the scenario below to answer Questions 26 to 28:

A selection panel for Research Psychology students is interested in the importance of numerical literacy for Research Psychologists. They want to determine whether there is a difference between the numerical literacy scores of Research Psychologists and the general population of Psychologists. The researchers hypothesized that Research Psychologists will have higher numerical literacy scores than Psychologists in general.

In a previous study the mean and standard deviation for Psychologists in general have been determined as 74 and of 5 on the Numerical Literacy Test (NLT). When a random sample of $n = 49$ Research Psychologists were tested on the NLT, a mean of 78 and a standard deviation of 10 was found.

Question 26

An alternative hypothesis for the above can be written as follows:

1. $H_1: \mu > 78$
2. $H_1: \mu > 74$
3. $H_1: \bar{x} > 74$
4. $H_1: \mu \neq \bar{x}$

→ **Answer:** Option 2 is correct.

The point of the test is to determine whether Research Psychologists are found to have (on average) a higher score on the NLT than the mean score for Psychologists in general. From the scenario, we can assume that the mean score for Psychologists in general is equal to 74. So the alternative hypothesis should state that the population mean for Research Psychologists is greater than this, which is correctly expressed in Option 2.

Option 1 compares the population mean for Research Psychologists with the mean found for Research Psychologists in the sample; 78 is in effect compared with itself. In Option 3 the comparison is stated in terms of a sample mean, which is wrong. The test should be one-sided (i.e., not in terms of whether Research Psychologists differ from Psychologists in general, but in terms of whether they do better), so the two-sided statement in Option 4 is not appropriate. In any case, since $\bar{x} = 78$, even the statement $\mu > \bar{x}$ would also be wrong as an alternative hypothesis, since it would be equivalent to the incorrect statement in Option 1.

Question 27

The researchers in the selection panel decide to use a z-test to test for statistical significance. Calculate this test statistic.

1. 39.2
2. 0.8
3. 2.8
4. 5.6

→ **Answer:** Option 4 is correct.

The formula for transforming a measurement to the equivalent value on standardised distribution (the z-value), is given on p. 80 of the PYC3704 Guide.

The formula is:
$$z_{\bar{x}} = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}}$$

From the problem statement above, we know that the sample mean for a sample of $n = 49$ Research Psychologists is $\bar{x} = 78$, and the sample standard deviation is $s = 10$. This sample mean has to be compared to a theoretical population mean for all Psychologists, namely $\mu = 74$, and a population standard deviation of $\sigma = 5$.

First, it is necessary to use this information to calculate the standard error ($\sigma_{\bar{x}}$) as follows:

$$\sigma_{\bar{x}} = \sigma / \sqrt{n} = 5 / \sqrt{49} = 5/7 = 0.714 \text{ (see p. 79 in the Guide).}$$

Then this should be substituted into the formula for the $z_{\bar{x}}$ test statistic:

$$z_{\bar{x}} = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = (78 - 74) / 0.714 = 4 / 0.714 = 5.6$$

If you chose one of the other options, you may have calculated the test statistic incorrectly. Below is an explanation of each *incorrect* option.

In Option 1, the standard error is calculated incorrectly: $\sigma_x = \sigma/n = 5/49 = 0.102$. (The *square root* of n should be used, according to the formula). Using this to calculate the statistic would give $z_x = 4/0.102 = 39.2$ (rounded).

Option 2 is based on calculation of z-transformation, using the value of σ rather than $\sigma_{\bar{x}}$ as calculated above. So you get $(78-74)/5 = 0.8$. But, as indicated on p. 80 of PYC3704 Guide, this should be adjusted: one should use the *standard error*, not the *population standard deviation*.

Option 3 incorrectly uses the sample standard deviation for Research Psychologists and not the population standard deviation for psychologists in general in calculating the standard error:

$$\sigma_{\bar{x}} = \sigma / \sqrt{n} = 10 / \sqrt{49} = 10/7 = 1.429 \text{ which leads to } z_{\bar{x}} = 4/1.429 = 2.799, \text{ which can be rounded to } 2.8.$$

Question 28

Even though the result is found to be very significant, the researchers wonder what the practical value of the outcome may be, if the sample size is compensated for. They decide to also calculate the effect size. They find the effect size is - - - - -.

Use these ranges in the table below to answer the question:

Effect size range	Meaning
0 – 0.3	Small
0.4 – 0.6	Medium
0.7 – 0.9	Large
1.0 and greater	Very large

1. small
2. medium
3. large
4. very large

➔ **Answer:** Option 3 is correct.

To calculate the relevant effect size (d), one should divide the *mean difference* by the population standard deviation (see Section 3.3.3 in the PYC3704 Guide; the formula is given on p. 88). The mean difference is difference between observed mean ($\bar{X} = 78$) and the expected mean as suggested in the hypothesis ($\mu = 74$).

$$\text{Effect size } d = d = \frac{\text{mean difference}}{\text{standard deviation}} = \frac{\bar{x} - \mu}{\sigma} = \frac{78 - 74}{5} = \frac{4}{5} = 0.8$$

Using table above, this can be classified as a 'large' effect size.

Note that when you calculate effect sizes, the sign (plus or minus) can be ignored. If you used $74 - 78 = -4$, the result would have been the same.

4. Assignment 02

You will find Assignment 02 for Semester 1 of 2018 in Tutorial Letter 101/3/2018. Note that the closing date for this assignment is 20 April 2018 for the first semester and that the assignment will count for 10% of your exam mark. We will not be able to give extensions for the assignment because feedback will be provided in the Tutorial Letter 202/1/2018, which will be made available shortly after the assignment deadline by post and via *myUnisa*.

Also note that there are different assignments for Semester 1 and Semester 2 students in the 101/2018 Tutorial Letter, *so make sure you submit the correct one!* You will get marks for the assignment which you submit and we will not accept alternative submissions of assignments once the feedback becomes available.