Tutorial letter 101/3/2018

Distribution Theory II

STA2603

Semesters 1 & 2

Department of Statistics

IMPORTANT INFORMATION:

This tutorial letter contains important information about your module and includes the assignment questions for both semesters.

Define tomorrow.

UNISA university of south africa

CONTENTS

Page

1 INTRODUCTION

Dear Student,

We wish to welcome you to the module STA2603 (Distribution theory II), and hope that you will enjoy studying it. We shall do our best to make your study of this module successful.

This tutorial letter contains important information about how to study this module. We urge you to read it carefully and to keep it at hand when working through the study material, preparing the assignments, preparing for the examination and addressing questions to your lecturers.

In this tutorial letter, you will find the assignments as well as instructions on the preparation and submission of the assignments. This tutorial letter also provides all the information about your study material. Please study this information carefully.

1.1 Tutorial matter

1.1.1 Tutorial letters

The Department of Despatch will supply you with the following study material for this module:

- A study guide, written by a lecturer to guide you through the relevant sections in the prescribed book. Use it together with the textbook as the guide indicates the relevant prescribed sections, explaining difficult concepts in more detail, giving additional examples and exercises, etc.
- This tutorial letter (Tutorial letter 101), as well as others which will be sent out during the semester.

The Department of Despatch should have supplied to you the tutorial letter 101 and the study guide shortly after your registration. The other tutorial letters will be sent to you throughout the semester. Follow the instructions in the brochure entitled *Study @ Unisa* if you have not received some of the material that should have been sent to you.

Take note that every tutorial letter you will receive is important and you should read them all immediately and carefully. Some information contained in these tutorial letters may be urgent, while others may, for example, contain examination information. So, it is wise to keep them all in a file!

If you have access to the Internet, you can view and print the study guide and tutorial letters for the modules for which you are registered on the University's online campus, myUnisa, at http://my.unisa.ac.za
As UNISA is moving increasingly towards online access to study material, please note that some tutorial letters may only be available for downloading on the module's myUnisa web site. We will attempt to send you all the material in printed copy, however this might only be later in the semester. All the study material is immediately available on myUnisa, often long before it reaches you via mail or courier. It is important that you activate your myLife email account, since a notification is sent to your myLife email address whenever a new resource is made available. Alternatively, you should regularly visit the module web site to check on any new available material.

2 PURPOSE OF AND OUTCOMES FOR THE MODULE

STA2603 is one of the compulsory modules of the *major in Statistics* and is the “middle” module in distribution theory. Once you have completed *STA2603: Distribution theory II*, you will be properly prepared to continue with *STA3703: Distribution theory III*.

Most of the modules in the Statistics major have *prerequisite modules.* These are either statistics or mathematics modules carefully selected by the staff in the department. These are modules you should have completed before you will be allowed to register for that particular module. Some modules have *co-requisite* modules, meaning that you have to register simultaneously for the module (if you have not passed it at a previous stage). Statistics knowledge accumulates in a specific order and you will not be able to do the particular module if you do not have the necessary pre-knowledge. Achieving statistical knowledge can therefore be compared to the building of a brick wall - you have to start at the bottom and not “skip” anything, as it can have disastrous effects at a later stage. Another example is to imagine a non-existing spider’s web because the spider was only interested in the centerpiece of its web!

For this particular module, the pre-requisites are STA1503 and MAT1512 or alternatively for BCom students, STA1502 and DSC1620/DSC1520 and co-registration for STA2610. This means that you are assumed to be familiar with basic statistics and distribution theory at first year level, and you are also assumed to be familiar with basic calculus (differentiation and integration). If necessary, be ready to revise this knowledge to ensure success in this module!

2.1 Purpose

Students credited with this unit must be able to gain insight into the role that formal theory plays in data analytic methods and to discuss a wide variety of discrete and continuous distributions.

2.2 Outcomes

Qualifying students will be able to:

- Understand the joint probability structure of two random variables (discrete and continuous case);
- calculate expectation and moment-generating functions; have insight into distributions of functions of random variables, extrema and order-statistics;
- apply the law of large numbers and the central limit theorem;
- understand how the chi-square-, t-, and F-distributions are derived from the normal distribution.

3 LECTURER AND CONTACT DETAILS

3.1 Lecturer

You are most welcome to contact your lecturer whenever you experience any difficulties with your studies. You may do this by writing a letter, by telephone, by fax, by e-mail, or by seeing the lecturer in person. If you wish to see the lecturer in person, then in order to avoid disappointment, you are advised to make an appointment by telephone or e-mail in advance to make sure that the lecturer is available to help you.

If you cannot get through to the lecturer by phone, PLEASE call the departmental secretary at 011 670 9255. She will be able to tell you when the lecturer will be available, or forward your call to another lecturer involved in this module who may be able to help you, or you can leave a message with her asking the lecturer to call you back!

Your STA2603 lecturer in 2015 will be Dr. E. Rapoo. You can reach the lecturer via email at

rapooe@unisa.ac.za

The phone number of the lecturer is 011 670 9259, and the office number is GJ Gerwel Floor 6, Office 6-08, at UNISA's Science campus in Florida, Roodepoort. You can also send a letter to the address

The Lecturer (STA2603)
Department of Statistics
University of South Africa
PO Box 392
UNISA
0003

There is also an e-mail link to the lecturer from the module's myUnisa page.

Please do not include your enquiries with your assignments as this will cause unnecessary delays.

3.2 Department

The phone number of the Department of Statistics is 011 670 9255.

3.3 University

If you need to contact the University about matters not related to the content of this module, please consult the publication *Study @ Unisa* that you received with your study material. This brochure contains information on how to contact the University (e.g. to whom you can write for different queries, important telephone and fax numbers, addresses and details of the times certain facilities are open).

Always have your student number at hand when you contact the University.

4 MODULE RELATED RESOURCES

4.1 Prescribed books

The prescribed book for this module is

Rice JA, Mathematical Statistics and data analysis (2007), 3rd edition, Cengage.

The following 5 chapters are relevant for this module:

Chapter 2: Random variables

Chapter 3: Joint distributions

Chapter 4: Expected values

Chapter 5: Limit theorems

Chapter 6: Distributions derived from the normal distribution

You have to buy this book. Please consult the list of official booksellers and their addresses listed in *Study @ Unisa*. If you have any difficulties in obtaining books from these bookshops, please see *Study @ Unisa* for more information.

4.2 Recommended books

There are no recommended books for this module.

4.3 Electronic Reserves (e-Reserves)

There are no e-Reserves for this module.

4.4 Library services and resources information

For brief information go to : http://www.unisa.ac.za/contents/studies/docs/myStudies-at-Unisa2017-brochure.pdf
For more detailed information, go to the Unisa website: http://www.unisa.ac.za/, click on Library
For research support and services of Personal Librarians, go to:
http://www.unisa.ac.za/Default.asp?Cmd=ViewContent&ContentID=7102
The Library has compiled numerous library guides:

- find recommended reading in the print collection and e-reserves -http://libguides.unisa.ac.za/request/undergrad
- request material - http://libguides.unisa.ac.za/request/request
- postgraduate information services - http://libguides.unisa.ac.za/request/postgrad
- finding , obtaining and using library resources and tools to assist in doing research http://libguides.unisa.ac.za/Research_Skills
- how to contact the Library/find us on social media/frequently asked questions - http://libguides.unisa.ac.za/ask

5 STUDENT SUPPORT SERVICES FOR THE MODULE

For information on the various student support systems and services available at Unisa (e.g. student counseling, tutorial classes, language support), please consult the publication *Study @ Unisa* that you received with your study material.

5.1 Tutors

To further assist you in mastering this module, you will be allocated an e-tutor. Communication with your e-tutor is via a special e-tutor website you will be linked to soon after registration. The tutor is there to support you throughout the semester as you work through the study material.

5.2 Contact with Fellow Students – Study Groups

It is advisable to have contact with fellow students. One way to do this is to form study groups. **Please consult the publication *Study@Unisa* to find out how to obtain the addresses of students in your region.**

5.3 myUnisa

If you have access to the internet, you can quickly access resources and information at the University. The *myUnisa* learning management system is Unisa's online campus that will help you to communicate with your lecturers, with other students and with the administrative departments of Unisa – all through the internet.

Joining *myUnisa* will offer you the following benefits:

- You have access to the additional resources on this module.
- You will be able to immediately download all your study material from this site, in electronic format.
- You can use the discussion forum to communicate with your fellow students.
- You can contact your lecturer through the e-mail link of your *myUnisa* module page.

For this module, the lecturer will add various resources onto the myUnisa web site during the semester. An announcement will be sent to your myLife email address whenever a new resource is uploaded, so do make sure your email address is activated.

To go to the *myUnisa* website, start at the main Unisa website, http://www.unisa.ac.za, and then click on the "Login to *myUnisa*" link on the right-hand side of the screen. This will take you to the *myUnisa* website. You can also go there directly by typing in http://my.unisa.ac.za. On the website you will find general Unisa related information, plus a module site for each module you are registered for. Please consult the publication *Study @ Unisa* which you received with your study material for more information on *myUnisa*.

5.4 Free computer and internet access

Unisa has entered into partnerships with establishments (referred to as Telecentres) in various locations across South Africa to enable you (as a Unisa student) free access to computers and the Internet. This access enables you to conduct the following academic related activities: registration; online submission of assignments; engaging in e-tutoring activities and signature courses; etc. Please note that any other activity outside of these are for your own costing e.g. printing, photocopying, etc. For more information on the Telecentre nearest to you, please visit www.unisa.ac.za/telecentres.

6 MODULE-SPECIFIC STUDY PLAN

The semester during which you study at UNISA consists of 15 weeks between the last day of registration and the beginning of the examination period, during which time you need to study and understand the contents of the module, complete and submit six assignments, and then prepare for the examination. Therefore it is important that you create a timetable for planning your studies for this module, and all the other modules you take this semester or year.

Please start studying as soon as you receive your study material. Note that if you are registered for Semester 1, then all your assignments need to be submitted by end of April and you will write your examination in May-June; and if you are registered for Semester 2, then your assignments need to be submitted by early October and you will write your examination in October-November.

6.1 Suggested time table

The following time tables are provided as a starting point for your personal schedule.

SEMESTER 1	Study units for preparing your assignments	Study from	To
Assignment 1	Pre-knowledge of probability theory	Registration	19 Feb
Assignment 2	Unit 2	19 Feb	26 Feb
Assignment 3	Unit 3	26 Feb	19 March
Assignment 4	Units 4, 5, 6	19 March	9 April
Assignments 5 & 6	The whole module	9 April	26 April
Exam	Prepare for the examination	26 April	Exam

SEMESTER 2	Study units for preparing your assignments	Study from	To
Assignment 1	Pre-knowledge of probability theory	Registration	6 August
Assignment 2	Unit 2	6 August	16 August
Assignment 3	Unit 3	16 August	3 Sept
Assignment 4	Units 4, 5, 6	3 Sept	26 Sept
Assignments 5 & 6	The whole module	26 Sept	5 Oct
Exam	Prepare for the examination	5 Oct	Exam

6.2 How to study this module

6.2.1 An overview of the module

The outcomes of the module are listed in Section 2.2 of this tutorial letter. To pass this module, you must achieve these outcomes.

To do this, you will need to study and work through the material in the study guide and the prescribed textbook, until you are able to understand and apply the concepts and principles involved. The study guide contains activities and problems, which are there to help you ensure that you have mastered the material. Another way to find out how you are doing is through the assignments that you are supposed to submit throughout the semester. The lecturer will mark your work and give individual feedback to you.

For even more help in case you need it, please join myUnisa — on the module web page at myUnisa, there will be more resources available. These will be explained on the web page.

The final decision on whether you have mastered the module outcomes well enough comes from your final mark for the module, which is calculated from your semester mark and the examination mark. (How exactly this is done is explained later on.)

Note that the examination date is fixed, and it is your duty to make sure that you are ready to write the examination when it comes! In Statistics, it is often very hard to catch up again with the work if you fall behind, since you need to understand previous material thoroughly before learning new things.

Although you do need to take responsibility for your studies, remember that you are not alone. Your lecturer and your e-tutor are there to help you, and you can also contact your fellow students and use Unisa's student support systems. Details of all of these are listed elsewhere in this tutorial letter!

6.2.2 Guidelines for studying this module

Guidelines of what you should do while studying for this module are therefore as follows:

- There is quite a bit of work to be done in the 15 weeks of study time. Make a timetable for yourself, to make sure you know what amount of work you need to do by what time to keep up to date with the work.
- Work through the textbook and the study guide. This includes doing the activities, and working on more exercises from the textbook if you feel you need more practice.
- You will need to use a calculator for this module. Make sure you know how to use your calculator! You will be allowed to bring a non-programmable calculator to the examination.
- Submit the assignments by their due dates. The due dates of the assignments are chosen in such a way that you will need to work steadily through the semester. When you receive back your marked assignments, make note and take advantage of the lecturer's feedback on your work.
- Prepare well for the examination.

7 MODULE PRACTICAL WORK AND WORK-INTEGRATED LEARNING

There is no practical work or work-integrated learning in this module.

8 ASSESSMENT

Assessment is the process where the lecturer assesses your work by comparing it to the module outcomes and the related assessment criteria. The assessment in this module consists of formative assessment and summative assessment.

Formative assessment means assessment of your work while you are still studying. This is particularly important in distance learning since it might sometimes be the only way you can get feedback on how you are doing, while you can still benefit from it. In this module, formative assessment is through the assignments. The lecturer marks your work and gives you individual feedback on how you are doing, as well as suggestions for improvement. Make sure to take advantage of the lecturer's feedback! In addition to your marked assignment, all students will also receive the answers to the assignment questions as well as comments on the assignment in a tutorial letter sent out after the closing date of each assignment. You will also receive model solutions to the assignments that you submitted.

Summative assessment refers to the final mark you receive for this module. In this module, your final mark is calculated from you examination mark (which counts for 80%) and from your semester mark (which counts for 20%). The semester mark is determined by how well you did in your assignments. Details of how this works are given in the following.

The semester mark and the final mark

Your **final mark** will be calculated from your **semester mark** and the **examination mark.**

The **semester mark** is calculated from your assignment results (the percentages you receive for the assignments). The weights of the different assignments differ: Assignment 1 counts for 0%, Assignments 2, 3 and 4 for 15% each, Assignment 5 for 10% and Assignment 6 for 45%. That is, the semester mark is calculated as

$$\text{semester mark} = \frac{1}{100} * (15 \cdot A_2 + 15 \cdot A_3 + 15 \cdot A_4 + 10 \cdot A_5 + 45 \cdot A_6)$$

where A_2 to A_6 are the percentages you received in assignments 2 to 6, respectively. Assignments not submitted, or submitted late, will give you 0%.

- The **examination mark** is the percentage mark you get in the examination.

- The examination mark contributes 80% to the final mark, and the semester mark contributes 20%. That is, your **final mark** is calculated as

$$\text{final mark} = 0.8 * (\text{examination mark}) + 0.2 * (\text{semester mark}).$$

You pass the module if your final mark is ≥ 50, and you pass it with distinction if your final mark is ≥ 75. There is also a subminimum rule, which says that you must get at least 40% in the examination to pass the module.

IMPORTANT: Please note that a poor semester mark will lower your final mark! It is therefore important that you try to complete all the assignments as well as your can – if your year mark is zero, you must get 63% in the exam to pass the module! Also, you must make sure that you submit all the assignments on time, since if we receive your assignment too late, we have to give you 0% for it.

8.1 Assessment criteria

The outcomes of this module are given in Section 2.2 of this tutorial letter. These outcomes describe what you should be able to do in order to successfully pass this module. Assignments, examinations, and in some modules projects and portfolios are the ways we use to assess whether you have reached the outcomes.
The criteria we use to assess your work can be summarised as follows:

- You must apply the correct and appropriate formulas, presentations, methods, rules, laws, values from tables, and so on, as required in the question.
- Applying of formulas, methods etc. must be done correctly.
- Results, tests, computer printouts etc. should be interpreted correctly, when you are asked to do so.
- Calculations must be correct and accurate.

The following general comments are valid to all our modules. In some cases the lecturers will give further instructions to keep in mind when completing your work; these will be given in the tutorial letters for that particular module.

8.1.1 Written assignment and examination questions

Please keep the following in mind when answering questions.

- Read the question carefully – you will get zero marks if you end up answering what was not asked for!
- Give full calculations, marks will usually not be given for the end results only.
- Present your solutions clearly. A collection of disjointed formulas and numbers is not the right way to answer questions, please use words to explain what you are doing and why. Use correct mathematical notation and remember that lines of mathematical equations must always be linked to each other – for example with the = sign if they are a series of continuing calculations, or otherwise maybe by the signs for “equals” or “therefore”. See your textbooks and/or study guides for examples.

8.1.2 Multiple choice questions

- Only one of the given answers is correct. If you believe several to be correct, check your work again!
- We suggest you keep copies of your calculations, so that when you get the results, you can check where you went wrong.

8.2 Assessment plan

There are six assignments in this module.

- The first assignment is compulsory: You must submit Assignment 01 by its due date, otherwise you will not get examination admission. To make it easier for you to be able to submit it on time, I have made the first assignment shorter than all the other assignments. What is more, the first assignment tests things you should be able to do already – that is, it tests how well you are prepared for this module. You can answer this assignment before you receive you study guide or your text book. If you struggle with any of the questions, you must make sure to revise probability theory from your previous modules! The first assignment does not count for the semester mark. Assignment 01 is a multiple choice assignment, and all students will receive full solutions to this assignment.
- The next three assignments 2, 3 and 4 will help you work through the module and will give you an idea on which topics you understand correctly, and where you are struggling. Please do view these assignments as a chance for you to get feedback from your lecturer! These assignments each count for 15% towards the semester mark, so these 3 assignments together make up 45% of the semester mark. Assignments 02, 03 and 04 are written assignments.
- Assignment number 5 is a survey questionnaire, seeking your opinions on this module. The questionnaire will be sent out to all students later in the semester, and any student who fills it in and submits it gets 10% for the semester mark – which means 2 percentage points in the final mark, absolutely free!
- Assignment number 6 is meant to help you prepare for the examination. It consists of questions from a previous year's examination paper. In order for you to be able to complete Assignment 6 closer to the examination, we have made it a multiple choice assignment. All students will receive detailed model solutions to the examination questions in this assignment. Assignment 06 counts for 45% of the year mark, so it is well worth doing as well as you can!

In conclusion, you should complete all assignments as well as you can: To get admission to the examination; and because of the semester mark system which means that how well you do your assignments will also have a direct effect on the final mark you get for this module; and most importantly, because submitting the assignments gives you a chance to find out how well you have mastered the course contents, and for us to give you feedback on your progress!

Marking of the Assignments

Written assignments (Assignments 02, 03 and 04)
After you have submitted your assignment, we will mark it, give you a percentage mark for it (a number between 0% and 100%), and send it back to you together with the model solutions to the assignment. The percentage mark you received will be indicated in your marked assignment. Your marked assignment will contain detailed feedback on your work. This feedback is very important, so make sure to read through the comments when you receive your assignment back.

Multiple-choice assignments (Assignments 01 and 06)
These assignments must be done on optical mark reading sheets and will be marked by a computer. Instructions on how to submit these assignments appear with the assignment questions. Again, you will get a percentage mark, between 0% and 100%, for these assignments.

The survey questionnaire (Assignment 05)
The survey questionnaire will be sent to you later in the semester; you should fill it in and submit it as assignment, and as soon as we receive it, you will get 100% for it.

Feedback to assignments
Comments and the answers to the questions in each assignment will be automatically sent out to all students a few days after the closing date of the assignment, and therefore we have to give 0% for assignments which reach us too late. If you are struggling to meet the closing dates, please contact your lecturer before the closing date!

However, for the written assignments, even if we receive your assignment too late, we will mark it and provide you feedback for it. So, it will still be a good idea to submit your assignment even if you know it might be too late for you to receive a percentage mark for it.

If you are genuinely unable to submit an assignment at all, please try to answer the questions in it anyway by yourself, before looking at the solutions. You will learn much more in this way than by simply reading through the correct solutions we send to you.

8.3 General assignment numbers

The assignments are numbered 01 to 06. Please remember to give your assignment the correct number in the assignment cover. The assignment questions for Semester 1 and for Semester 2 are listed in Appendix A and Appendix B at the end of this tutorial letter.

8.3.1 Unique assignment numbers

Please note that each assignment has its unique six-digit assignment number which has to be written on the cover of your assignment or on the mark reading sheet upon submission. The unique numbers are given in the table in the next section of this tutorial letter; you will also find them in the heading of each set of assignment questions.

8.3.2 Due dates for assignments

For each assignment there is a **FIXED CLOSING DATE**, which is the date by which the assignment **must reach** the university. The closing dates for submission of the assignments are given in the following table. We also give the contribution of each assignment to the semester mark.

SEMESTER 1				
Assignment no.	**Type**	**Fixed closing date**	Semester mark %	Unique number
01	Multiple choice	19 February 2018	0	640876
02	Written	26 February 2018	15	686856
03	Written	19 March 2018	15	824146
04	Written	9 April 2018	15	674158
05	Written (questionnaire)	26 April 2018	10	891254
06	Multiple choice	26 April 2018	45	851336
SEMESTER 2				
Assignment no.	**Type**	**Fixed closing date**	Semester mark %	Unique number
01	Multiple choice	6 August 2018	0	832585
02	Written	16 August 2018	15	797853
03	Written	3 September 2018	15	739762
04	Written	26 September 2018	15	845842
05	Written (questionnaire)	5 October 2018	10	797137
06	Multiple choice	5 October 2018	45	646825

8.4 Submission of assignments

Enquiries about assignments, such as whether they have been received by the university, what mark you obtained, when they were returned to you, etc., should be addressed to the Assignments section. For detailed information and requirements as far as assignments are concerned, see *Study @ Unisa*, which you received with your study package.

You are strongly recommended to submit your assignments electronically via myUnisa, rather than in hard copy: the turnaround times of marking your assignments will be much shorter, and your assignments cannot get lost. To submit an assignment **via myUnisa:**

- Go to *myUnisa*.
- Log in with your student number and password.
- Select the module.
- Click on assignments in the left–hand menu.
- Click on the assignment number you want to submit.
- Follow the instructions on the screen.

You can submit statistics assignments in electronic format, but please note that you must still use all the correct mathematical notation, and include all necessary graphs, diagrams, and so on, just as if you were submitting a hand-written assignment! Your final submission file must be in the PDF format. You can use a word-processing program with an equation editor (e.g. MSWord) or you can use special mathematical typesetting programs such as LaTeX, and at the end convert your assignment to PDF; or you can scan your hand-written assignment into a PDF file.

Please note: Although students may work together when preparing assignments, each student must write and submit his or her own individual assignment. It is unacceptable for students to submit identical assignments on the basis that they worked together. That is copying (a form of plagiarism) and none of these assignments will be marked. Furthermore, in such cases students may be penalised or subjected to disciplinary proceedings by the University.

8.5 Assignments

This tutorial letter 101 contains the assignment questions for both semesters, so select the semester you are enrolled for and do the set of assignments for that semester only. The assignments for Semester 1 are in Appendix A, pages 18 – 29. The assignments for Semester 2 are in Appendix B, pages 30 – 42.

9 OTHER ASSESSMENT METHODS

All the other assessment methods in this module are by self-assessment. To find out whether you are on the right track, you can: Do the activities in the study guide and compare your answers with the feedback; do exercises in the text book and compare your answers with the given ones; and take the self-assessment quizzes which will be made available on myUnisa.

10 EXAMINATION

10.1 Examination Admission

To be admitted to the examination you must submit the compulsory assignment, i.e. Assignment 01, by its due date. Note that admission therefore does not rest with the department and if you do not submit that particular assignment in time, we can do nothing to give you admission. Although you are most probably a part time student with many other responsibilities, work circumstances will not be taken into consideration for exemption from assignments or the eventual admission to the examination.

No concession will be made to students who do not qualify for the examination

10.2 Examination Period

This module is offered in a semester period of fifteen weeks. This means that

- if you are registered for the first semester, you will write the examination in May/June 2018 and should you fail and qualify for a supplementary examination, that supplementary examination will be written in October/November 2018.
- if you are registered for the second semester, you will write the examination in October/November 2018 and should you fail and qualify for a supplementary examination, that supplementary examination will be written in May/June 2019.

The examination section will provide you with information regarding the examination in general, examination venues, examination dates and examination times. Eventually, your results will also be processed by them and sent to you.

10.3 Examination Paper

The examination consists of a two hour paper. You are allowed to use a non-programmable calculator in the examination. Should you have a final mark of less than 50%, it implies that you failed the module. However, should your results be within a specified percentage (from 40% to 49%), you will be given a second chance in the form of a supplementary examination. There is also an examination subminimum rule, which states that if you fail the examination with less than 40%, the semester mark will not count to help you pass.

10.4 Previous Examination Papers

Previous examination papers are available to students on myUnisa. In addition, Assignment 6 in each semester is based on a previous year's examination paper, and model solutions to that paper will be sent out to you in a tutorial letter.

10.5 Tutorial Letter with Information on the Examination

To help you in your preparation for the examination, you will receive a tutorial letter that will set out clearly what material you have to study for examination purposes and what the assessment criteria are.

You are automatically admitted to the exam on the submission of Assignment 01 by a specific date – see Section 10.1. Please note that lecturers are not responsible for exam admission, and ALL enquiries about exam admission should be directed by e-mail to exams@unisa.ac.za.

11 FREQUENTLY ASKED QUESTIONS

The my Studies @ Unisa brochure contains an A-Z guide of the most relevant study information. Please refer to this brochure for any other questions.

12 SOURCES CONSULTED

No books or other sources were consulted in preparing this tutorial letter.

13 CONCLUSION

We hope that you will enjoy this module and wish you all the best!

Your lecturer,
Dr E Rapoo

ADDENDUM A: FIRST SEMESTER ASSIGNMENTS

The questions for each assignment for Semester 1 follow.

SEMESTER 1
COMPULSORY ASSIGNMENT FOR EXAM ADMISSION

ASSIGNMENT 01
Multiple Choice Assignment
Based on your previous knowledge
Fixed closing date: 19 February 2018
Unique Assignment Number: 640876

This multiple-choice assignment will be marked by computer. Hence the closing date is **fixed** and no extension of time can be granted.

You can submit your multiple choice assignments via myUnisa. Alternatively, if you choose to submit in hard copy, please note that your answers must be entered on an optical mark reading sheet. But before you attempt that, please study in detail the relevant chapter of the publication *Study @ Unisa.* Please make sure that you know how to handle the optical mark reading sheets, since sheets which are marked incorrectly and which are rejected by the computer will not be marked by hand and students will not receive marks for such assignments.

The unique number appearing in the box above links your assignment to the corresponding set of answers in the computer. It must therefore be filled in correctly on the optical mark reading sheet.

Note that your assignment will not be returned to you. Please keep a record of your answers so that you can compare them with the correct answers.

In each of the following four questions, choose the number of the answer that you think is correct. Each correct answer gives you 25%, adding up to a total of 100%.

Question 1: Which one of the following statements does NOT hold true for ALL random variables X, Y?

1. $E(X+2) = E(X) + 2$
2. $Var(-X) = -Var(X)$
3. $E(X+Y) = E(X) + E(Y)$
4. $Var(X+2) = Var(X)$

Question 2: Assume that $F(x)$ is the distribution function of some random variable X. Which one of the following statements is NOT true?

1. If $F(2) = 0$ then $P(X = 2) = 0$.
2. If $F(4) = 1$ then $P(X = 5) = 0$.
3. If $a > b$ then $F(a) > F(b)$.
4. $F(0) \geq 0$.

Question 3: Which one of the following statements is true for all random variables X, Y?

1. $P(X = 0) = 0$
2. $P(X = 0) \leq P(X = 1)$
3. $P(X + Y = 2) = P(X = 2) + P(Y = 2)$
4. $P(X = 1) \leq P(X \leq 1)$

Question 4: Which one of the following statements is true?

1. If $E(X) \geq 0$ then always $X \geq 0$.
2. If $P(X = 1) = 1$ then $E(X) = 1$.
3. If $E(X) = 2$ then $E(X^2) = 4$.
4. If $X < 0$ then $Var(X) < 0$.

SEMESTER 1

ASSIGNMENT 02
Written Assignment
Based on Unit 2
Fixed closing date: 26 February 2018
Unique Assignment Number: 686856

Question 1

The probability mass function of a random variable X is as follows:

$$p(x) = \begin{cases} 1/3, & x = 1 \\ 2/3, & x = 2 \end{cases}$$

(a) Find the distribution function $F_X(x)$ of X.

(b) Find $P(X \leq 1)$.

Question 2

Suppose that a continuous random variable X has the following distribution function:

$$F_X(x) = \begin{cases} 0 & & x < 0 \\ cx^3 & \text{for} & 0 \leq x < 2 \\ 1 & & x \geq 2 \end{cases}$$

(a) Find the value of c.

(b) Determine the density function $f_X(x)$ of X.

(c) Find $P(1/2 < X < 1)$.

(d) Find the density function of the random variable $Y = 2 + 2X$,

(i) by using the transformation method;

(ii) by first finding the distribution function of Y.

Question 3

Let X have the following density function:

$$f_X(x) = \begin{cases} \frac{1}{4}, & -1 \le x < 0, \\ \frac{1}{2}, & 0 \le x < 1, \\ \frac{1}{4}, & 1 \le x < 2, \end{cases} \qquad \text{and zero elsewhere.}$$

(a) Does X have a uniform distribution? Justify your answer!

(b) Find $P(X < 0)$.

(c) Find $P(X > 1 \mid X > 0)$.

(d) Find the distribution function of X.

(e) Find the lower quartile of the distribution.

Question 4

Suppose that X, the number of serious fires in a city has the Poisson distribution with an average of λ fires per month. Let $p(k) = P(X = k)$ denote the probability mass function.

(a) Show that the ratio of successive probabilities satisfies

$$\frac{p(k)}{p(k-1)} = \frac{\lambda}{k}, \quad \text{for} \quad k = 1, 2, \ldots$$

(b) For which values of k is $p(k) > p(k-1)$?

(c) What is the probability of having n fires in *one year*, for $n = 0, 1, 2, \ldots$?

SEMESTER 1

ASSIGNMENT 03
Written Assignment
Based on Unit 3
Fixed closing date: 19 March 2018
Unique Assignment Number: 824146

Question 1

The joint density function for the random variables X and Y for $0 < x < 1, -1 < y < 0$ is

$$f_{X,Y}(x, y) = c\,(5 - x - y)\,.$$

(a) Find the value of the constant c.

(b) Find the joint distribution function $F_{X,Y}(x, y)$.

(c) Find the marginal distribution function $F_X(x)$ of X.

(d) Calculate $P(X \leq 1/2)$.

(e) Find the condition density function $f_{Y/X}(y|x)$.

(f) Calculate $P(Y < X)$.

Question 2

Assume that two independent random variables X and Y have the density functions

$$f_X(x) = \begin{cases} 1/2, & 0 < x < 2 \\ 0 & \text{elsewhere,} \end{cases} \qquad f_Y(y) = \begin{cases} 1/4, & -2 < y < 2 \\ 0 & \text{elsewhere.} \end{cases}$$

(a) Identify the distributions of X and Y.

(b) Find the joint density function $f_{X,Y}(x, y)$.

(c) Find the joint cumulative distribution function of X and Y in all parts of the xy plane.

(d) Find the conditional density function $f_{Y|X}(y \mid x)$.

(e) Calculate $P(Y > X)$.

(f) Find the joint density function of the random variables U and V, if $U = 2X + Y$, $V = Y - X$.

Question 3

Let

$$f_{X,Y}(x,y) = \begin{cases} cy^2, & -1 < x < 1, \quad x+1 < y < 2, \\ 0 & \text{elsewhere.} \end{cases}$$

(a) Draw a sketch of the region A where the joint density function of the variables is non-zero.

(b) Find the value of c.

(c) Find the joint distribution function $F_{X,Y}(x,y)$ in the region $-1 < x < 1, \quad x+1 < y < 2$.

(d) Calculate $P(X > 0)$.

SEMESTER 1

ASSIGNMENT 04
Written Assignment
Based on Units 4, 5, 6
Fixed closing date: 9 April 2018
Unique Assignment Number: 674158

Question 1

Suppose the random variable X has the following distribution function:

$$F_X(x) = \begin{cases} 0 & \text{for} \quad x < -1 \\ \frac{1}{3} & \quad -1 \le x < 1 \\ \frac{2}{3} & \quad 1 \le x < 2 \\ 1 & \quad x \ge 2. \end{cases}$$

(a) Determine $E(X)$, $E\left(X^2\right)$ and $Var(X)$.

(b) Determine the moment generating function $M_X(t)$ of X.

(c) Use $M_X(t)$ to calculate $E\left(X^4\right)$.

Question 2

Assume that random variable X has the gamma distribution with parameters $\alpha = 2$ and $\lambda = 3$. Calculate:

(a) $E(X)$

(b) $Var(X)$

(c) $E\left(\frac{1}{X}\right)$

Question 3

Let U and V be independent random variables, each with mean μ and variance σ^2. Let

$$Z = \alpha U + V\sqrt{1-\alpha^2}.$$

Find $E(Z)$ and $Var(Z)$.

Question 4

Suppose that X_i, $i = 1, \ldots, 5$ is a random sample from a $N(-4; 4)$ distribution and Y_i, $i = 1, \ldots, 5$ is an independent random sample from a $N(0, 100)$ distribution.

(a) Give the names of the distributions of the following random variables, as well as the values of the parameters of the distributions. Justify your answers.

(i) $U = \sum_{i=1}^{2} \left(\frac{X_i}{2} + 2\right)^2$

(ii) $V = \sum_{i=1}^{5} \frac{(Y_i + 10)}{5}$

(b) Explain how you can construct a random variable which has the $F_{4,6}$-distribution from the random variables X_i and Y_i. Explain also why the construction is not unique.

Question 5

Let

$$T = \frac{W}{\sqrt{\frac{V}{R}}}$$

where the independent variables W and V are, respectively, normal with mean zero and variance 1, and Chi-squared with r degrees of freedom. Prove that T^2 has an F distribution with parameters $m = 1$ and $n = r$.

SEMESTER 1

ASSIGNMENT 05
Written Assignment
Questionnaire
Fixed closing date: 26 April 2018
Unique Assignment Number: 891254

The questionnaire will be sent out to you later in the semester. Fill it in, and submit it in the assignment covers by the due date, to get 100% for this assignment.

SEMESTER 1

ASSIGNMENT 06
Multiple Choice Assignment
Based on the whole module
Fixed closing date: 26 April 2018
Unique Assignment Number: 851336

This multiple-choice assignment will be marked by computer. Hence the closing date is **fixed** and no extension of time can be granted.

You can submit your multiple choice assignments via myUnisa. Alternatively, if you choose to submit in hard copy, please note that your answers must be entered on an optical mark reading sheet. But before you attempt that, please study in detail the relevant chapter of the publication *Study @ Unisa.* Please make sure that you know how to handle the optical mark reading sheets, since sheets which are marked incorrectly and which are rejected by the computer will not be marked by hand and students will not receive marks for such assignments.

The unique number appearing in the box above links your assignment to the corresponding set of answers in the computer. It must therefore be filled in correctly on the optical mark reading sheet.

In the following we give questions from a previous year's examination paper. Answer first the examination questions, working through the questions on paper. Then answer the multiple choice questions based on your work. In each of the multiple choice questions, up to four possible answers are given. In each case, mark the number of the answer that you think is correct. Note that you will be sent full model solutions to the examination questions later on, so do keep your written work where you answered the examination questions so that you can compare it against the model solutions later on! Make also a note of your choices in the multiple choice questions, so that you can see where you went wrong.

Each multiple choice question counts for 5%, with a total of 100% from the twenty questions.

Examination Question 1
Two discrete random variables X and Y have a joint probability mass function given in the table below.

$p_{X,Y}(x,y) =$

		x = 1	x = 2	x = 3
	1	0.1	0.1	0.1
y	2	0 .1	0.2	0.1
	3	0.1	0.1	0.1

1.1 Prove that $p_{X,Y}(x,y)$ given above is the joint probability mass function of two random variables, X and Y.

1.2 Are the random variables X and Y independent? Justify your answer!

1.3 Find $F_X(x)$, the cumulative distribution function of X.

1.4 Calculate $P(X \leq 2 \mid Y \leq 2)$.

1.5 Calculate $E\left(\frac{1}{Y}\right)$.

Question 1: The answer to 1.2 is:

1. Yes, they are independent
2. No, they are not independent.

Question 2: The answer to 1.3 is:

1. $F_X(x) = \begin{cases} 0.3, & x = 1 \\ 0.4, & x = 2 \\ 0.3, & x = 3 \end{cases}$
2. $F_X(x) = \begin{cases} 0.3, & x = 1 \\ 0.7, & x = 2 \\ 1, & x = 3 \end{cases}$
3. $F_X(x) = \begin{cases} 0.3, & 1 \leq x < 2 \\ 0.4, & 2 \leq x < 3 \\ 0.3, & x \geq 3. \end{cases}$
4. $F_X(x) = \begin{cases} 0 & x < 1 \\ 0.3, & 1 \leq x < 2 \\ 0.7, & 2 \leq x < 3 \\ 1, & x \geq 3. \end{cases}$

Question 3: The answer to 1.4 is:

1. $\frac{1}{2}$
2. $\frac{5}{7}$
3. $\frac{5}{6}$
4. 1

Question 4: The answer to 1.5 is:

1. $\frac{3}{5}$
2. 1
3. $\frac{55}{6}$
4. $\frac{55}{3}$

Examination Question 2
Assume that the cumulative distribution function of a continuous random variable X is given by

$$F_X(x) = \begin{cases} 0, & x < -1 \\ \frac{1}{4}(x+1), & -1 \le x < a \\ \frac{1}{2}x, & a \le x < 2 \\ 1, & x \ge 2. \end{cases}$$

2.1 Prove that this defines a cumulative distribution function only if $a = 1$.

2.2 Calculate $P(X < 0 \mid X < 1)$.

2.3 Calculate $E(X)$.

2.4 Find the distribution function $F_U(u)$ of $U = \dfrac{1}{2}X$.

Question 5: The answer to 2.2 is:

1. 0 2. $1/8$

3. $1/4$ 4. $1/2$

Question 6: The answer to 2.3 is:

1. $7/12$ 2. $3/4$

3. 1 4. $3/2$

Question 7: According to the answer to 2.4, the value of $F_U(u)$ for $-\frac{1}{2} < u < \frac{1}{2}$ is:

1. $\frac{1}{8}$ 2. $\frac{1}{4}(2u+1)$

3. $\frac{1}{4}$ 4. $\frac{1}{4}\left(\frac{1}{2}u+1\right)$

Examination Question 3
Assume that the joint density function of continuous random variables X and Y is given by

$$f_{X,Y}(x,y) = \begin{cases} \frac{1}{8}(x+y+2), & -1 < x < 1,\ -1 < y < 1 \\ 0 & \text{elsewhere.} \end{cases}$$

3.1 Find the marginal density function of X.

3.2 Calculate $P(Y > 0 \mid X > 0)$.

Question 8: The answer to 3.1 is:

1. $f_X(x) = \frac{3}{16}(x + 2x + 1), -1 < x < 1.$

2. $f_X(x) = \frac{1}{4}x + \frac{1}{2}, -1 < x < 1.$

Question 9: The answer to 3.2 is:

1. $3/8$ 2. $1/2$

3. $3/5$ 4. 1

Examination Question 4
Assume that the joint density function of continuous random variables X and Y is given by

$$f_{X,Y}(x,y) = \begin{cases} 1, & 0 < x < 1,\ 2x < y < 2 \\ 0 & \text{elsewhere.} \end{cases}$$

4.1 Draw a sketch of the region A where the joint density function of the two variables is non-zero.

4.2 Find $P(Y < 1)$.

4.3 Are the random variables X and Y independent? Justify your answer!

Question 10: The region in the answer to 4.1 is:

1. A square. 2. A triangle. 3. Neither a square nor a triangle.

Question 11: Is the line $y = 2 - 2x$ one of the boundaries of the region in 4.1?

1. Yes. 2. No.

Question 12: The answer to 4.2 is:

1. 0 2. $1/4$

3. $3/4$ 4. 1

Question 13: The answer to 4.3 is:

1. Yes, they are independent. 2. No, they are not independent.

Examination Question 5

5.1 Suppose that X_1 and X_2 are two independent random variables from the same distribution with the moment generating function $M_{X_i}(t) = m(t)$. Find the moment generating function $M_Y(t)$ of the random variable

$$Y = 2X_1 - X_2 + 2.$$

5.2 State if each of the following statements is true or false. If the statement is false, explain why it is false.

(a) If X is a discrete random variable then its cumulative distribution $F_X(x)$ must be a step function.

(b) If $F_X(x)$ is a cumulative distribution function of a random variable X, then there must be some value x such that $F_X(x) = 1$.

Question 14: The answer to 5.1 is:

1. $2X_1e^t - X_2e^t + 2e^t$ 2. $X_1e^{2t} + X_2e^{-t} + e^{2t}$

3. $m(t) + 2$ 4. $e^{2t}m(2t)\,m(-t)$

Question 15: Is the following statement true or false? "To find the answer to 5.1, we need to substitute $e^t = 1 + t + t^2/2 + ...$ into the expression Ye^t."

1. True. 2. False.

Question 16: The answer to 5.2 (a) is

1. True. 2. False.

Question 17: The answer to 5.2 (b) is

1. True. 2. False.

Examination Question 6

Let $X_1, X_2, \cdots, X_{10}$ be a random sample from a $N(-1, 1)$ distribution, independent of the random sample $Y_1, Y_2, \cdots, Y_{10}$ coming from a $N(0, 100)$ distribution.

6.1 Give the name and parameters of the distributions of the following random variables. Justify your answers!

(a)

$$U = \frac{1}{200}\left(Y_1 + 10X_1 + 10\right)^2.$$

(b)

$$V = X_1 + Y_1 + 10.$$

6.2 Find the value of A for which the random variable

$$W = A \frac{\sum_{i=1}^{6} (X_i + 1)^2}{\sum_{j=1}^{3} \left(Y_j\right)^2}$$

has the $F_{m,n}$-distribution with some degrees of freedom m and n. Give also the values of m and n. Justify your answer!

Question 18: The distribution in 6.1 (a) is:

1. A standard normal distribution.
2. Normal distribution with $\mu = 0$, $\sigma^2 = 10$.
3. Chi-squared distribution with one degree of freedom.
4. Chi-squared distribution with 10 degrees of freedom.

Question 19: The distribution in 6.1 (b) is:

1. Normal distribution with $\mu = -1$, $\sigma^2 = 100$.
2. Normal distribution with $\mu = 9$, $\sigma^2 = 101$.
3. Chi-squared distribution with one degree of freedom.
4. Chi-squared distribution with 10 degrees of freedom.

Question 20: The value of A in 6.2 is

1. $\frac{1}{2}$ 2. 2 3. 5 4. 50

ADDENDUM B: SECOND SEMESTER ASSIGNMENTS

The questions for each assignment for Semester 2 follow. .

SEMESTER 2
COMPULSORY ASSIGNMENT FOR EXAM ADMISSION

ASSIGNMENT 01
Multiple Choice Assignment
Based on your previous knowledge
Fixed closing date: 6 August 2018
Unique Assignment Number: 832585

This multiple-choice assignment will be marked by computer. Hence the closing date is **fixed** and no extension of time can be granted.

You can submit your multiple choice assignments via myUnisa. Alternatively, if you choose to submit in hard copy, please note that your answers must be entered on an optical mark reading sheet. But before you attempt that, please study in detail the relevant chapter of the publication *Study @ Unisa.* Please make sure that you know how to handle the optical mark reading sheets, since sheets which are marked incorrectly and which are rejected by the computer will not be marked by hand and students will not receive marks for such assignments.

The unique number appearing in the box above links your assignment to the corresponding set of answers in the computer. It must therefore be filled in correctly on the optical mark reading sheet.

Note that your assignment will not be returned to you. Please keep a record of your answers so that you can compare them with the correct answers.

In each of the following four questions, choose the number of the answer that you think is correct. Each correct answer gives you 25%, adding up to a total of 100%.

Question 1: Which one of the following statements does NOT hold true for ALL random variables X, Y?

1. $E\left(X^2\right) = E\left(X\right)^2$
2. $Var\left(2Y\right) = 4Var\left(Y\right)$
3. $E\left(X+2\right) = E\left(X\right) + 2$
4. $Var\left(X\right) \geq 0$

Question 2: Assume that X and Y are independent random variables. Which of the following statements is always true?

1. If $P\left(X=1\right) = 0$ then $P\left(X=1, Y=1\right) = 0$
2. $P\left(X=Y\right) = 0$
3. $P\left(X=0\right) = P\left(Y=0\right)$
4. X and Y cannot have the same distribution function.

Question 3: Which one of the following statements is true for all random variables X, Y?

1. $P(X = 2) < P(X = 3)$
2. $P(X > 0) = 1 - P(X \leq 0)$
3. $P(XY = 0) = P(X = 0)\, P(Y = 0)$
4. $P(X \leq 1) = P(X = 0) + P(X = 1)$

Question 4: Assume that $f(x)$ is the density function of some continuous random variable X. Which one of the following statements is always true?

1. $f(x) > 0$ for all $x > 0$.
2. $f(0) = 0$
3. $f(x) > 0$ must hold for at least some values of x
4. If $a > b$ then $f(a) > f(b)$

SEMESTER 2

ASSIGNMENT 02
Written Assignment
Based on Unit 2
Fixed closing date: 16 August 2018
Unique Assignment Number: 797853

Question 1

An item is manufactured on an assembly line and each item is either defective or not defective. Assume that the defective items occur independently with probability 0.2 and that we start observing at an arbitrary point on the assembly line. Let the random variable X denote the number of items we inspect before we find the first defective item.

(a) Give the name of the distribution of X and give the value of its parameter(s).

(b) Determine the probability that the first defective item is found after we have inspected 10 items.

Question 2

Suppose the random variable X has the following cumulative distribution function:

$$F_X(x) = \begin{cases} 0 & \text{for} \quad x < 2 \\ \frac{1}{3}(x-2) & \text{for} \quad 2 \leq x < 5 \\ 1 & \text{for} \quad x \geq 5 \end{cases}$$

(a) Does X have a uniform distribution? Justify your answer!

(b) Find $P(X > 3)$.

Question 3

Suppose the random variable X has the following density function:

$$f_X(x) = \begin{cases} 0 & \text{for} \quad x < 0 \\ c\left(1 - x^2\right) & \text{for} \quad 0 \leq x \leq 1 \\ 0 & \text{for} \quad x > 1 \end{cases}$$

(a) Find the value of c.

(b) Determine the cumulative distribution function $F_X(x)$ of X.

(c) Find the density function of the random variable $Y = 4 - 2X$,

(i) by using the transformation method;

(ii) by first calculating the distribution function of Y.

Question 4

If the random variable X has the following cumulative distribution function

$$F_X(x) = \begin{cases} 0 & \text{for } x < 0 \\ \frac{1}{8}x^3 & \text{for } 0 \leq x < 2 \\ 1 & \text{for } x \geq 2, \end{cases}$$

(a) Find the density function of X.

(b) Find $P(1 < X < 2)$.

Question 5

Assume that a random variable X has the following density function:

$$f_X(x) = \begin{cases} \frac{b}{x^2} & \text{for } x \geq b \\ 0 & \text{elsewhere.} \end{cases}$$

(a) Prove that $f_X(x)$ is a density function.

(b) Find the cumulative distribution function of X.

(c) Find $P(X > b + x)$ for $x > 0$.

Question 6

X is uniformly distributed in the interval $(-1, 3)$ and Y is exponential with parameter λ. Find λ such that $Var\,(X) = Var\,(Y)$.

SEMESTER 2

ASSIGNMENT 03
Written Assignment
Based on Unit 3
Fixed closing date: 3 September 2018
Unique Assignment Number: 739762

Question 1

Let X and Y be two random variables with the following joint density function:

$$f_{X,Y}(x, y) = \begin{cases} a\,(x + y + 1) & \text{for} \quad -1 < x < 1, \quad 0 < y < 2 \\ 0 & \text{elsewhere} \end{cases}$$

(a) Find the value of a.

(b) Find the joint distribution function $F_{X,Y}\,(x, y)$.

(c) Find the marginal density functions of X and Y.

(d) Find the conditional density of Y given $X = x$.

(e) Calculate $P\,(X > 0, Y > 1)$.

(f) Calculate $P\,(X > 0 \mid Y > 1)$.

(g) Calculate $P\,(X > Y)$.

Question 2

Let the joint distribution function of two random variables X and Y be given by

$$F_{X,Y}(x, y) = (xy)^{1/2} + xy - (xy)^2\,, \quad x,\; y \geq 0.$$

(a) Find the joint density function.

(b) Calculate $P\,(X > 1, Y > 3)$.

Question 3

Let X and Y be two random variables with the joint density function

$$f_{X,Y}(x,y) = \begin{cases} cx & \text{for} \quad 0 < x < 1, \quad 1 - x \leq y \leq 1 \\ 0 & \text{elsewhere.} \end{cases} .$$

(a) Draw a sketch of the area A in which the joint density function is non-zero.

(b) Find the value of c.

(c) Find the marginal density function f_X of X and the marginal density function f_Y of Y.

(d) Find the joint density function $F_{X,Y}(x, y)$ of X and Y in the following two regions:

  (i) $0 < x < 1, \quad 1 - x \leq y \leq 1$

  (ii) $0 < x < 1, \quad y < 1 - x$

(e) Calculate $P(X > 1/2)$ and $P(Y < 1/2)$.

SEMESTER 2

ASSIGNMENT 04
Written Assignment
Based on Units 4, 5, 6
Fixed closing date: 26 September 2018
Unique Assignment Number: 845842

Question 1

Suppose the random variable X has the following cumulative distribution function:

$$F_X(x) = \begin{cases} 0 & \text{for} \quad x < 0 \\ \frac{1}{2}x & \text{for} \quad 0 \leq x < 1 \\ \frac{1}{4}(x-1) + \frac{1}{2} & \text{for} \quad 1 \leq x < 3 \\ 1 & \text{for} \quad x \geq 3 \end{cases}$$

(a) Determine $E(X)$, $E\left(X^2\right)$ and $Var(X)$.

(b) Determine the moment generating function $M_X(t)$ of X.

(c) Use $M_X(t)$ to verify the values of $E(X)$ and $E\left(X^2\right)$ found in (a).

Question 2

Let X and Y the two independent random variables and $Z = Y - X$. Find expressions for the covariance and the correlation of X and Z in terms of the variances of X and Y.

Question 3

Assume that X has a binomial distribution with $n = 25$ and $p = 0.4$.

(a) Find the exact probabilities that $X \leq 8$ and $X = 8$.

(b) Use the normal approximation to find the approximate values of the two probabilities.

(c) Do you consider this an acceptable approximation? Justify your answer!

Question 4

Suppose that $X_i,\ i = 1, \ldots, 5$ is a random sample from a $N(0; 100)$ distribution, and $Y_j,\ j = 1, \ldots, 10$ is an independent random sample from a $N\,(-1; 4)$ distribution.

(a) Given the name of the distribution of the following random variables as well as the value(s) of the parameter(s) of the distribution. Justify your answers!

(i)

$$R = X_1 + X_2 + Y_1$$

(ii)

$$Q = \frac{1}{400}\left(\sum_{i=1}^{10}(Y_i + 1)\right)^2$$

(b) Explain how you can construct a random variable which has the t_4-distribution from the random variables $X_i,\ i = 1, \ldots, 5$.

SEMESTER 2

ASSIGNMENT 05
Written Assignment
Questionnaire
Fixed closing date: 5 October 2018
Unique Assignment Number: 797137

The questionnaire will be sent out to you later in the semester. Fill it in, and submit it in the assignment covers by the due date, to get 100% for this assignment.

SEMESTER 2

ASSIGNMENT 06
Multiple Choice Assignment
Based on the whole module
Fixed closing date: 5 October 2018
Unique Assignment Number: 646825

This multiple-choice assignment will be marked by computer. Hence the closing date is **fixed** and no extension of time can be granted.

You can submit your multiple choice assignments via myUnisa. Alternatively, if you choose to submit in hard copy, please note that your answers must be entered on an optical mark reading sheet. But before you attempt that, please study in detail the relevant chapter of the publication *Study @ Unisa.* Please make sure that you know how to handle the optical mark reading sheets, since sheets which are marked incorrectly and which are rejected by the computer will not be marked by hand and students will not receive marks for such assignments.

The unique number appearing in the box above links your assignment to the corresponding set of answers in the computer. It must therefore be filled in correctly on the optical mark reading sheet.

In the following we give questions from a previous year's examination paper. Answer first the examination questions, working through the questions on paper. Then answer the multiple choice questions based on your work. In each of the multiple choice questions, up to four possible answers are given. In each case, mark the number of the answer that you think is correct. Note that you will be sent full model solutions to the examination questions later on, so do keep your written work where you answered the examination questions so that you can compare it against the model solutions later on! Make also a note of your choices in the multiple choice questions, so that you can see where you went wrong.

Each multiple choice question counts for 5%, with a total of 100% from the twenty questions.

Examination Question 1
Two discrete random variables X and Y have a joint probability mass function given in the table below.

$p_{X,Y}(x,y) =$

		x	
		1	**2**
	1	0.1	0.4
y	**2**	0.4	0.1

1.1 Find the marginal probability mass functions $p_X(x)$ and $p_Y(y)$ of X and Y.

1.2 Are the random variables X and Y independent? Justify your answer!

1.3 Calculate $P(X = 1 \mid Y = 2)$.

1.4 Find the conditional probability mass function of Y given that $X = 1$.

1.5 Find the probability mass function $p_Z(z)$ of the random variable $Z = XY$.

Question 1: The answer to 1.2 is:

1. Yes. 2. No.

Question 2: The answer to 1.3 is:

1. $\frac{2}{5}$ 2. $\frac{1}{2}$ 3. $\frac{4}{5}$ 4. 1

Question 3: The answer to 1.4 is:

1. 0.5 2. 1 3. 1.8 4. None of these answers is correct.

Question 4: The answer to 1.5 is:

1. 0.25 2. 1 3. 2.1 4. None of these answers is correct.

Examination Question 2
Suppose that X is a random variable with the following density function:

$$f_X(x) = \begin{cases} \frac{3}{2}x^2 & \text{for } -1 \leq x \leq 1, \\ 0 & \text{elsewhere.} \end{cases}$$

2.1 Find the cumulative distribution function of the random variable X.

2.2 Calculate $P(X < 0 \mid X < 1/2)$.

2.3 Find the density function of the random variable $U = \frac{1}{4}X$.

Question 5: The answer to 2.1 gives:

1. $F_X(x) = 1, \quad -1 \le x \le 1$
2. $F_X(x) = 3x, \quad -1 \le x \le 1$
3. $F_X(x) = \frac{1}{2}x^3, \quad -1 \le x \le 1$
4. $F_X(x) = \frac{1}{2}x^3 + \frac{1}{2}, \quad -1 \le x \le 1$

Question 6: The answer to 2.2 is:

1. $\frac{1}{2}$
2. $\frac{8}{9}$
3. $\frac{8}{7}$
4. 1

Question 7: In 2.3, the value of the density function $f_U(u)$ in the interval $-1/4 < u < 1/4$ is:

1. 24
2. $24u$
3. $24u^2$
4. $96u^2$

Examination Question 3

Suppose that the random variables X and Y have the joint density function given by

$$f_{XY}(x, y) = \begin{cases} x - xy & \text{for } 0 < x < 2, \quad 0 < y < 1, \\ 0 & \text{elsewhere.} \end{cases}$$

3.1 Prove that the random variables X and Y are independent.

3.2 Determine the joint distribution function of X and Y for $0 < x < 2, \quad 0 < y < 2$.

3.3 Find $P\left(X < \frac{1}{2}\right)$.

Question 8: One possible way to prove the claim is 3.1 is:

1. Find f_X and f_Y and prove that $f_X \cdot f_Y = 1$.
2. Find f_X and f_Y and prove that $f_X \cdot f_Y = f_{XY}$.
3. Find f_X and f_Y and prove that $f_X = f_Y$.

Question 9: The answer to 3.2 is found by evaluating the integral:

1. $\int_0^2 \left(\int_0^2 (x - xy)\, dx\right) dy$
2. $\int_0^2 \left(\int_0^y (x - xy)\, dx\right) dy$

3. $\int_0^x \left(\int_0^y (u - uv)\, du\right) dv$ 4. $\int_0^y \left(\int_0^x (u - uv)\, du\right) dv$

Question 10: The answer to 3.3 is:

1. $\frac{1}{4}$ 2. $\frac{1}{16}$ 3. $\frac{3}{16}$ 4. $\frac{7}{64}$

Examination Question 4
State if each of the following statements is true or false. If the statement is false, explain why it is false.

(a) If X and Y are two continuous random variables such that the joint density function $f_{XY}(x, y)$ depends on y then X and Y cannot be independent.

(b) If X and Y are two discrete random variables for which the marginal probability mass functions of X and Y both add up to 1, then X and Y are independent.

(c) If X and Y are two random variables such that $F_{XY}(a, b) = 1$ for some values a and b, where F_{XY} is their joint distribution function, then we know that always $X \leq a$ and $Y \leq b$.

(d) If X and Y are two continuous random variables such that $f_{XY}(a, b) = 0$ for some values a and b, where f_{XY} is their joint density function, then we know that always $X \leq a$ and $Y \leq b$.

(e) Let X be a random variable with the distribution function $F(x)$. If X is continuous then $\frac{dF(x)}{dx}$ gives the density function of X; and if X is discrete then $\frac{dF(x)}{dx}$ gives the probability mass function of X

Question 11: The answer to 4.2 (a) is:

1. True 2. False

Question 12: The answer to 4.2 (b) is:

1. True 2. False

Question 13: The answer to 4.2 (c) is:

1. True 2. False

Question 14: The answer to 4.2 (d) is:

1. True 2. False

Question 15: The answer to 4.2 (e) is:

1. True 2. False

Examination Question 5

1. Let X and Y be two random variables with the joint density function

$$f_{X,Y}(x,y) = \begin{cases} \frac{1}{2}, & 0 < x < 2,\ 0 < y < x \\ 0 & \text{elsewhere.} \end{cases}$$

5.1 Draw a sketch of the area A in which the joint density function is non-zero.

5.2 Find the joint distribution function of X and Y in the region $0 < x < 2,\ y > x$.

Question 16: The area A in 5.1 is:

1. A square 2. A triangle 2. Neither a square nor a triangle

Question 17: The answer to 5.2 is found by evaluating the integral:

1. $\int_0^2 \left(\int_0^x \frac{1}{2} dv\right) du$ 2. $\int_0^2 \left(\int_x^\infty \frac{1}{2} dv\right) du$

3. $\int_0^x \left(\int_0^y \frac{1}{2} dv\right) du$ 4. $\int_0^x \left(\int_0^u \frac{1}{2} dv\right) du$

Examination Question 6

Let $X_1, X_2, \cdots, X_4$ be a random sample from a normal distribution with mean -1 and variance 9 and $Y_1, Y_2, \cdots, Y_6$ be an independent random sample from a normal distribution with mean 0 and variance 16.

6.1 Give the distributions and degrees of freedom of the following random variables. Justify your answers!

(a) $T = \frac{1}{25}(X_1 + Y_1 + 1)^2$

(b) $U = \frac{1}{6}\sum_{i=1}^{6}(Y_i + 1)$

6.2 Let

$$V = A\,\frac{Y_4}{\sqrt{\sum_{i=1}^{4}(X_i+1)^2}}.$$

What should the value of the constant A be for V to have the t_4 distribution? Justify your answer!

Question 18: The answer to 6.1 (a) is:

1. $N\left(\mu=-1,\sigma^2=25\right)$ 2. t_{25} 3. χ^2_1 4. χ^2_2

Question 19: The answer to 6.1 (b) is:

1. $N\left(\mu=0,\sigma^2=16\right)$ 2. $N\left(\mu=1,\sigma^2=\frac{16}{6}\right)$ 3. χ^2_1 4. χ^2_6

Question 20: The value of A in 6.2 is:

1. $\frac{1}{4}$ 2. $\frac{3}{2}$ 3. 2 4. 4