Tutorial Letter 101/3/2018

Mathematics III (Engineering)

MAT3700

Semesters 1 and 2

Department of Mathematical Sciences

This tutorial letter contains important information about your module.
Any updates and follow-up tutorial letters will be posted on myUNISA under additional resources.

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Dear Student

1 INTRODUCTION

Welcome as a student to **Mathematics III, MAT3700**.

Check your registration papers now to confirm for which semester you are registered. Call the lecturer if in doubt or check on myUNISA.

If you are registered for semester 1 you will write your final examination in May/June 2018 and qualify for this by doing assignments for semester 1.

If you are registered for semester 2 you will write your final examination in October/November 2018 and qualify for this by doing assignments for semester 2.

1.1 Study material

You will receive ***two*** study guides. Two topics are not covered in your study guides: Linear Algebra and Fourier Series. To master this content you need to buy the prescribed book named in paragraph 4 of this letter. The prescribed book will also provide more examples and exercises on the other topics in this module.

You need to work on your mathematics regularly. See an example of a study plan further on in this letter. The amount of time you study is not important, but how efficient you use that time is very important.

If you have access to the Internet, you can view the study guides and tutorial letters for the modules for which you are registered on the University's online campus, *myUnisa*, at http://my.unisa.ac.za under official study material. You will also find past papers under official study material. You should also check additional resources for hints and revision notes.

2 PURPOSE AND OUTCOMES

2.1 Purpose

Students completing this module will be able to solve first-order ordinary differential equations and second order ordinary differential equations using the method of undetermined coefficients, solve any order differential equations using d-operators and laplace transforms, to find the eigenvalues and eigenvectors of a matrix and write the Fourier series of a function.

This module will support students in their studies in the field of engineering and the physical sciences as part of a diploma.

2.2 Outcomes

Specific outcome 1**:**
Solving first order differential equations
Assessment criteria:

1. Use direct integration, separation of variables, substitution and the integrating factor method to solve exact, linear, Bernoulli and homogeneous first-order differential equations.

2. Apply knowledge to solve practical problems involving growth and decay, cooling, mixtures and falling bodies.

Specific outcome 2**:**

Solving second order differential equations of the form $P_0 \frac{d^2 y}{dx^2} + P_1 \frac{dy}{dx} + P_2 y = Q$ where Q is equal to zero, a constant, or a function of x only, and P_0, P_1 and P_2 are constants.

Assessment criteria

1. Use the method of undetermined coefficients to find the general solution of second order differential equations.
2. Be able to find the particular solution from the general solution when given the necessary conditions.

Specific outcome 3**:**

Solving second order differential equations of the form $P_0 \frac{d^n y}{dx^n} + \ldots + P_1 \frac{dy}{dx} + P_2 y = Q$ where Q is equal to zero, a constant, or a function of x only, and P_0, P_1 and P_2 are constants and n is a natural number.

Assessment criteria

1. Use D-operator methods to find the general or particular solution.
2. Use Laplace transforms and inverse Laplace transforms to find the particular solution.

Specific outcome 4**:**

Determine eigenvalues and eigenvectors of a matrix

Assessment criteria

1. Be able to calculate the eigenvalues of a 2x2 or 3x3 matrix.
2. Given an eigenvalue of a 2x2 or 3x3 matrix be able to find an eigenvector corresponding to the eigenvalue.

Specific outcome 5**:**

Representing a function as a Fourier series.

Assessment criteria

1. Be able to sketch a function over a given range and expand the sketch to represent a periodic function.
2. Obtain the Fourier series expansion of the periodic function.

3 LECTURER(S) AND CONTACT DETAILS

Please have your student number at hand before contacting any department at UNISA.

If you have access to a computer that is linked to the internet, you can quickly access resources and information at the University. The *myUnisa* learning management system is Unisa's online campus that will help you to communicate with your lecturers, with other students and with the administrative departments of Unisa .

To go to the *my*Unisa website, start at the main Unisa website, http://www.unisa.ac.za, and then click on the "Login to *myUnisa*" link on the right-hand side of the screen. This should take you to the *myUnisa* website. You can also go there directly by typing in http://my.unisa.ac.za.

You can access *myUnisa* at your regional office learner centre (without any cost) if you do not have internet at home.

3.1 Lecturer(s)

Your lecturer at the time of compiling this tutorial letter (July 2017) is Miss LE Greyling, Science Campus, Florida, Roodepoort.

If you experience any problems with the ***mathematical content*** you are welcome to contact the lecturer:

1) by e-mail (Lgreylin@unisa.ac.za)
2) by sending a message using the Questions and Answers function on myUNISA.
3) by telephone (011-471-2350) If you do not have sufficient funds on your phone you may ask the lecturer to return your call.
4) by fax (086 274 1520)
5) or personally. For a personal visit you must make an appointment by telephone or e-mail. You must be prepared to come to the Science Campus in Roodepoort for a personal visit.

You should spend time on a problem but do not brood over it, in most cases you only need a hint from the lecturer or tutor to solve the problem or to move forward with your studies. You can save valuable time if you contact your lecturer when needed. If you disagree with a solution in the study guide make contact so that we can work together to correct mistakes.

You must mention your **student number** and the code **MAT3700** in all enquiries about this module. **Any question without your student number and code MAT3700 will be ignored.**

3.2 Department

The department of Mathematical Sciences is situated on the UNISA Science Campus in Florida. The secretary of the department is available on 011-670-9147.

3.3 University

Read the brochure on S*tudy@Unisa2018.* Check this brochure on where to direct administrative enquiries. Your **student number** should be in the **subject line** of any e-mail to a service department at Unisa.

Tip: Do not write any administrative requests like the change of contact details or examination venue in your assignment. The markers cannot help you. Likewise any messages for the lecturer must be directed to the lecturer by telephone, fax or e-mail.

4 RESOURCES

4.1 Prescribed books

Your study guides do not cover the outcomes sufficiently.

First Author	Year	Title	Edition	Publisher	ISBN
Bird, John	2014	Higher Engineering Mathematics	7th	Routledge(Newnes)	978-0-415-66282-6

Note: The 6th or 8th edition will also be acceptable. Students who intend to proceed to EMT4801 are advised to buy the 8th edition.

Prescribed books can be obtained from the University's official booksellers. Please refer to the list of official booksellers and their addresses in the Study@Unisa brochure. If you have difficulty locating your book at these booksellers, please contact the Prescribed Books section at 012 429 4152 or e-mail Vospresc@unisa.ac.za.

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4.2 Recommended books

You may consult the following book in order to broaden your knowledge of MAT3700
A **limited** number of copies are available in the Library and at learning centers:

First Author	Year	Title	Edition	Publisher	ISBN
James,G	2011	Advanced Modern Engineering Mathematics	4th	Pearson-Prentice Hall	978-0-273-71923-6

4.3 Electronic reserves (e-reserves)

There are no e-reserves for this module.

4.4 Library services and resources information

For brief information, go to www.unisa.ac.za/brochures/studies

For detailed information, go to the Unisa website at http://www.unisa.ac.za/ and click on **Library**.

For research support and services of personal librarians, go to http://www.unisa.ac.za/Default.asp?Cmd=ViewContent&ContentID=7102.

The library has compiled a number of library guides:

- finding recommended reading in the print collection and e-reserves – http://libguides.unisa.ac.za/request/undergrad
- requesting material – http://libguides.unisa.ac.za/request/request
- postgraduate information services – http://libguides.unisa.ac.za/request/postgrad
- finding, obtaining and using library resources and tools to assist in doing research – http://libguides.unisa.ac.za/Research_Skills
- how to contact the library/finding us on social media/frequently asked questions – http://libguides.unisa.ac.za/ask

5 STUDENT SUPPORT SERVICES

5.1 Tutor Support

Currently there is no tutor support for level three modules. Contact the lecturer with any questions on the subject matter.

Note: E-tutors and classes at regional offices are organized by student support services. The lecturer is not responsible for organizing any tutorial activities.

5.2 Discussion classes

There are no discussion classes by the lecturer for this subject.

5.3 Other services @ Regional offices

For information on the various student support systems and services available at Unisa (e.g. student counselling, language support, academic writing), please consult the publication S*tudy@Unisa2018* that you received with your study material.

6 STUDY PLAN

To successfully prepare for submitting your assignments you have to work according to a time table. **Use the following table or draw up your own table** to schedule your studies for this subject.

Be realistic. You need to add additional factors like work, family commitments and rest. If you do not schedule your "play time" you will feel guilty and stressed instead of relaxing and building up energy for your studies.

Warning: Do not study selectively. To explain, do not take the assignment and try to find similar questions in the study guides and only work through those questions.

WEEK	STUDYGUIDES	TEXT BOOK (7^{th} ed) Chapter	Content
1	Module 1 Unit 1 and 2	49, 50 and 51	First Order Differential Equations Applications of First Order Differential Equations
		Complete assignment 1	
	Module 1 Unit 3	52	Numerical Methods
2	Module 2 Unit 1 and 2	53 and 54	Solving second-order differential equations with constant coefficients Method of undetermined coefficients
3	Module 3 Unit 1 and 2		Introduction to Differential Operators Differential Equations and D-operator Methods
4	Module 3 Unit 3 and 4		Simultaneous equations and numerical methods. Practical Problems solved with D-operator methods
5	Module 4 Unit 1	67 and 68	Laplace Transforms
6	Module 4	69	Inverse Laplace Transforms

	Unit 2		
7	Module 4 Unit 3	70	Special Functions: Unit step and impulse functions
8	Module 4 Unit 4	71	Solving differential equations with Laplace transforms
9	Module 4 Unit 5	72	Solving simultaneous differential equations with Laplace transforms
	Module 2 Unit 6		Practical Problems solved with Laplace Transforms
		Complete assignment 2	
10	Module 5 Unit 1 and Unit 2	25.4 and 25.5	Gauss Elimination Eigenvalues and eigenvectors
11	Module 6 Unit 1	73	Fourier Series for periodic functions of period 2 π
	Module 6 Unit 2	74	Fourier Series for non-periodic functions over a range of 2 π
12	Module 6 Unit 3	75	Even and Odd functions and Half-range Fourier series
13	Module 6 Unit 4	76	Fourier Series over any range
		Complete assignment 3	
Revision: Do preparation papers			

7 PRACTICAL WORK AND WORK-INTEGRATED LEARNING

There is no practical work for this module

8 ASSESSMENT

8.1 Assessment criteria

See the assessment criteria listed with each outcome under point 2 in this letter.

8.2 Assessment plan

Your performance in your assignments plays a vital part in your final mark. Submission of an assignment gives you admission to the examination. Your assignment marks will be used to calculate your year mark. Your year mark will form part of your final mark for the module.

Assignment	% of Year mark	Description and Instructions
01	10	Written assignment on module 1, unit 1
02	70	Written assignment on module 1 and 2
03	20	Written assignment on module 3 and 4

Your final mark will be calculated as follows: 20% × Year mark + 80% × Examination mark

You need a final mark of **50%** in order **to pass** the subject with a subminimum of 40% on your examination mark. A **subminimum of 40%** means that if you receive less than 40% in the exam you fail and in this case your year mark does not count.

Each module in your study guides contains a self-test with solutions on MyUnisa (additional resources) to help you prepare for the compulsory assignments and the final examination.

8.3 Assignment numbers

8.3.1 General assignment numbers

You must submit assignment 01, 02 and 03 for the semester you are registered. Assignment questions per semester are given in 8.6 below.

8.3.2 Unique assignment numbers

All assignments have their own unique number per assignment and per semester given in the table below

8.4 Assignment due dates

		Semester1	*Unique nr.*	Semester 2	Unique nr.
Assignment 01	Written	*14 February*	*659077*	15 August	890287
Assignment 02	Written	*22 March*	*812211*	18 September	766813
Assignment 03	Written	*12 April*	*671911*	27 September	754300

8.5 Submission of assignments

Plan your programme so that study problems can be sorted out in time. The dates on *myUNISA* may differ from dates in your tutorial letter and messages from the university. You may confirm dates with the lecturer if a message seems suspect.

For Mathematics III you have to send in three assignments to the university before the given closing dates. You will not be permitted to write the examination if you do not send in at least one assignment.

Submit the assignments linked to your registration period (semester 1 or 2).
Note: use the correct unique number, assignment number and module code.

8.5.1 Written assignments

The rules for assignments are:

- **Please keep a copy of your answers**.
- Submit answers in numerical order.
- Keep to the due dates.
- Write clearly with a black pen.
- Marks may be deducted or not given if answers are scratched out or difficult to read.

Write your answers down in the correct order and make sure that every answer is numbered clearly. Make sure that your answers are clear and unambiguous.

Do not string a series of numbers together without any indication of what you are calculating. Be careful with the use of the equal sign (=). The correct units must be shown in you answer.

Note that we are not only interested in whether you can get the correct answer, but also in whether you can **formulate your thoughts correctly.** Mere calculations are not good enough – you have to make sure that what you have written down consists of mathematically correct notation, which makes sense to the marker.

Students must send in their own work. Of course, it is a good thing to discuss problems with fellow students. However, where copying has clearly taken place disciplinary action will be taken.

An information sheet containing the formulas is enclosed at the end of this letter for your convenience. Keep this sheet at hand when completing your assignments. The same sheet will be supplied during the examination. Consult this sheet regularly, it may mean the difference between success and failure in this module. Make sure you know how to use the table of integrals in reverse to find derivatives. You need not memorize all formulas and can check memorized formulas.

You may submit written assignments either **by post** or **electronically** via *myUnisa*. Choose one way to submit, do not use both. Double submissions waste not only my time but also cause extra work for the assignment department. If the marker cannot access an electronic submission we will send you an e-mail. Assignments may **not** be submitted by fax, e-mail or registered post.

Assignments by post:

Ensure that you complete the assignment cover. If the subject or assignment number is incorrect your assignment cannot be noted as received. Each assignment must have a separate cover with the unique number. Submit one assignment per envelope. All regional offices have Unisa post boxes. Only use the SA postal services if you cannot get to a regional office or one of the drop-off boxes listed in Study@Unisa2018.

Assignments should be addressed to: The Registrar, PO Box 392, UNISA, 0003

Your marked assignment will be posted back to you.

To submit via *myUnisa*:

You can scan your handwritten assignment answers to be submitted electronically. **Don't scan the assignment cover** as the system will create a cover for you when you upload the assignment. You assignment will be marked electronically. Please make sure that it is easy to read. Your assignment must be combined in one pdf document. Only one document can be uploaded per assignment. Login with your student number and password. Select the module. Click on assignments. Click on the assignment number you want to submit. Follow the instructions on the screen.

Your marked assignment will be available on myUNISA for viewing.

8.6 The assignments

8.6.1 Assignment 01 Semester 1

ONLY FOR SEMESTER 1 STUDENTS **Assignment 01** **Due Date: 14 February** **Unique number: 659077**

Submission of this assignment by the due date will give you admission to the examination This assignment contributes 10% to your year mark. *Primary Source: Paper October 2016*

QUESTION 1

Solve the following differential equations:

1.1 $x^2dy + \sqrt{y^2+4}dx + dy = 0$ (3)

1.2 $x\dfrac{dy}{dx} - y = xe^{\frac{y}{x}}$ (6)

1.3 $3\dfrac{dy}{dx} + \dfrac{y}{x} = -y^4\cos(\ell n\ x)$ (7)

QUESTION 2

Show that $2xy^3 + 2 + \left(3x^2y^2 + 8e^{4y}\right)\dfrac{dy}{dx} = 0$ is an exact differential equation. (4)

[20]

Maximum: [20]

8.6.2 Assignment 02 Semester 1

ONLY FOR SEMESTER 1 STUDENTS **Assignment 02(Compulsory)** **Due Date: 22 March** **Unique number: 812211**

This assignment is a written assignment based on Study Guide 1 and 2 *This assignment contributes 70% to your year mark.* *Primary Source: Paper October 2016*

QUESTION 1

*Find the general solutions of the following differential equations using **D-operator** methods:*

1.1 $\left(D^2 - 2D + 2\right)y = 3e^x\cos 2x$ (6)

1.2 $\left(D^2+D-2\right)y=x^2+e^{2x}$ (6)

[12]

QUESTION 2

Solve for **only** y in the following set of simultaneous differential equations by using **D-operator** methods:

$$\begin{aligned}(D+1)x+(2D+7)y &= e^t+2\\ -2x+(D+3)y &= e^t-1\end{aligned} \qquad (8)$$

[8]

QUESTION 3

3.1 Determine $L\{\cos 4(t-3)H(t-3)\}$ (2)

3.2 Determine $L\{2\sin 2x.\cosh 2x\}$ (3)

3.3 Determine $L^{-1}\left\{\dfrac{2}{(s-3)^5}\right\}$ (3)

[8]

QUESTION 4

Determine the unique solution of the following differential equation by using **Laplace transforms:**

4.1 $y''-7y'+6y=\delta(t-2)$, if $y(0)=0$ and $y'(0)=0$. (6)

4.2 $y''(t)+6y'(t)+13y(t)=0$, if $y(0)=3$ and $y'(0)=7$. (7)

[13]

QUESTION 5

The equation of motion of a body performing damped forced oscillations is:

$\dfrac{d^2x}{dt^2}+5\dfrac{dx}{dt}+6x=\cos t$, with initial conditions $x(0)=\frac{1}{10}$ and $x'(0)=0$

Determine the unique solution for the displacement, x in terms of the time, t. (9)

[9]

Maximum: [50]

8.6.3 Assignment 03 Semester 1

ONLY FOR SEMESTER 1 STUDENTS
Assignment 03 (Compulsory)
Due Date: 12 April
Unique number: 671911

This assignment contributes 20% to your year mark.

Source: Paper October 2016

QUESTION 1

If $A = \begin{bmatrix} 1 & 0 & 4 \\ 0 & 2 & 0 \\ 3 & 1 & -3 \end{bmatrix}$, find the eigenvalues of A and then find an eigenvector corresponding to the eigenvalue $\lambda = 2$. (9)

[9]

QUESTION 2

Determine the Fourier series expansion of a function *f(x)* with period 2π defined by

$$f(x) = \begin{cases} 0 & -\pi < x < -\frac{\pi}{2} \\ 4 & -\frac{\pi}{2} < x < \frac{\pi}{2} \\ 0 & \frac{\pi}{2} < x < \pi \end{cases}$$ (11)

[11]

Maximum: [20]

8.6.4 Assignment 01 Semester 2

ONLY FOR SEMESTER 2 STUDENTS
Assignment 01 (Compulsory)
Due Date: 15 August
Unique number: 890287

This assignment contributes 10% to your year mark.

Primary Source: Paper May 2017

QUESTION 1

Solve the following differential equations:

1.1 $x^3 + (y+1)^2 \dfrac{dy}{dx} = 0$ (3)

1.2 $\dfrac{dy}{dx} + y\tan x = y^3 \sec^4 x$ (7)

1.3 $x\dfrac{dy}{dx} - y = x\tan\left(\dfrac{y}{x}\right)$ [Hint: Put $y = vx$] (5)

1.4 $e^x \sin y - 2x + \left(e^x \cos y + 1\right)\dfrac{dy}{dx} = 0$ (5)

[20]

Maximum: [20]

8.6.5 Assignment 02 Semester 2

ONLY FOR SEMESTER 2 STUDENTS
Assignment 02(Compulsory)
Due Date: 18 September
Unique number: 766813

This assignment is a written assignment based on Study Guide 1 and 2

This assignment contributes 70% to your year mark.

Source: Paper May 2017

QUESTION 1

Find the general solutions of the following differential equations using **D-operator** methods:

1.1 $\left(D^2 - 4D + 4\right)y = x + e^{2x}\sinh 2x$ (8)

1.2 $\left(D^2-3D+2\right)y=\sin x$, **given** that if $x=0$, then $y=0$ and $\dfrac{dy}{dt}=0$. (10)

[18]

QUESTION 2

Solve for **x** and ***y*** in the following set of simultaneous differential equations by using **D-operator** methods:

$$2\frac{dx}{dt}-5x+\frac{dy}{dt}=e^t$$
$$\frac{dx}{dt}-x+\frac{dy}{dt}=5e^t$$

(9)

[9]

QUESTION 3

Given $f\left(t\right)=\begin{cases}2 & 0\le t\le 2\\ -1 & 2\le t<3\\ 0 & 3<t\end{cases}$

3.1 Sketch the function *f (t).* (1)

3.2 Rewrite the function in terms of the Heaviside(unit) step function. (2)

3.3 Find the **Laplace transform** of *f (t).* (2)

[5]

QUESTION 4

Determine the unique solution of the following differential equation by using **Laplace transforms:** $y''\left(t\right)+2y'\left(t\right)+y\left(t\right)=te^{-t}$, given $y\left(0\right)=1$, $y'\left(0\right)=-2$ (8)

[8]

QUESTION 5

For a certain electrical circuit the applicable differential equation is: $\dfrac{d^2q}{dt^2}+4\dfrac{dq}{dt}+5q=5\cos 2t$

Determine the general solution for the charge $q(t)$ if at $t=0, q(t)=0$ and $q'(t)=0$. Find the steady state solution for the current $i(t)=\dfrac{dq}{dt}$. (10)

[10]

Maximum: [50]

8.6.6 Assignment 03 Semester 2

ONLY FOR SEMESTER 2 STUDENTS
Assignment 03 (Compulsory)
Due Date: 27 September
Unique number: 754300

This assignment contributes 20% to your year mark.

Source: Paper May 2016

QUESTION 1

Find the eigenvalues of $A = \begin{bmatrix} 1 & 2 & 0 \\ 1 & 2 & -1 \\ 0 & 0 & 4 \end{bmatrix}$, and an eigenvector corresponding to $\lambda = 0$.

(9)

[9]

QUESTION 2

A function $f(t)$) is defined by:

$f(t) = \pi - t \qquad 0 < t < \pi$

Determine the half range Fourier cosine series of $f(t)$ with period 2π. (11)

[11]

Maximum: [20]

8.7 Other assessment methods

There are no other assessment methods for this module.

8.8 The examination

Particulars about the examination will be sent to you by the examinations division during the semester. To write the examination you must have one assignment registered on the student system before 30 March for semester 1 students and before 8 September for semester 2 students. Note that lecturers cannot give admission to the examination if you failed to obtain access to the examination nor can we change the examination date or your chosen venue.

Please check your permission to write the examination a month before the examination on myUNISA. Also check your examination date and center.

Copies of previous examination papers are not available on request from the lecturer. The most recent paper is available on MyUnisa under official study material without memorandum**.**

Results can be viewed on *myUnisa* and will be posted to you.

Included are preparation examination papers. These are old papers showing the kind of questions as well as the topics covered. The papers must be done after you have submitted your second compulsory assignment. The answers to these papers are in addendum 12.2 in this letter.

When attempting a preparation paper, work under examination conditions. Your paper will be two hours and 80 marks. Divide the paper into two sections of one hour each. Sit down and attempt to do all the questions in that section without referring to your notes. Use the answers to mark your work. If you cannot find your own mistakes e-mail your answer to the lecturer for correction. Determine which areas you need to concentrate on before the examination.

8.8.1 Preparation Paper I (October 2009)

QUESTION 1

1.1 Solve the following differential equations:

a) $\left(x^3+y^3\right)=3xy^2\dfrac{dy}{dx}$ [Hint: Let $y=vx$] (7)

b) $\cos y+\left(\dfrac{1+e^x}{e^x}\right)(\sin y)\dfrac{dy}{dx}=0$ (5)

1.2 Consider a tank full of water which is being drained out through an outlet.

The following differential equation is applicable:

$$\frac{dH}{dt}=-\left(2.8\times10^{-3}\right)\sqrt{H}$$

where H is height of the water inside the tank in meters(m) and t is time in seconds(s) to drain the water. Find an expression for the height in terms of time, given that when t = 0, then H = 4m. (5)

[17]

QUESTION 2

2.1 Determine the *general solutions* of each the following differential equations using **D-operator methods**:

a) $\left(D^2+D-2\right)y=2\cosh 2x$ (9)

b) $\left(D-2\right)^2 y=e^{2x}x^{-3}$ (5)

2.2 Solve for ***x only*** in the following set of simultaneous differential equations by using **D-operator methods:**

$$\frac{dx}{dt}=3x-y-1$$

$$\frac{dy}{dt}=x+y+4e^t$$

(7)

2.3 In an L-C circuit, L = 1 henry, C = $\dfrac{1}{16}$ farad and E(t) = 60 volts. The differential equation $\dfrac{d^2q}{dt^2}+16q=60$ represents the capacitor charge at any time t.

If $q(0)=0$ and $i(0)=0$ use **D-operator methods** and find:

2.3.1 the charge q on the capacitor at any time t.

2.3.1 the current i. $\left[\text{Hint: } i=\dfrac{dq}{dt}\right]$ (8)

[29]

QUESTION 3

3.1 Determine the following:

a) $L\left\{10t.H\left(t+\pi\right)\right\}$ (3)

b) $L^{-1}\left\{\frac{1}{3p^2+1}\right\}$ (2)

3.2 Determine the ***unique*** solution of the following differential equation by using **Laplace transforms**:

$y''(t)-9y(t)=\cosh 3t$, given $y(0)=0$ and $y(0)=4$ (7)

Hint: $\left[\frac{s}{\left(s^2-9\right)^2}=\frac{1}{12}\frac{1}{\left(s-3\right)^2}-\frac{1}{12}\frac{1}{\left(s+3\right)^2}\right]$

3.3 Use **Laplace transforms** to determine the current for the circuit i that is represented by the differential equation

$$20\frac{d^2i}{dt^2}+80\frac{di}{dt}+\frac{i}{0{,}01}=0$$

for which the initial conditions are so that $i(0)=0$ and $i'(0)=5$.

Discuss the motion if $t\to\infty$. (6)

[18]

QUESTION 4

A function *f*(*x*) is defined by:

$$f(x)=\begin{cases}-6 & -\pi<x<0\\ 6 & 0<x<\pi\end{cases} \qquad \left[f(x)=f(x+2\pi)\right]$$

4.1 Sketch *f*(*x*) and state whether the function is odd, even or neither. (2)

4.2 Find the Fourier expansion of *f*(*x*). (8)

[10]

QUESTION 5

In control engineering the system poles, λ, of a system are the eigenvalues of a given matrix *A.*

Determine the system poles for $A=\begin{bmatrix}1 & 2\\ -4 & -3\end{bmatrix}$. (5)

[5]

FULL MARKS: 80

8.8.2 Preparation Paper II (October 2010)

QUESTION 1

Solve the following differential equations:

1.1 $(\cos x+\sin x)dy+(\cos x-\sin x)dx=0$ (4)

1.2 $x\dfrac{dy}{dx}=y+xe^{\frac{y}{x}}$ given that $y(1)=1$ (7)

1.3 $y'-y=xy^2e^{2x}$ (7)

[18]

QUESTION 2

Find the general solutions of the following differential equations using **D-operator** methods:

2.1 $\left(D^2+9\right)y=72xe^{3x}$ (7)

2.2 $\left(D^2+2D+2\right)y=2e^{-x}\sin 2x$ (7)

[14]

QUESTION 3

The conditions in a certain electrical circuit is represented by the following differential equation:

$$\frac{1}{50}\frac{d^2q}{dt^2}+\frac{dq}{dt}+8q=50\cos 30t$$

By using **D-operator** methods determine:

3.1 An expression for q in terms of t. (7)

3.2 An expression for the current i. $\left(\text{Hint } i=\dfrac{dq}{dt}\right)$. (1)

3.3 The amplitude and the frequency of the steady-state current. (2)

[10]

QUESTION 4

Solve for x by using **D-operator** methods in the following set of simultaneous equations:

$$2\frac{dx}{dt}-5x+\frac{dy}{dt}=e^t$$
$$\frac{dx}{dt}-x+\frac{dy}{dt}=5e^t$$
(7)

[7]

QUESTION 5

Determine the following:

5.1 $L\left\{(2t-3).H(t-1)\right\}$ (3)

5.2 $\quad L^{-1}\left\{\dfrac{1}{(s-1)^3}+\dfrac{1}{s^2+2s-8}\right\}$ (4)

5.3 $\quad L^{-1}\left\{\dfrac{e^{-\frac{\pi}{2}s}}{s^2+9}\right\}$ (3)

[10]

QUESTION 6

The equation given applies to a certain beam:

$$\frac{d^4y}{dx^4}=\frac{1}{5}\delta(x-5)$$

Determine a unique solution for *y* (the sag) by using Laplace transforms and hence determine the sag at the point x = 5 m. The boundary values of the equation are $y(0)=y'(0)=y''(10)=y'''(10)=0$. (11)

[11]

QUESTION 7

Find the eigenvalues of $A=\begin{bmatrix}-4 & 1 & 1\\ 1 & 5 & -1\\ 0 & 1 & -3\end{bmatrix}$. (4)

[4]

QUESTION 8

A periodic function *f(x)* with period 2π is defined by:

$$f(x)=\begin{cases}\dfrac{x+\pi}{2} & -\pi<x<0\\ \dfrac{x-\pi}{2} & 0<x<\pi\end{cases}$$

Determine the Fourier expansion of the periodic function *f(x).* (8)

[8]

(Total:82) **Full marks = 80**

8.8.3 Preparation Paper III (May 2011)

QUESTION 1

Solve the following differential equations:

1.1 $\dfrac{dy}{dx} + y\cot x = \cos x$ (5)

1.2 $\left(x^2 + y^2\right)dx + \left(x^2 - xy\right)dy = 0$ (6)

[11]

QUESTION 2

Find the general solutions of the following differential equations using **D-operator** methods:

2.1 $\left(D^2 - 3D + 2\right)y = \sin 3x$ (8)

2.2 $\left(D^2 + 6D + 9\right)y = e^{-2x}\cosh 2x$ (6)

[14]

QUESTION 3

In an R-L-C series circuit, the differential equation for the instantaneous charge $q(t)$ on the capacitor is $L\dfrac{d^2q}{dt^2} + R\dfrac{dq}{dt} + \dfrac{q}{C} = E(t)$. Determine the charge $q(t)$ and current $i(t)$ for a circuit with $R = 10$ ohm, $L = 1$ henry, C $= 10^{-2}$ farad and E(t) = $50\cos 10t$ volts by using **D-operator methods.** What is the steady-state current for this circuit?

(9)

[9]

QUESTION 4

Solve the following set of simultaneous equations by using **D-operator** methods:

$$\frac{dx}{dt} = 3x - y - 1$$
$$\frac{dy}{dt} = x + y + 4e^t$$

(10)

[10]

QUESTION 5

Determine the following:

5.1 $L\left\{e^t\cos 2t\right\}$ (2)

5.2 $L^{-1}\left\{\frac{8se^{-2s}}{s^2-9}\right\}$ (2)

[4]

QUESTION 6

6.1 Solve the given equation by using **Laplace transforms**:

$$\frac{d^2y}{dt^2}+y=\sin t$$

The initial values of the equation are $y(0)=1$ and $y'(0)=0$. (7)

6.2 The equation of motion of a system is

$$\frac{d^2x}{dt^2}+5\frac{dx}{dt}+4x=3.\delta(t-2)$$

If $t=0,$ *then* $x=2$ and $x'(t)=-2$.

Use Laplace transform methods to find an expression for the displacement *x* in terms of *t.*

(12)

[19]

QUESTION 7

The period, T, of natural vibrations of a building is given by $T=\frac{2\pi}{\sqrt{-\lambda}}$ where λ is an eigenvalue of matrix *A.* Find the period(s) if $A=\begin{bmatrix}-2 & 1\\ 1 & -2\end{bmatrix}$. (5)

[5]

QUESTION 8

Determine the half-range Fourier cosine expansion to represent the function *f(t)* defined by $f(t)=t,\quad 0<t<\pi$. (8)

[8]

Full marks = 80

8.8.4 Preparation Paper IV (October 2011)

QUESTION 1

Solve the following differential equations:

1.1 $(\cos y + y\cos x)dx + (\sin x - x\sin y)dy = 0$ (4)

1.2 $\cos^2 x\dfrac{dy}{dx} = y + 3$ (3)

1.3 $\dfrac{dy}{dx} + y\tan x = -y^3 \sec x$ (7)

[14]

QUESTION 2

Find the general solutions of the following differential equations using **D-operator** methods:

2.1 $(D^2 - 6D + 9)y = e^{3x} + e^{-3x}$ (6)

2.2 $(D^2 + 2D + 5)y = e^{-x}\cos 3x$ (6)

[12]

QUESTION 3

In an R-L-C series circuit, the differential equation for the instantaneous charge $q(t)$ on the capacitor is $\dfrac{d^2q}{dt^2} + 6\dfrac{dq}{dt} + \dfrac{q}{0,04} = 204\sin t$. Determine the charge $q(t)$ subjected to the initial conditions $q(0) = q'(0) = 0$ by using **D-operator methods.** Show that the steady-state solution for the current $i(t)$ is given by $2\sin t + 8\cos t$.

(11)

[11]

QUESTION 4

Solve for ***y* only** in the following set of simultaneous equations by using **D-operator** methods:

$$\begin{aligned}(D-1)x + (D+9)y &= t\\ (D-2)x + (2D+9)y &= 4\end{aligned}$$ (7)

[7]

QUESTION 5

Determine the following:

5.1 $L\left\{\left(1-e^{t}\right)^{2}\right\}$ (3)

5.2 $L^{-1}\left\{\dfrac{e^{-2s}}{s^{4}}\right\}$ (2)

[5]

QUESTION 6

6.1 Solve the given equation by using **Laplace transforms**:

$$y''(t)++4y'(t)+4y(t)=4e^{-2t}$$

The initial values of the equation are $y\left(0\right)=-1$ and $y'\left(0\right)=4$. (10)

6.2 The motion of a spring is given by the equation

$$\frac{d^2x}{dt^2}+9x=3\delta\left(t-\pi\right)$$

If $x\left(0\right)=1,$ and $x'\left(0\right)=0$, use Laplace transform methods to find an expression for the displacement x in terms of *t.* (8)

[18]

QUESTION 7

Find all the eigenvalues of matrix **A** and the eigenvector corresponding to $\lambda=1.$ $\mathbf{A}=\begin{bmatrix} 2 & 0 & 0 \\ 4 & 1 & 0 \\ -1 & 2 & -1 \end{bmatrix}$

. (5)

[5]

QUESTION 8

A function *f(x)* is defined by

$$f\left(x\right)=\begin{cases} -\pi & -2<x<0 \\ \pi & 0<x<2 \end{cases}.$$

Determine Fourier series of the periodic *f(x)* (8)

[8]

8.8.5 Preparation Paper V (May 2012)

QUESTION 1

Solve the following differential equations:

1.1 $\left(x\ln x\right)dy + ydx = 0$ (3)

1.2 $\cos x\dfrac{dy}{dx} + y\sin x = 1$ (7)

1.3 The rate of decay of radium is proportional to the amount present.

If half of the original amount decomposes in 1600 years, what percentage decomposes in 100 years? (7)

[17]

QUESTION 2

Find the general solutions of the following differential equations using **D-operator** methods:

2.1 $\left(D^2 - 2D + 1\right)y = xe^x + 7x - 2$ (7)

2.2 $D\left(D^2 + 1\right)y = 2\sinh x$ (6)

[13]

QUESTION 3

A system vibrates according to the equation $\dfrac{d^2x}{dt^2} + 4\dfrac{dx}{dt} + 13x = \sin t$ .where x is the displacement and t is the time. Determine $x(t)$ by using **D-operator methods.**

Discuss the motion of the system if $t \to \infty$. (7)

[7]

QUESTION 4

Solve the following set of simultaneous equations by using **D-operator** methods:

$$\begin{aligned} Dx + (D+2)y &= 0 \\ (D-3) - 2y &= 0 \end{aligned}$$ (7)

[7]

QUESTION 5

Determine the following:

5.1 $L\left\{\sin^2 t\right\}$ (2)

5.2 $L^{-1}\left\{\dfrac{2(s+1)}{s^2+2s+10}\right\}$ (2)

[4]

QUESTION 6

Given $y''+4y=\begin{cases}0, & 0<t<4\\ 3, & 4<t\end{cases}$

6.1 Rewrite the right hand side of the given ordinary differential equation using the Heaviside step function. (1)

6.2 Use **Laplace transforms** to find *Y(s)* for the given equation if the initial values of the equation are $y(0)=1$ and $y'(0)=0$. (4)

6.3 Solve the given equation for *y*. $\left[\text{Hint: } \dfrac{1}{s\left(s^2+4\right)}=\dfrac{1}{s}-\dfrac{s}{s^2+4}\right]$ (3)

[8]

QUESTION 7

The equation of a particular electrical circuit is given by:

$5\dfrac{di}{dt}+4i=20$, if $t=0$, then $i=2$.

Use **Laplace transform methods** and determine the current at any time *t > 0* and discuss the solution as $t\to\infty$. (8)

[8]

QUESTION 8

Find the eigenvalues of $A = \begin{bmatrix} 2 & -3 & 1 \\ 2 & -2 & 1 \\ 1 & -3 & 2 \end{bmatrix}$ and an eigenvector corresponding to $\lambda = 1$. (8)

[8]

QUESTION 9

Determine the Fourier series for the function defined by $f(t) = t^2, \quad -\pi < t < \pi$.

The function has a period of 2π. (8)

[8]

Full marks = 80

9 FREQUENTLY ASKED QUESTIONS

Question: Can I get an extension on the due date for my assignment 01?

Answer: No, this assignment gives you admission to the examination and for administrative purposes no extension can be given.

Question: May I use a calculator in the examination?

Answer: Yes, it must be non-programmable.

Question: Can I request more past papers from the lecturers?

Answer: No, past papers are on MyUnisa under official Study material. This site is maintained by the examination department.

Question: Where do I get memorandums for past papers?

Answer: No memorandums are supplied, but the lecturer may decide to post some solutions or answers only under additional resources on myUNISA.

Question: Why don't I receive any follow-up Tutorial letters from UNISA?

Answer: In some modules all follow-up letters are only posted on myUnisa, so no post is sent to students. If a tutorial letter is send by post and you owe money on your student account it will not be posted to you however you will still be able to access the letter on myUNISA.

Question: What does the letters FC after my module code mean?

Answer: FC stands for Financial Cancellation. You may continue your studies and even write the examination but no results will be released before your student account has been paid in full.

10 SOURCES CONSULTED

Study Guides, prescribed book and past papers for MAT3700

11 IN CLOSING

The semester system affords you more flexibility in planning your studies. The guideline is that you must be able to spend *72 minutes* per day (including weekends) *per module.* Planning is the key to success.

May you achieve your dreams!

Ms L E Greyling

12 ADDENDUM

12.1 Errata Study guides

Please forward possible mistakes to the lecturer to be included in future letters to students. Give the page number and the correction.

Study Guide	Page	Error	Correction
1	53	Example 2	$\frac{d^2y}{dx^2} - \frac{3dy}{dx} + 2y = 0$
1	89	By theorem 4 (middle of page)	By theorem 5.2
1	102	Sign error	Equation 7.6 $x = -\frac{5Ae^{2t} + 2Be^{-t}}{5}$
1	102	Typing	Equation 7.7 $2\frac{dx}{dt} + \frac{dy}{dt} - 6x - y = e^t$

12.2 Answers to Preparation Papers

12.2.1 Answers Preparation Paper 1

QUESTION 1

1.1 a) $\ln x = -\frac{1}{2}\ln\left(1-2\left(\frac{y}{x}\right)^3\right) = C$ (7)

b) $\ln\cos y = \ln\left(1+e^x\right)+C$ (5)

1.2 $2\sqrt{H} = -2{,}8\times10^{-3}t + C$ (5)

$C = 4$

$2\sqrt{H} = -2{,}8\times10^{-3}t + 4$

$\sqrt{H} = -1{,}4\times10^{-3}t + 2$

$H = \left(-1{,}4\times10^{-3}t + 2\right)^2$

[17]

QUESTION 2

2.1 a) $y_{gen} = Ae^{-2x} + Be^{x} + \frac{1}{4}e^{2x} - \frac{1}{3}xe^{-2x}$ (9)

b) $y_{gen} = \left(A+Bx\right)e^{2x} + \frac{e^{2x}}{2x}$ (5)

2.2 $x_{gen} = \left(A+Bt\right)e^{2t} + \frac{1}{4} - 4e^{t}$ (7)

2.3

2.3.1 $q_{gen} = A\sin 4t + B\cos 4t + 3.75$

2.3.1 $i = \frac{dq}{dt} = 15\sin 4t$ (8)

[29]

QUESTION 3

3.2 Determine the following:

a) $L\left\{10t.H(t+\pi)\right\}$

$= L\left\{10(t+\pi-\pi)H(t+\pi)\right\}$

$= L\left\{10\left(t+\pi\right)H(t+\pi) - 10\pi.H(t+\pi)\right\}$

$= L\left\{10\left(t+\pi\right)H(t+\pi)\right\} - L\left\{10\pi.H(t+\pi)\right\}$

$= \frac{10e^{\pi s}}{s^2} - 10\pi L\left\{H\left(t+\pi\right)\right\}$

$= \frac{10e^{\pi s}}{s^2} - \frac{10\pi e^{\pi s}}{s}$ (3)

b) $$L^{-1}\left\{\frac{1}{3p^2+1}\right\}=\frac{\sqrt{3}}{3}\sin\frac{\sqrt{3}}{3}t$$ (2)

3.2 Determine the ***unique*** solution of the following differential equation by using **Laplace transforms**:

$y''(t)-9y(t)=\cosh 3t$, given $y(0)=0$ and $y(0)=4$ (7)

Hint: $\left[\dfrac{s}{(s^2-9)^2}=\dfrac{1}{12}\dfrac{1}{(s-3)^2}-\dfrac{1}{12}\dfrac{1}{(s+3)^2}\right]$

3.3 $i(t)=5e^{-2t}\sin t$ (6)

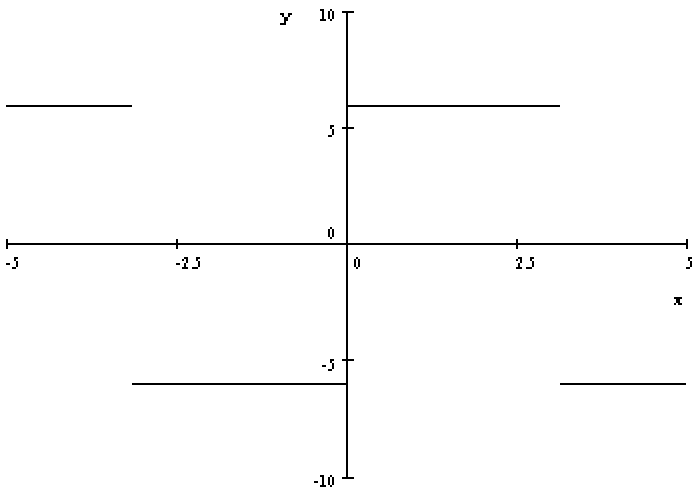
$i\to 0 \text{ if } t\to\infty$

[18]

QUESTION 4

A function *f*(*x*) is defined by: $f(x)=\begin{cases}-6 & -\pi<x<0\\ 6 & 0<x<\pi\end{cases}$ $\quad[f(x)=f(x+2\pi)]$

4.1 Odd



(2)

4.2 $$f(x)=\frac{24}{\pi}\left\{\sin x+\frac{1}{3}\sin 3x+\frac{1}{5}\sin 5x+.......\right\}$$ (8)

[10]

QUESTION 5

$\lambda=-1-2i$ or $\lambda=1+2i$. (5)

[5]

FULL MARKS: 80

12.2.2 Answers Preparation Paper 2

QUESTION 1

1.1 $y = \ell n\left|\sin x + \cos x\right| + C$ (4)

1.2 $e^{\frac{y}{x}} = \ell n\left|x\right| + 1$ (7)

1.3 $\dfrac{1}{y} = \dfrac{-xe^{2x}}{3} + \dfrac{e^{2x}}{9} + \dfrac{c}{e^{x}}$ (7)

[18]

QUESTION 2

2.1 $y_{gen} = A\cos 3x + B\sin 3x + 4e^{3x}\left(x - \dfrac{1}{3}\right)$ (7)

2.2 $y_{gen} = e^{-x}\left(A\cos x + B\sin x\right) - \dfrac{2}{3}e^{-x}\sin 2x$ (7)

[14]

QUESTION 3

3.1 $q_{gen} = Ae^{-10t} + Be^{-40t} - \dfrac{1}{2}\left(\cos 30t - 3\sin 30t\right)$ (7)

3.2 $\dfrac{dq_{gen}}{dt} = i = -10Ae^{-10t} - 40Be^{-40t} + \left(15\sin 30t + 45\cos 30t\right)$ (1)

3.3 The amplitude and the frequency of the steady-state current. (2)

$$q_{steadystate} = -\frac{1}{2}\left(\cos 30t - 3\sin 30t\right)$$
$$f = \frac{30}{2\pi} = 4{,}774$$
$$A = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{3}{2}\right)^2} = 1$$

[10]

QUESTION 4

$x_{gen} = A + Be^{4t} + \dfrac{4}{5}e^{t}$ (7)

[7]

QUESTION 5

5.1 $L\left\{\left(2t-3\right).H(t-1)\right\} = e^{-s}L\left\{2t-1\right\} = e^{-s}\left(\dfrac{2}{s^2} - \dfrac{1}{s}\right)$ (3)

5.2 $L^{-1}\left\{\frac{1}{(s-1)^3}+\frac{1}{s^2+2s-8}\right\}=\frac{1}{2}e^t t^2+\frac{1}{3}e^{-t}\sinh 3t$ (4)

5.3 $L^{-1}\left\{\frac{e^{-\frac{\pi}{2}p}}{s^2+9}\right\}=\frac{1}{3}\sin 3\left(t-\frac{\pi}{2}\right)H\left(t-\frac{\pi}{2}\right)$ (3)

[10]

QUESTION 6

$$y=\begin{cases}\frac{1}{2}Ax^2+\frac{1}{6}Bx^3 & if \quad x\le 5\\ \frac{1}{30}(x-5)^3+\frac{1}{2}Ax^2+\frac{1}{6}Bx^3 & if \quad x>5\end{cases}$$

apply conditions

$$y=\begin{cases}\frac{1}{2}x^2-\frac{1}{30}x^3 & if \quad x\le 5\\ \frac{1}{30}(x-5)^3+\frac{1}{2}x^2-\frac{1}{30}x^3 & if \quad x>5\end{cases}$$

$y(5)=8{,}33\ m$ (11)

[11]

QUESTION 7

eigenvalues $\lambda=-4$ or -3 or 5. (4)

[4]

QUESTION 8

Odd Function $a_o=a_n=0$

$f(x)=\sin x-\frac{1}{2}\sin 2x+\frac{1}{3}\sin 3x+......$ (8)

[8]

Total:82

Full marks = 80

12.2.3 Answers Preparation Paper 3

QUESTION 1

1.1 $y=\frac{\sin x}{2}+\frac{C}{\sin x}$ (linear) (5)

1.2 $\frac{y}{x}-2\ln\left|\frac{y}{x}+1\right|=\ln x+C$ (homogeneous) (6)

[11]

QUESTION 2

2.1 $y_{gen}=Ae^{x}+Be^{2x}+\frac{9}{130}\cos 3x-\frac{7}{130}\sin 3x$ (8)

2.2 $y=(A+Bx)e^{-3x}+\frac{1}{18}+\frac{e^{-4x}}{2}$ using substitution $\cosh 2x=\frac{e^{2x}+e^{-2x}}{2}$ (6)

$=(A+Bx)e^{-3x}-\frac{e^{-2x}}{9}(4\sinh 2x-5\cosh 2x)$ without substitution

[14]

QUESTION 3

$q(t)=e^{-5t}\left(A\cos 5\sqrt{3}\,t+B\sin 5\sqrt{3}\,t\right)+\frac{1}{2}\sin 10t$

$i(t)=\frac{dq}{dt}$

$=-5e^{-5t}\left(A\cos 5\sqrt{3}t+B\sin 5\sqrt{3}t\right)+e^{-5t}\left(-5\sqrt{3}A\sin 5\sqrt{3}\,t+5\sqrt{3}B\cos 5\sqrt{3}\right)+\frac{1}{2}(10)\cos 10t$

Steady-state t $\to\infty$: $i(t)=5\cos 10t$

(9)

[9]

QUESTION 4

$x_{gen}=Ae^{2t}+Bte^{2t}+\frac{1}{4}-4e^{t}$

$y_{gen}=Ce^{2t}+Dte^{2t}-\frac{1}{4}-8e^{t}$ (10)

[10]

QUESTION 5

5.1 $L\left\{e^{t}\cos 2t\right\}=\frac{s-1}{(s-1)^2+4}$ (2)

5.2 $L^{-1}\left\{\dfrac{8se^{-2s}}{s^2-9}\right\} = 8H(t-2)\cosh 3(t-2)$ (2)

[4]

QUESTION 6

6.1 $y = \dfrac{1}{2}\sin t - \dfrac{1}{2}t\cos t + \cos t$ (7)

6.2 $x(t) = H(t-2)e^{2-t} - H(t-2)e^{4(2-t} + 2e^{-t}$ (12)

[19]

QUESTION 7

$T = \dfrac{2\pi}{\sqrt{3}}$ or 2π (5)

[5]

QUESTION 8

$f(t) = \dfrac{\pi}{2} + \sum_{n=1}^{\infty} \dfrac{2\left[(-1)^n - 1\right]}{\pi n^2}\cos nt$. (8)

[8]

12.2.4 Answers Preparation Paper 4

QUESTION 1

1.1 Exact equation. (4)
$x\cos y + y\sin x = C$

1.2 $ln\left|y+3\right|$=tanx+C (3)

1.3 $y^{-2} = \sec^2 x\left(2\sin x + C\right)$ (7)

[14]

QUESTION 2

Find the general solutions of the following differential equations using **D-operator** methods:

2.1 $y_{gen} = Ae^{3x} + Bxe^{3x} + \frac{x^2e^{3x}}{2} + \frac{e^{-3x}}{36}$ (6)

2.2 $y_{gen} = e^{-x}\left(A\cos 2x + B\sin 2x\right) - \frac{e^{-x}\cos 3x}{5}$ (6)

[12]

QUESTION 3

$q(t) = e^{-3t}\left(2\cos 4t - \frac{1}{2}\sin 4t\right) - 2\cos t + 8\sin t$. (11)

[11]

QUESTION 4

$y_{gen} = A\cos 3t + B\sin 3t + \frac{2t}{9} - \frac{5}{9}$ (7)

[7]

QUESTION 5

5.1 $L\left\{\left(1-e^t\right)^2\right\} = \frac{1}{s} - \frac{2}{s-1} + \frac{1}{s-2}$ (3)

5.2 $L^{-1}\left\{\frac{e^{-2s}}{s^4}\right\} = \frac{\left(t-2\right)^3}{6}H(t-2) = \begin{cases}\frac{\left(t-2\right)^3}{6} & t > 2\\ 0 & t \le 2\end{cases}$ (2)

[5]

QUESTION 6

6.1 $y(t) = 2t^2e^{-2t} - e^{-2t} + 2te^{-2t}$ (10)

6.2 $x(t) = \sin 3(t-\pi)H(t-\pi) + \cos 3t$ (8)

[18]

QUESTION 7

$\lambda = 1, -1$ or 2. An eigenvector $X_1 = \begin{bmatrix} 0 \\ k \\ k \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$. (5)

[5]

QUESTION 8

$f(x) = \sum_{n=1}^{\infty} \frac{2}{n}\left(1-(-1)^n\right)\sin\frac{n\pi x}{2}$. (8)

[8]

Total: 80

12.2.5 Answers Preparation Paper 5

QUESTION 1

1.1 $y^{-1} = A\ln x$ (3)

1.2 $y = \sin x + C\cos x$ (7)

1.3 After 100 years 95,8% is still present. 4,2% decomposes in 100 years. (7)

[17]

QUESTION 2

2.1 $y_{gen} = Ae^x + Bxe^x + \frac{x^3}{6}e^x + 7x + 12$ (7)

2.2 $y_{gen} = A + B\cos x + C\sin x + \cosh x$ (6)

[13]

QUESTION 3

$x(t) = e^{-2t}(A\cos 3t + B\sin 3t) - \frac{1}{40}(\cos t - 3\sin t)$ (7)

[7]

QUESTION 4

$$x(t) = Ae^{-3t} + Be^{2t}$$
$$y(t) = Ce^{-3t} + De^{2t}$$
(7)

[7]

QUESTION 5

5.1 $L\left\{\sin^2 t\right\} = L\left\{\frac{1}{2}(1-\cos 2t)\right\} = \frac{2}{s\left(s^2+4\right)}$ (2)

5.2 $L^{-1}\left\{\frac{2(s+1)}{s^2+2s+10}\right\} = 2e^{-t}\cos 3t$ (2)

[4]

QUESTION 6

Given $y'' + 4y = \begin{cases} 0, & 0 < t < 4 \\ 3, & 4 < t \end{cases}$

6.1 *3H(t - 4)* . (1)

6.2 $Y(s) = \dfrac{3e^{-4s}}{s\left(s^2+4\right)} + \dfrac{s}{s^2+4}$ (4)

6.3 $y(t) = 3H(t-4) - 3H(t-4)\cos\left(2(t-4)\right) + \cos 2t$ (3)

[8]

QUESTION 7

$i(t) = 5 - 3e^{\frac{-4}{5}t}$, $t \to \infty$ then *i(t) = 5.* is the steady state solution. (8)

[8]

QUESTION 8

eigenvalues $\lambda = 1$ or $\lambda = \dfrac{-1 \pm \sqrt{11}i}{2}$ and an eigenvector corresponding to $\lambda = 1$ is $\begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}$. (8)

[8]

QUESTION 9

$f(t) = \dfrac{\pi^2}{3} + \sum_{n=1}^{\infty} \dfrac{4}{n^2}(-1)^n \cos nt$. (8)

[8]

Full marks = 80

12.3 Formula Sheets

ALGEBRA

Laws of indices

1. $a^m \times a^n = a^{m+n}$
2. $\dfrac{a^m}{a^n} = a^{m-n}$
3. $\left(a^m\right)^n = a^{mn} = \left(a^n\right)^m$
4. $a^{\frac{m}{n}} = \sqrt[n]{a^m}$
5. $a^{-n} = \dfrac{1}{a^n}$ and $a^n = \dfrac{1}{a^{-n}}$
6. $a^0 = 1$
7. $\sqrt{ab} = \sqrt{a}\sqrt{b}$
8. $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$

Logarithms

Definitions:

If $y = a^x$ then $x = \log_a y$

If $y = e^x$ then $x = \ell n\, y$

Laws:

1. $\log(A \times B) = \log A + \log B$
2. $\log\left(\dfrac{A}{B}\right) = \log A - \log B$
3. $\log A^n = n \log A$
4. $\log_a A = \dfrac{\log_b A}{\log_b a}$
5. $a^{\log_a f} = f \quad \therefore \quad e^{\ell n f} = f$

Factors

$a^3 - b^3 = (a-b)\left(a^2 + ab + b^2\right)$

$a^3 + b^3 = (a+b)\left(a^2 - ab + b^2\right)$

Partial Fractions

$$\frac{f(x)}{(x+a)(x+b)(x+c)} = \frac{A}{(x+a)} + \frac{B}{(x+b)} + \frac{C}{(x+c)}$$

$$\frac{f(x)}{(x+a)^3(x+b)} = \frac{A}{(x+a)} + \frac{B}{(x+a)^2} + \frac{C}{(x+a)^3} + \frac{D}{(x+b)}$$

$$\frac{f(x)}{\left(ax^2 + bx + c\right)(x+d)} = \frac{Ax+B}{\left(ax^2 + bx + c\right)} + \frac{C}{(x+d)}$$

Quadratic Formula

If $\quad ax^2 + bx + c = 0$

then $\quad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

DETERMINANTS

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$$= a_{11}\left(a_{22}a_{33} - a_{32}a_{23}\right) - a_{12}\left(a_{21}a_{33} - a_{31}a_{23}\right) + a_{13}\left(a_{21}a_{32} - a_{31}a_{22}\right)$$

SERIES

Binomial Theorem

$$(a+b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{2!}a^{n-2}b^2 + \frac{n(n-1)(n-2)}{3!}a^{n-3}b^3 + \ldots.$$

and $|b| < |a|$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots$$

and $-1 < x < 1$

Maclaurin's Theorem

$$f(x) = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \ldots + \frac{f^{n-1}(0)}{(n-1)!}x^{n-1} + \ldots$$

Taylor's Theorem

$$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \ldots + \frac{f^{n-1}(a)}{(n-1)!}(x-a)^{n-1} + \ldots$$

$$f(a+h) = f(a) + \frac{h}{1!}f'(a) + \frac{h^2}{2!}f''(a) + \ldots + \frac{h^{n-1}}{(n-1)!}f^{n-1}(a) + \ldots$$

COMPLEX NUMBERS

1. $z = a + bj = r(\cos\theta + j\sin\theta) = r\underline{|\theta} = re^{j\theta}$,
   where $j^2 = -1$
   Modulus: $r = |z| = \sqrt{(a^2 + b^2)}$
   Argument: $\theta = \arg z = arc\tan\frac{b}{a}$
2. Addition:
   $(a + jb) + (c + jd) = (a + c) + j(b + d)$
3. Subtraction:
   $(a + jb) - (c + jd) = (a - c) + j(b - d)$
4. If $m + jn = p + jq$, then $m = p$ and $n = q$
5. Multiplication: $z_1 z_2 = r_1 r_2 \underline{|(\theta_1 + \theta_2)}$
6. Division: $\frac{z_1}{z_2} = \frac{r_1}{r_2}\underline{|(\theta_1 - \theta_2)}$
7. De Moivre's Theorem
   $[r\underline{|\theta}]^n = r^n \underline{|n\theta} = r^n(\cos n\theta + j\sin n\theta)$
8. $z^{\frac{1}{n}}$ has n distinct roots:
   $z^{\frac{1}{n}} = r^{\frac{1}{n}}\underline{\left|\frac{\theta + k360^\circ}{n}\right.}$ with $k = 0, 1, 2, \ldots, n-1$
9. $re^{j\theta} = r(\cos\theta + j\sin\theta)$
   $\therefore \Re(re^{j\theta}) = r\cos\theta$ and $\Im(re^{j\theta}) = r\sin\theta$
10. $e^{a+jb} = e^{a}(\cos b + j\sin b)$
11. $\ell n\, re^{j\theta} = \ell n\, r + j\theta$

GEOMETRY

1. Straight line:

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

Perpendiculars, then $m_1 = \dfrac{-1}{m_2}$

2. Angle between two lines:

$$\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2}$$

3. Circle:

$$x^2 + y^2 = r^2$$

$$(x-h)^2 + (y-k)^2 = r^2$$

4. Parabola:

$$y = ax^2 + bx + c$$

axis at $x = \dfrac{-b}{2a}$

5. Ellipse:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

6. Hyperbola:

$$xy = k$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad (\text{round } x\text{-axis})$$

$$-\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad (\text{round } y\text{-axis})$$

MENSURATION

1. Circle: (θ in radians)

$$\text{Area} = \pi r^2$$

$$\text{Circumference} = 2\pi r$$

$$\text{Arc length } \ell = r\theta$$

$$\text{Sector area} = \frac{1}{2}r^2\theta = \frac{1}{2}\ell r$$

$$\text{Segment area} = \frac{1}{2}r^2(\theta - \sin\theta)$$

2. Ellipse:

$$\text{Area} = \pi ab$$

$$\text{Circumference} = \pi(a+b)$$

3. Cylinder:

$$\text{Volume} = \pi r^2 h$$

$$\text{Surface area} = 2\pi rh + 2\pi r^2$$

4. Pyramid:

$$\text{Volume} = \frac{1}{3}\text{area base} \times \text{height}$$

5. Cone:

$$\text{Volume} = \frac{1}{3}\pi r^2 h$$

$$\text{Curved surface} = \pi r\ell$$

6. Sphere:

$$A = 4\pi r^2$$

$$V = \frac{4}{3}\pi r^3$$

7. Trapezoidal rule:

$$A = h\left(\frac{y_1 + y_n}{2} + y_2 + y_3 + \ldots + y_{n-1}\right)$$

8. Simpsons rule:

$$A = \frac{s}{3}\left((F+L) + 4E + 2R\right)$$

9. Prismoidal rule

$$V = \frac{h}{6}(A_1 + 4A_2 + A_3)$$

HYPERBOLIC FUNCTIONS

Definitions:

$$\sinh x = \frac{e^x - e^{-x}}{2}$$
$$\cosh x = \frac{e^x + e^{-x}}{2}$$
$$\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

Identities:

$$\cosh^2 x - \sinh^2 x = 1$$
$$1 - \tanh^2 x = \operatorname{sech}^2 x$$
$$\coth^2 x - 1 = \operatorname{cosech}^2 x$$
$$\sinh^2 x = \frac{1}{2}(\cosh 2x - 1)$$
$$\cosh^2 x = \frac{1}{2}(\cosh 2x + 1)$$
$$\sinh 2x = 2\sinh x \cosh x$$
$$\cosh 2x = \cosh^2 x + \sinh^2 x$$
$$= 2\cosh^2 x - 1$$
$$= 1 + 2\sinh^2 x$$

TRIGONOMETRY

Identities

$$\sin^2\theta + \cos^2\theta = 1$$
$$1 + \tan^2\theta = \sec^2\theta$$
$$\cot^2\theta + 1 = \operatorname{cosec}^2\theta$$
$$\sin(-\theta) = -\sin\theta$$
$$\cos(-\theta) = +\cos\theta$$
$$\tan(-\theta) = -\tan\theta$$
$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

TRIGONOMETRY

Compound angle addition and subtraction formulae:

$\sin(A + B) = \sin A \cos B + \cos A \sin B$
$\sin(A - B) = \sin A \cos B - \cos A \sin B$
$\cos(A + B) = \cos A \cos B - \sin A \sin B$
$\cos(A - B) = \cos A \cos B + \sin A \sin B$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$
$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Double angles:

$\sin 2A = 2 \sin A \cos A$
$\cos 2A = \cos^2 A - \sin^2 A$
$= 2\cos^2 A - 1$
$= 1 - 2\sin^2 A$
$\sin^2 A = \tfrac{1}{2}(1 - \cos 2A)$
$\cos^2 A = \tfrac{1}{2}(1 + \cos 2A)$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Products of sines and cosines into sums or differences:

$\sin A \cos B = \tfrac{1}{2}(\sin(A + B) + \sin(A - B))$
$\cos A \sin B = \tfrac{1}{2}(\sin(A + B) - \sin(A - B))$
$\cos A \cos B = \tfrac{1}{2}(\cos(A + B) + \cos(A - B))$
$\sin A \sin B = -\tfrac{1}{2}(\cos(A + B) - \cos(A - B))$

Sums or differences of sines and cosines into products:

$$\sin x + \sin y = 2\sin\left[\frac{x+y}{2}\right]\cos\left[\frac{x-y}{2}\right]$$
$$\sin x - \sin y = 2\cos\left[\frac{x+y}{2}\right]\sin\left[\frac{x-y}{2}\right]$$
$$\cos x + \cos y = 2\cos\left[\frac{x+y}{2}\right]\cos\left[\frac{x-y}{2}\right]$$
$$\cos x - \cos y = -2\sin\left[\frac{x+y}{2}\right]\sin\left[\frac{x-y}{2}\right]$$

DIFFERENTIATION

1. $\frac{dy}{dx} = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

2. $\frac{d}{dx}k = 0$

3. $\frac{d}{dx}ax^n = anx^{n-1}$

4. $\frac{d}{dx}f.g = f.g' + g.f'$

5. $\frac{d}{dx}\frac{f}{g} = \frac{g.f' - f.g'}{g^2}$

6. $\frac{d}{dx}[f(x)]^n = n[f(x)]^{n-1}.f'(x)$

7. $\frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dv}.\frac{dv}{dx}$

8. Parametric equations

$$\frac{dy}{dx} = \frac{dy}{dt} \Big/ \frac{dx}{dt}$$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$$

9. Maximum/minimum
   For turning points: $f'(x) = 0$
   Let $x = a$ be a solution for the above
   If $f''(a) > 0$, then a is a minimum point
   If $f''(a) < 0$, then a is a maximum point
   For points of inflection: $f''(x) = 0$
   Let $x = b$ be a solution for the above
   Test for inflection: $f(b - h)$ and $f(b + h)$
   Change sign or $f'''(b) \neq 0$ if $f'''(b)$ exists.

10. $\frac{d}{dx}\sin^{-1} f(x) = \frac{f'(x)}{\sqrt{1-[f(x)]^2}}$

11. $\frac{d}{dx}\cos^{-1} f(x) = \frac{-f'(x)}{\sqrt{1-[f(x)]^2}}$

12. $\frac{d}{dx}\tan^{-1} f(x) = \frac{f'(x)}{1+[f(x)]^2}$

13. $\frac{d}{dx}\cot^{-1} f(x) = \frac{-f'(x)}{1+[f(x)]^2}$

14. $\frac{d}{dx}\sec^{-1} f(x) = \frac{f'(x)}{f(x)\sqrt{[f(x)]^2-1}}$

15. $\frac{d}{dx}\mathrm{cosec}^{-1} f(x) = \frac{-f'(x)}{f(x)\sqrt{[f(x)]^2-1}}$

16. $\frac{d}{dx}\sinh^{-1} f(x) = \frac{f'(x)}{\sqrt{[f(x)]^2+1}}$

17. $\frac{d}{dx}\cosh^{-1} f(x) = \frac{f'(x)}{\sqrt{[f(x)]^2-1}}$

18. $\frac{d}{dx}\tanh^{-1} f(x) = \frac{f'(x)}{1-[f(x)]^2}$

19. $\frac{d}{dx}\coth^{-1} f(x) = \frac{f'(x)}{1-[f(x)]^2}$

20. $\frac{d}{dx}\mathrm{sech}^{-1} f(x) = \frac{-f'(x)}{f(x)\sqrt{1-[f(x)]^2}}$

21. $\frac{d}{dx}\mathrm{cosech}^{-1} f(x) = \frac{-f'(x)}{f(x)\sqrt{[f(x)]^2+1}}$

22. Increments: $\delta z = \frac{\partial z}{\partial x}.\delta x + \frac{\partial z}{\partial y}.\delta y + \frac{\partial z}{\partial w}.\delta w$

23. Rate of change:

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}.\frac{dx}{dt} + \frac{\partial z}{\partial y}.\frac{dy}{dt} + \frac{\partial z}{\partial w}.\frac{dw}{dt}$$

INTEGRATION

1. By parts: $\int udv = uv - \int vdu$

2. $\int_a^b f(x)dx = F(b) - F(a)$

3. Mean value $= \frac{1}{b-a}\int_a^b ydx$

4. $(\text{R.M.S.})^2 = \frac{1}{b-a}\int_a^b y^2\,dx$

13 TABLE OF INTEGRALS

1.	$\int ax^n \, dx$	$= \dfrac{ax^{(n+1)}}{n+1} + c,$	$n \neq -1$
2.	$\int [f(x)]^n . f'(x) \, dx$	$= \dfrac{[f(x)]^{n+1}}{n+1} + c,$	$n \neq -1$
3.	$\int \dfrac{f'(x)}{f(x)} dx$	$= \ell n \lvert f(x) \rvert + c$	
4.	$\int f'(x).e^{f(x)} \, dx$	$= e^{f(x)} + c$	
5.	$\int f'(x).a^{f(x)} \, dx$	$= \dfrac{a^{f(x)}}{\ell n \, a} + c$	
6.	$\int f'(x).\sin f(x) \, dx$	$= -\cos f(x) + c$	
7.	$\int f'(x).\cos f(x) \, dx$	$= \sin f(x) + c$	
8.	$\int f'(x).\tan f(x) \, dx$	$= \ell n \lvert \sec f(x) \rvert + c$	
9.	$\int f'(x).\cot f(x) \, dx$	$= \ell n \lvert \sin f(x) \rvert + c$	
10.	$\int f'(x).\sec f(x) \, dx$	$= \ell n \lvert \sec f(x) + \tan f(x) \rvert + c$	
11.	$\int f'(x).\mathrm{cosec}\, f(x) \, dx$	$= \ell n \lvert \mathrm{cosec}\, f(x) - \cot f(x) \rvert + c$	
12.	$\int f'(x).\sec^2 f(x) \, dx$	$= \tan f(x) + c$	
13.	$\int f'(x).\mathrm{cosec}^2 f(x) \, dx$	$= -\cot f(x) + c$	
14.	$\int f'(x).\sec f(x).\tan f(x) \, dx$	$= \sec f(x) + c$	
15.	$\int f'(x).\mathrm{cosec}\, f(x).\cot f(x) \, dx$	$= -\mathrm{cosec}\, f(x) + c$	
16.	$\int f'(x).\sinh f(x) \, dx$	$= \cosh f(x) + c$	
17.	$\int f'(x).\cosh f(x) \, dx$	$= \sinh f(x) + c$	
18.	$\int f'(x).\tanh f(x) \, dx$	$= \ell n \cosh f(x) + c$	

19. $\int f'(x).\coth f(x)\, dx \quad = \ell n \sinh f(x) + c$

20. $\int f'(x).\text{sech}^2 f(x)\, dx \quad = \tanh f(x) + c$

21. $\int f'(x).\text{cosech}^2 f(x)\, dx \quad = -\coth f(x) + c$

22. $\int f'(x).\text{sech} f(x).\tanh f(x)\, dx \quad = -\text{sech } f(x) + c$

23. $\int f'(x).\text{cosech } f(x).\coth f(x)\, dx \quad = -\text{cosech } f(x) + c$

24. $\displaystyle\int \frac{f'(x)}{\sqrt{a^2 - [f(x)]^2}}dx \quad = arc\sin\left(\frac{f(x)}{a}\right) + c$

25. $\displaystyle\int \frac{f'(x)}{[f(x)]^2 + a^2}dx \quad = \frac{1}{a} arc\tan\left(\frac{f(x)}{a}\right) + c$

26. $\displaystyle\int \frac{f'(x)}{\sqrt{[f(x)]^2 + a^2}}dx \quad = arc\sinh\left(\frac{f(x)}{a}\right) + c$

27. $\displaystyle\int \frac{f'(x)}{\sqrt{[f(x)]^2 - a^2}}dx \quad = arc\cosh\left(\frac{f(x)}{a}\right) + c$

28. $\displaystyle\int \frac{f'(x)}{a^2 - [f(x)]^2}dx \quad = \frac{1}{a} arc\tanh\left(\frac{f(x)}{a}\right) + c$

29. $\displaystyle\int \frac{f'(x)}{[f(x)]^2 - a^2}dx \quad = -\frac{1}{a} arc\coth\left(\frac{f(x)}{a}\right) + c$

30. $\displaystyle\int f'(x)\sqrt{a^2 - [f(x)]^2}\; dx = \frac{a^2}{2} arc\sin\left(\frac{f(x)}{a}\right) + \frac{f(x)}{2}\sqrt{a^2 - [f(x)]^2} + c$

31. $\displaystyle\int f'(x)\sqrt{[f(x)]^2 + a^2}\; dx = \frac{a^2}{2} arc\sinh\left(\frac{f(x)}{a}\right) + \frac{f(x)}{2}\sqrt{[f(x)]^2 + a^2} + c$

32. $\displaystyle\int f'(x)\sqrt{[f(x)]^2 - a^2}\; dx = -\frac{a^2}{2} arc\cosh\left(\frac{f(x)}{a}\right) + \frac{f(x)}{2}\sqrt{[f(x)]^2 - a^2} + c$

TABLE OF LAPLACE TRANSFORMS

$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
a	$\frac{a}{s}$
t^n	$\frac{n!}{s^{n+1}}, \quad n = 1, 2, 3, \ldots$
e^{bt}	$\frac{1}{s-b}$
$\sin at$	$\frac{a}{s^2+a^2}$
$\cos at$	$\frac{s}{s^2+a^2}$
$\sinh at$	$\frac{a}{s^2-a^2}$
$\cosh at$	$\frac{s}{s^2-a^2}$
$t^n e^{bt}$	$\frac{n!}{(s-b)^{n+1}}, \quad n = 1, 2, 3, \ldots$
$t \sin at$	$\frac{2as}{(s^2+a^2)^2}$
$t \cos at$	$\frac{s^2-a^2}{(s^2+a^2)^2}$
$t \sinh at$	$\frac{2as}{(s^2-a^2)^2}$
$t \cosh at$	$\frac{s^2+a^2}{(s^2-a^2)^2}$
$e^{bt} \sin at$	$\frac{a}{(s-b)^2+a^2}$
$e^{bt} \cos at$	$\frac{(s-b)}{(s-b)^2+a^2}$
$e^{bt} \sinh at$	$\frac{a}{(s-b)^2-a^2}$
$e^{bt} \cosh at$	$\frac{(s-b)}{(s-b)^2-a^2}$
$H(t-c)$	$\frac{e^{-cs}}{s}$
$H(t-c).F(t-c)$	$e^{-cs}.f(s)$
$\delta(t-a)$	e^{-as}

$\mathcal{L}\{f(t)\} = F(s)$
$\mathcal{L}\{f'(t)\} = sF(s) - f(0)$
$\mathcal{L}\{f''(t)\} = s^2F(s) - sf(0) - f'(0)$
$\mathcal{L}\{f'''(t)\} = s^3F(s) - s^2f(0) - sf'(0) - f''(0)$