Tutorial letter 101/3/2017

REAL ANALYSIS

MAT3711

Semesters 1 & 2

Department of Mathematical Sciences

IMPORTANT INFORMATION:

This tutorial letter contains important information about your module.

BARCODE

Define tomorrow.

UNISA
university of south africa

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1 INTRODUCTION

Dear Student

Welcome to the MAT3711 module in the Department of Mathematical Sciences at Unisa. We trust that you will find this module both interesting and rewarding.

Some of this tutorial matter may not be available when you register. Tutorial matter that is not available when you register will be posted to you as soon as possible, but is also available on *my*Unisa.

1.1 *my*Unisa

You must be registered on *my*Unisa (`http://my.unisa.ac.za`) to be able to submit assignments online, gain access to the library functions and various learning resources, download study material, "chat" to your lecturers and fellow students about your studies and the challenges you encounter, and participate in online discussion forums. *my*Unisa provides additional opportunities to take part in activities and discussions of relevance to your module topics, assignments, marks and examinations.

1.2 Tutorial matter

A tutorial letter is our way of communicating with you about teaching, learning and assessment. You will receive a number of tutorial letters during the course of the module. This particular tutorial letter contains important information about the scheme of work, resources and assignments for this module as well as the admission requirements for the examination. We urge you to read this and subsequent tutorial letters carefully and to keep it at hand when working through the study material, preparing and submitting the assignments, preparing for the examination and addressing queries that you may have about the course (course content, textbook, worked examples and exercises, theorems and their applications in your assignments, tutorial and textbook problems, etc.) to your MAT3711 lecturers.

2 PURPOSE AND OUTCOMES FOR THE MODULE

2.1 Purpose

The purpose of this module is to equip students with a sound grounding in the theory of metric spaces, linear analysis and Riemann-Stieltjies integration. The expected outcomes include, but are not limited to:

- The ability to construct metric spaces.
- Grasp the rudiments of the key notions such as compactness and its various variants.
- Familiarity with basic concepts in Banach spaces.

2.2 Outcomes

2.2.1 Be able to determine if a set is countable.

2.2.2 Be able to compute suprema and infima of sets.

2.2.3 Determine when a metric space is complete.

2.2.4 Understanding of the notion of compactness and all it characterisations.

2.2.5 Good grasp of continuity and its variants.

2.2.6 Ability to use convergence concepts to construct continuous functions.

2.2.7 Thorough understanding of Banach spaces and linear transformations.

2.2.8 Understanding of the abstract notion of integration.

3 LECTURER(S) AND CONTACT DETAILS

3.1 Lecturer(s)

The contact details for the lecturer responsible for this module is

Postal address: The MAT3711 Lecturers
Department of Mathematical Sciences
Private Bag X6
Florida
1709
South Africa

Additional contact details for the module lecturers will be provided in a subsequent tutorial letter.

All queries that are not of a purely administrative nature but are about the content of this module should be directed to your lecturer(s). Tutorial letter 301 will provide additional contact details for your lecturer. Please have your study material with you when you contact your lecturer by telephone. If you are unable to reach us, leave a message with the departmental secretary. Provide your name, the time of the telephone call and contact details. If you have problems with questions that you are unable to solve, please send your own attempts so that the lecturers can determine where the fault lies.

Please note: Letters to lecturers may not be enclosed with or inserted into assignments.

3.2 Department

The contact details for the Department of Mathematical Sciences are:

Departmental Secretary: (011) 670 9147 (SA) +27 11 670 9147 (International)

3.3 University

If you need to contact the University about matters not related to the content of this module, please consult the publication *Studies @ Unisa* that you received with your study material. This booklet contains information on how to contact the University (e.g. to whom you can write for different queries, important telephone and fax numbers, addresses and details of the times certain facilities are open). Always have your student number at hand when you contact the University.

4 RESOURCES

4.1 Prescribed books

There are no prescribed books for this module.

4.2 Recommended books

Title	Author
1. Principles of Mathematical Analysis	Walter Rudin
2. Real Mathematical Analysis	Charles C. Pugh
3. Real Analysis	John M. Howie
4. Introduction to Real Analysis	Manfred Stoll
5. Mathematical Analysis	Tom M. Apostol

4.3 Electronic reserves (e-Reserves)

There are no e-Reserves for this module.

4.4 Library services and resources information

For brief information go to:
`http://www.unisa.ac.za/brochures/studies`

For more detailed information, go to the Unisa website: `http://www.unisa.ac.za/`, click on Library. For research support and services of Personal Librarians, go to:
`http://www.unisa.ac.za/Default.asp?Cmd=ViewContent&ContentID=7102`

The Library has compiled numerous library guides:

- find recommended reading in the print collection and e-reserves
  - `http://libguides.unisa.ac.za/request/undergrad`
- request material
  - `http://libguides.unisa.ac.za/request/request`
- postgraduate information services
  - `http://libguides.unisa.ac.za/request/postgrad`
- finding , obtaining and using library resources and tools to assist in doing research
  - `http://libguides.unisa.ac.za/Research_Skills`

- how to contact the Library/find us on social media/frequently asked questions
  - http://libguides.unisa.ac.za/ask

5 STUDENT SUPPORT SERVICES

For information on the various student support services available at Unisa (e.g. student counseling, tutorial classes, language support), please consult the publication *Studies @ Unisa* that you received with your study material.

6 STUDY PLAN

The following table provides an outline of the outcomes and ideal dates of completion, and other study activities.

Study plan	**Semester 1**	**Semester 2**
Outcomes 2.2.1 to 2.2.4 to be achieved by	16 March	22 August
Outcomes 2.2.5 to 2.2.8 to be achieved by	25 April	22 September
Work through previous exam paper	30 April	25 September
Revision	5 May	5 October

See the brochure *Studies @ Unisa* for general time management and planning skills.

7 PRACTICAL WORK AND WORK INTEGRATED LEARNING

There are no practicals for this module.

8 ASSESSMENT

8.1 Assessment criteria

Specific outcome 1: Demonstrate the concept of countability of sets.

Assessment criteria

You must be able to do the following.

- The definition and concept of countability of sets is given accurately.
- The ability to determine if a set is countable is sufficiently demonstrated.
- The ability to determine if two sets have the same cardinality is sufficiently demonstrated.
- Problems on determining if two sets are equipotent are solved.

Specific outcome 2: Demonstrate the concepts of upper and lower bounds, suprema and infima.

Assessment criteria

You must be able to do the following.

- The definitions of upper and lower bound, suprema, and infima are given accurately.
- The concepts of suprema and infima are explained sufficiently.
- Problems on finding suprema and infimaof sets are solved.

Specific outcome 3: Demonstrate the concept of a metric space.

Assessment criteria

You must be able to do the following.

- The definition of a metric space is given accurately.
- The concepts of open sets, closed sets, interior, closure, boundary, Cauchy sequence, bounded subset are explained sufficiently.
- Problems on finding closures and interiors are solved.
- Problems on determining completeness of metric spaces are solved.
- Openness and /or closedness of subspaces are verified.

Specific outcome 4: Demonstrate the concept of a compact metric space.

Assessment criteria

The student must be able to:

- The definition of a compact metric space is given accurately.
- Various variants of compactness (such as sequential compactness and countable compactness) are explained sufficiently.
- Problems on determining if metric spaces and subspaces are compact are solved.

Specific outcome 5: Demonstrate the concept of continuity.

Assessment criteria

The student must be able to:

- The definition and the characterisations of continuous functions are given accurately.
- The concept of uniform continuity is explained sufficiently,
- Problems on determining if functions are continuous or uniformly continuous are solved.
- Connections between continuity and compactness, continuity and connectedness are clearly understood.
- The concept of contraction mappings (and especially The Banachs Fixed Point Theorem)is well understood.
- Problems on contraction mappings and fixed points are solved.

Specific outcome 6: Demonstrate the concept of convergence of function spaces.

Assessment criteria

The student must be able to:

- The definitions of pointwise and uniform convergence of functions are given accurately.
- Problems on determining if sequences of functions converge uniformly are solved.

Specific outcome 7: Demonstrate the concept of linear analysis.

Assessment criteria

The student must be able to:

- The definition of a normed vector space is given accurately.
- The definition of a Banach space is fully understood.
- Linear maps are clearly defined.
- Problems on determining the norm of a bounded linear map are solved.

Specific outcome 8: Demonstrate the concept of the Riemann-Stieltjies Integral.

Assessment criteria

The student must be able to:

- The definitions of the upper Stieltjies and lower Stieltjies sums, upper Stieltjies and lower Stieltjies integrals are given accurately.
- Problems on the Riemann-Stieltjies integrals are solved.
- The Fundamental Theorem of Calculus is understood clearly.
- The connection between integration and convergence is sufficiently explained.

8.2 Assessment plan

A final mark of at least 50% is required to pass the module. If a student does not pass the module then a final mark of at least 40% is required to permit the student access to the supplementary examination. The final mark is composed as follows:

Year mark	
Assignment 01:	50%
Assignment 02:	50%

→

Final mark	
Year mark:	20%
Exam mark:	80%

8.3 Assignment numbers

8.3.1 General assignment numbers

The assignments for this module are Assignment 01, Assignment 02, etc.

8.3.2 Unique assignment numbers

Please note that each assignment has a unique assignment number which must be written on the cover of your assignment.

8.3.3 Assignment due dates

The dates for the submission of the assignments are:

Semester 1

Assignment 01: Thursday, 16 March 2017
Assignment 02: Tuesday, 25 April 2017

Semester 2

Assignment 01: Tuesday, 22 August 2017
Assignment 02: Friday, 22 September 2017

8.4 Submission of assignments

You may submit written assignments either by post or electronically via *my*Unisa. Assignments may **not** be submitted by fax or e-mail.

For detailed information on assignments, please refer to the *Studies @ Unisa* brochure which you received with your study package.

Please make a copy of your assignment before you submit!

To submit an assignment via *my*Unisa:

- Go to *my*Unisa.
- Log in with your student number and password.
- Select the module.
- Click on "Assignments" in the menu on the left-hand side of the screen.
- Click on the assignment number you wish to submit.
- Follow the instructions.

8.5 The assignments

Please make sure that you submit the correct assignments for the 1st semester, 2nd semester or year module for which you have registered. For each assignment there is a **fixed closing date**, the date at which the assignment must reach the University. When appropriate, solutions for each assignment will be dispatched, as Tutorial Letter 201 (solutions to Assignment 01) and Tutorial Letter 202 (solutions to Assignment 02) etc., a few days after the closing date. They will also be made available on *my*Unisa. Late assignments **will not** be marked!

Note that Assignment 01 is the compulsory assignment for admission to the examination and must reach us by the due date.

8.6 Other assessment methods

There are no other assessment methods for this module.

8.7 The examination

During the relevant semester, the Examination Section will provide you with information regarding the examination in general, examination venues, examination dates and examination times. For general information and requirements as far as examinations are concerned, see the brochure *Studies @ Unisa* which you received with your study material.

Registered for . . .	Examination period	Supplementary examination period
1st semester module	May/June 2017	October/November 2017
2nd semester module	October/November 2017	May/June 2018
Year module	January/February 2018	May/June 2018

9 FREQUENTLY ASKED QUESTIONS

The *Studies @ Unisa* brochure contains an A–Z guide of the most relevant study information.

10 IN CLOSING

We hope that you will enjoy MAT3711 and we wish you all the best in your studies at Unisa!

ADDENDUM A: ASSIGNMENTS – FIRST SEMESTER

ASSIGNMENT 01
Due date: Thursday, 16 March 2017
Total Marks: 40
UNIQUE ASSIGNMENT NUMBER: 713225

ONLY FOR SEMESTER 1

This assignment is based on Chapter 0, 1 and 2

Question 1: 20 Marks

(1.1) Let A and B be subsets of positive numbers of $\mathbb{R}$ each bounded above. Define the subset C of $\mathbb{R}$ by

$$C = \{xy \mid x \in A \text{ and } y \in B.\}$$

(a) Prove that C is bounded above. (5)

(b) Prove that $\sup C = \sup A \sup B$. (7)

(1.2) Let (X, d) be a metric space. Define (8)

$$d'(x, y) = \frac{d(x, y)}{1 + d(x, y)}.$$

Prove that d' is also a metric for X. Note that $0 \leq d' < 1$ for all $x, y \in M$.

Question 2: 20 Marks

(2.1) Let A be a subset of a metric space X. Prove that $\overline{X \backslash A} = X \backslash A^0$. (10)

(2.2) Let X be a nonempty set, and d a metric on X. Let $\{x_n\}$ be a Cauchy sequence in (X, d). Show that there exists $k \in \mathbb{N}$ such that $x_n = x_k$ for all $n \geq k$. (10)

ASSIGNMENT 02
Due date: Tuesday, 25 April 2017
Total Marks: 40
UNIQUE ASSIGNMENT NUMBER: 666824

ONLY FOR SEMESTER 1

This assignment is based on Chapters 0, 1, 2, 3 and 5

Question 1: 20 Marks

(1.1) Let $\{a_n\}$ be a sequence in a metric space $(X,\ d)$. Show that $a_n \to a$ if and only if the sequence $\{d\,(a_n,\ a)\}$ of real numbers converges to 0. (10)

(1.2) Prove that every compact subset of a metric space is complete. (10)

Question 2: 20 Marks

(2.1) Let A be a subset of a metric space X. Show that A is dense in X if and only if $\text{int}(X \backslash A) = \emptyset$. (10)

(2.2) Let (X, d) be a complete metric space, and let $T \colon X \to X$ be a contraction. Let $\alpha \in \mathbb{R}$ be such that $0 < \alpha < 1$ and $d(T(x_1), T(x_2)) \leq \alpha d(x_1, x_2)$ for all $x_1, x_2 \in X$, and let x' be the fixed point of T. Show that for all $x \in X$ and $n \geq 0$, $d(T^n(x), x') \leq (\alpha^n/(1-\alpha))d(x, Tx)$. (10)

ADDENDUM B: ASSIGNMENTS – SECOND SEMESTER

ASSIGNMENT 01
Due date: Tuesday, 22 August 2017
Total Marks: 40
UNIQUE ASSIGNMENT NUMBER: 851607

ONLY FOR SEMESTER 2

This assignment is based on Chapter 1

Question 1: 20 Marks

(1.1) Let A and B be subsets of positive numbers of $\mathbb{R}$ each bounded below. Define the subset C of $\mathbb{R}$ by

$$C = \{xy \mid x \in A \text{ and } y \in B.\}$$

(a) Prove that C is bounded below. (5)

(b) Prove that $\inf C = \inf A \inf B$. (7)

(1.2) Let (X, d) be a metric space. Define $d' \colon X \times X \to \mathbb{R}$ by $d' = \min\{d(x, y), 1\}$. Show that d' is a bounded metric. (8)

Question 2: 20 Marks

(2.1) $(X \backslash A)^0 = X \backslash \overline{A}$. (10)

(2.2) Let A be a subset of a metric space (X, d). Prove that a point $a \in X$ is an accumulation point of A if and only if $d(a; A\backslash\{a\}) = 0$. (10)

ASSIGNMENT 02
Due date: Friday, 22 September 2017
Total Marks: 40
UNIQUE ASSIGNMENT NUMBER: 598717

ONLY FOR SEMESTER 2

This assignment is based on Chapters 1, 2, 3 and 5

Question 1: 20 Marks

(1.1) Let $\{a_n\}$ and $\{b_n\}$ be sequences of real numbers. Suppose there exists $n_0 \in \mathbb{N}$ such that $a_n \leq b_n$ for all $n \geq n_0$. Prove that $\liminf\limits_{n\to\infty} a_n \leq \liminf\limits_{n\to\infty} b_n$. (10)

(1.2) Prove that a closed subset of a metric space is compact if and only if it is totally bounded. (10)

Question 2: 20 Marks

(2.1) Let $\{t_n\}$ and $\{s_n\}$ be Cauchy sequences in a metric space $(X,\ d)$. For each $n \in \mathbb{N}$, let $u_n = d(t_n, s_n)$. Show that $\{u_n\}$ converges in $\mathbb{R}$. (10)

(2.2) Let (X, d) be a metric space, and let $T\colon X \to X$ be a contraction. Show that T has at most one fixed point. (10)