DEPARTMENT OF FINANCE, RISK MANAGEMENT AND BANKING
SCHOOL OF MANAGEMENT SCIENCES

Investments:
DERIVATIVES

ONLY STUDY GUIDE FOR
INV3703

UNIVERSITY OF SOUTH AFRICA, PRETORIA

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CONTENTS

ADDITIONAL INFORMATION

The Study guide is a replication of the information on myUnisa. However as myUnisa is an online platform the information on the website may be presented differently to that in the Study guide. Further, the information contained in this document remains unchanged for the year, however new information may be released on myUnisa which was not captured in this document.

Please make use of the myUnisa site as the site will promote interaction between the lecturer and students via the online platform.

1 Learning unit 1

Study unit 1 Derivative markets and instruments

CONTENTS

LEARNING OUTCOMES and ASSESSMENT CRITERIA

Once you have worked through this study unit, you should be able to do the following:

- Argue the concept of a derivative.
- Differentiate between exchange traded and over-the-counter (OTC) derivatives.
- Assess a forward commitment and identify the different type of forward commitments.
- Contrast the basic characteristics of forward contracts, futures contracts and swaps.
- Assess a contingent claim and identify the different types of contingent claims.
- Identify the basic characteristics of options and distinguish between an option to buy and an option to sell.
- Differentiate between the ways of measuring the size of the global derivatives market.
- Evaluate the purposes and criticisms of derivative markets.
- Interpret the concept of arbitrage and critique the role it plays in determining prices and in promoting market efficiency.

(Adapted from the Learning Outcomes as given on page 1 of the prescribed book.)

KEY CONCEPTS

derivatives	underlying	spot price
forward commitments	forward contract	futures contract
swap	contingent claims	option
exchange-traded	OTC	derivatives dealers
price discovery	risk management	arbitrage

OVERVIEW

Accepting risk is essential to building wealth. Investors have to take risks to exploit potentially profitable opportunities. Risks that are not profitable must be avoided in order to take on more risks that are advantageous to the investor. Derivatives enable the shedding and economical retention of risk, which in combination constitute the management of risk.

A derivative instrument is a financial instrument whose value depends on some underlying (spot market) asset. The price and/or value of a derivative security is based on (derived from) the price of its underlying asset.

The four main derivatives classes that will be introduced in this study unit and explained in detail in the next four study units are: forwards and futures contracts, options and swaps.

Read the Introduction on pages 1 and 2 of the prescribed book. The introduction gives you an overview of risk management and explains how derivatives are defined and used in the risk management process.

STUDY

Study chapter 1 of the prescribed book. Read through the chapter at least twice before attempting to achieve the learning outcomes and answer the assessment questions.

ACTIVITY and FEEDBACK

There are no activities for this study unit.

ASSESSMENT

Work through the problems found at the end of chapter 1 in the prescribed book. This should be done without looking at the solutions on pages 23 and 24. The problems serve as a test on the contents of the chapter.

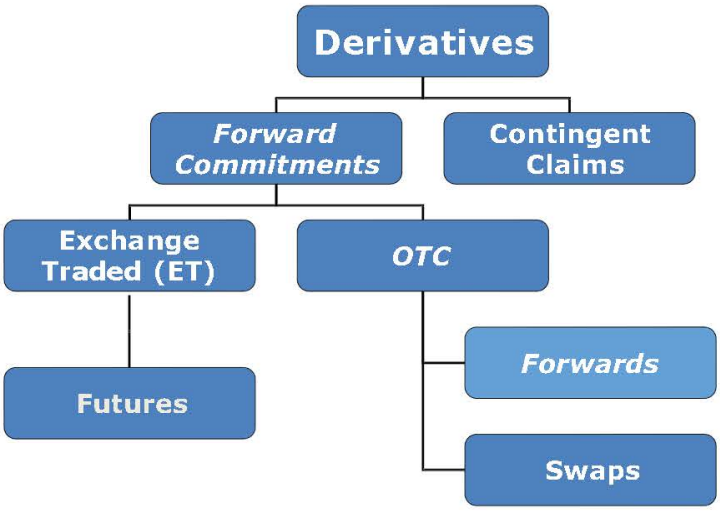
SUMMARY

The inherent danger in utilising derivatives (leveraged instruments) is their capacity to not only reduce risk but also create risk owing to an enhanced uncovered exposure. Ultimately, the primary purpose of derivative instruments remains the protection of an existing exposure, either by containing any potential upside or downside (**hedge**), retaining the upside while limiting the downside (**insurance**) or aligning the exposure to future expectations or needs (**transforming** assets or liabilities).

Forward and futures contracts are used for hedging, options for insurance and swaps to transform assets or liabilities. These different instruments will be discussed in more detail in the following study units. The classification of derivatives is set out in diagram 1.

Refer to the Key Points as presented on page 20 of the prescribed book for a more detailed discussion of the contents of chapter 1.

Diagram 1: Classification of derivative instruments:



CHECKLIST

- [ ] Did you read the chapter in full to get an overall impression of the content?
- [ ] Did you complete the activity?
- [ ] Did you complete the assessment?
- [ ] Did you contact your lecturer with specific questions on problem areas?
- [ ] Have you studied (learned, understood and practised) the content of this chapter?
- [ ] Have you achieved the stated learning outcomes?
- [ ] Would you be able to satisfy the stated assessment criteria?

Study unit 2 Forward markets and contracts

CONTENTS

LEARNING OUTCOMES and ASSESSMENT CRITERIA

Once you have worked through this study unit, you should be able to do the following:

- Distinguish between the positions held by the long and short parties to a forward contract.
- Identify the procedures for selling a forward contract at expiration.
- Demonstrate how a party to a forward contract can terminate a position prior to expiration and how credit risk is affected by the way in which a position is terminated.
- Differentiate between a dealer and an end user of a forward contract.
- Identify the essential characteristics of equity forward contracts.
- Identify the essential characteristics of forward contracts on zero-coupon and coupon bonds.
- Identify the characteristics of the Eurodollar time deposit market.
- Interpret LIBOR and Euribor.
- Identify the essential characteristics of forward rate agreements (FRAs).
- Calculate the payment at expiration of an FRA and evaluate each of the component terms.
- Identify the essential characteristics of currency forward contracts.
- Determine the price of a forward contract.

- Determine the value of a forward contract at initiation, during the life of the contract, and at expiration.
- Assess why valuation of a forward contract is important.
- Contrast an off-market forward contract with the standard type of forward contract.
- Calculate the price and value of an equity forward contract, given the different possible patterns of dividend payments.
- Calculate the price and value of a forward contract on a fixed-income security.
- Calculate the price and value of an FRA.
- Calculate the price and value of a forward contract on a currency.
- Determine how credit risk arises in a forward contract and how market value is a measure of the credit risk to a party in a forward contract.

(Adapted from the Learning Outcomes as given on page 25 of the prescribed book.)

KEY CONCEPTS

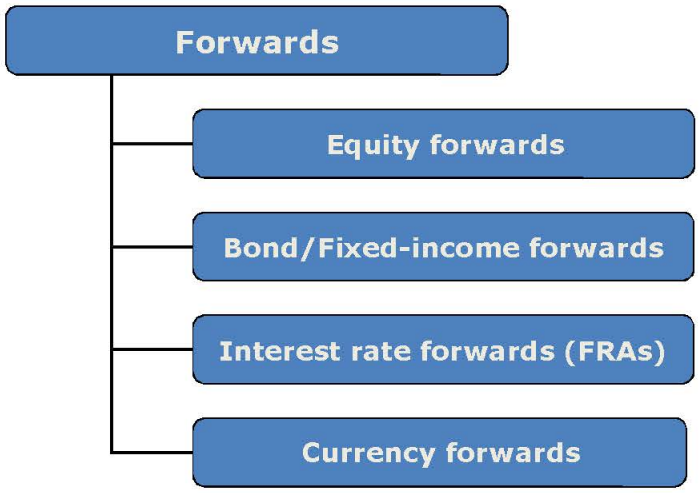
forward contract	default risk	credit risk
equity forwards	currency forwards	fixed-income forwards
individual stocks	stock portfolios	stock indices
interest rate forwards (FRAs)	Forward price	LIBOR
Eurodollar	interest rate parity	off-market FRA
covered interest arbitrage	marking to market	global forward market

OVERVIEW

A **forward contract** is an agreement between two parties in which one party, the buyer, agrees to buy from the other party, the seller, an underlying asset at a future date at a price established today. The contract is customised and each party is subject to the possibility that the other party will default.

The forward contracts that will be discussed in this study unit include equity forwards, bond/fixed-income forwards, interest rate forwards (FRAs) and currency forwards as illustrated in diagram 2.

Diagram 2: Forward contracts



Read the Introduction on pages 26 to 28 of the prescribed book.

STUDY

Study chapter 2 of the prescribed book. Read through the chapter at least twice before attempting to achieve the learning outcomes and answer the assessment questions.

ACTIVITY and FEEDBACK

Work through Practice Problems 1 to 5 in the prescribed book.

ASSESSMENT

Work through the problems found at the end of chapter 2 of the prescribed book. This should be done without looking at the solutions on pages 72 to 79. The problems can serve as a test on the contents of the study unit.

SUMMARY

Refer to the Key Points as presented on pages 64 to 66 of the prescribed book.

CHECK-LIST

- ☐ Did you read the chapter in full to get an overall impression of the content?
- ☐ Did you complete the activity?
- ☐ Did you complete the assessment?
- ☐ Did you contact your lecturer with specific questions on problem areas?
- ☐ Have you studied (learned, understood and practised) the content of this chapter?
- ☐ Have you achieved the stated learning outcomes?
- ☐ Would you be able to satisfy the stated assessment criteria?

Study unit 3 Futures markets and contracts

CONTENTS

LEARNING OUTCOMES and ASSESSMENT CRITERIA

Once you have worked through this study unit, you should be able to do the following:

- Identify the institutional features that distinguish futures contracts from forward contracts
- Determine the origins of modern futures markets.
- Identify the primary characteristics of futures contracts.
- Differentiate between margin in the securities markets and margin in the futures markets.
- Demonstrate how a futures trade takes place and how a futures position may be closed out prior to expiration.
- Interpret initial margin, maintenance margin, variation margin, and settlement price.
- Formulate the process of marking to market.
- Calculate the margin balance given the previous day's balance and the new futures price.
- Interpret price limits, limit move, limit up, limit down and locked limit.
- Demonstrate how a futures contract can be terminated by a closeout at expiration, a delivery, an equivalent cash settlement, or an exchange for physical.
- Identify delivery options in futures contracts.
- Distinguish between scalpers, day traders, and position traders.

- Identify the primary characteristics of the following types of futures contracts: Treasury bill, Eurodollar, Treasury bond, stock index, and currency.
- Argue why the futures price must converge to the spot price at expiration.
- Determine the value of a futures contract.
- Contrast forward and futures prices.
- Demonstrate how an arbitrage transaction is constructed to derive the futures price.
- Identify the different types of monetary and nonmonetary benefits and costs associated with holding the underlying asset, and determine how they affect the futures price.
- Contrast backwardation and contango.
- Argue whether futures prices equal expected spot prices.
- Demonstrate how to price Treasury bill futures.
- Assess the concept of an implied repo rate.
- Identify and illustrate the difficulties in determining the price of Eurodollar futures.
- Identify and illustrate how to price Treasury bond futures.
- Identify and illustrate how to price stock index futures.
- Identify and illustrate how to price currency futures.
- Evaluate the role of futures markets and exchanges in financial systems and in society.

(Adapted from the Learning Outcomes as given on pages 81 and 82 of the prescribed book.)

KEY CONCEPTS

standardised	clearinghouse	daily settlement
marking to market	futures trading	futures exchanges
initial margin	variation margin	maintenance margin
locked limit	limit move	limit up
limit down	Treasury notes	bond futures
interest rate futures	T-bill futures	Eurodollar futures
currency futures	stock index futures	stock index futures
conversion factor	futures arbitrage	convenience yield
cost-of-carry model	contango	backwardation

OVERVIEW

In the previous study unit you learned that a **forward contract** is an agreement between two parties in which one party, the buyer, agrees to buy from the other party, the seller, an

underlying asset at a future date at a price established today. The contract is customised and each party is subject to the possibility that the other party will default. A **futures contract** is therefore defined as variation of a forward contract that has essentially the same basic definition, but some clearly distinguishable additional features, the most important being that it is not a private and customised transaction. It is a public, standardised transaction that takes place on a futures exchange. The difference between forwards and futures contracts are illustrated in diagram 3.

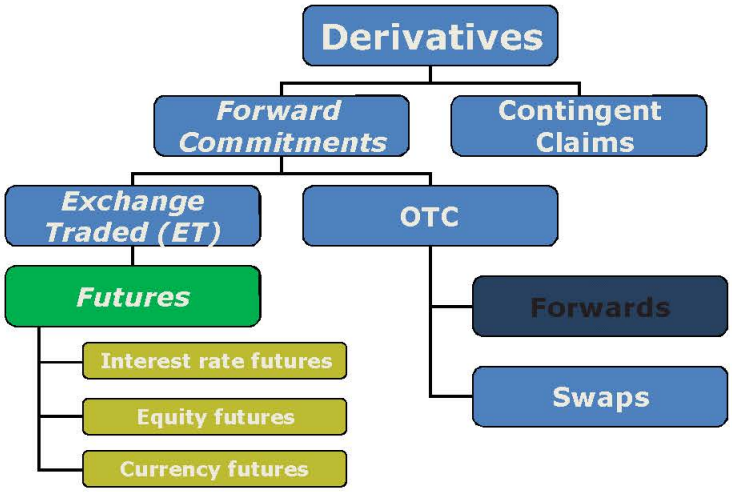
Diagram 3 Difference between forward and futures contracts

Forward contract	Futures contract
Over the counter	Futures exchange
Private	Public
Customised	Standardised
Default risk	Default free
Not marked to market	Marked to market
Held until expiration	Offset possible
Not liquid	Liquid
Unregulated	Regulated

Read the Introduction on pages 82 to 84 of the prescribed book.

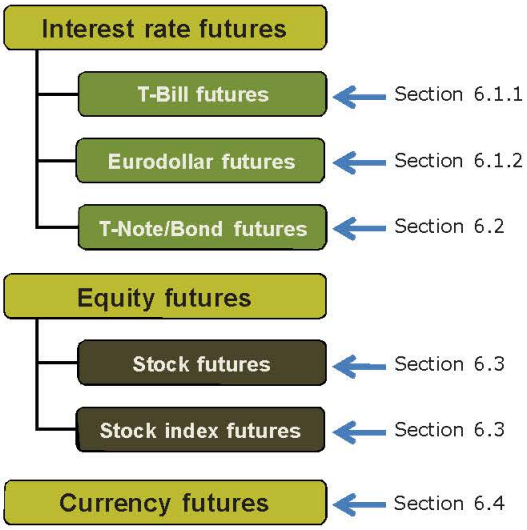
The way futures contracts fits into the derivative framework in diagram 1 is illustrated as follows:

Diagram 4 Classification of futures contracts



Futures contracts are divided into three sections, namely interest rate futures, equity futures and currency futures (as seen in diagram 4) and then subdivided into the different types of futures contracts available in the market. These are graphically set out in diagram 5.

Diagram 5 Type of futures contracts



STUDY

Study chapter 3 of the prescribed book. Read through the chapter at least twice before attempting to answer the learning outcomes and assessment questions.

ACTIVITY and FEEDBACK

Work through Practice Problems 1 to 6 of the prescribed book.

ASSESSMENT

Work through the problems found at the end of chapter 3 of the prescribed book. This should be done without looking at the solutions on pages 147 to 157. The problems can serve as a test on the contents of the study unit.

SUMMARY

Refer to the Key Points as presented on pages 139 to 142 of the prescribed book.

CHECK-LIST

- ☐ Did you read the chapter in full to get an overall impression of the content?
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Study unit 4 Option markets and contracts

CONTENTS

LEARNING OUTCOMES and ASSESSMENT CRITERIA

Once you have worked through this study unit, you should be able to do the following:

- Identify the basic elements and characteristics of option contracts.
- Assess European option, American option, moneyness, payoff, intrinsic value, and time value.
- Differentiate between exchange-traded options and over-the-counter options.
- Identify the different varieties of options in terms of the types of instruments underlying them.
- Compare and contrast interest rate options with forward rate agreements (FRAs).
- Determine option payoffs, and differentiate between interest rate option payoffs and payoffs of other types of options.
- Contrast interest rate caps and floors.
- Identify the minimum and maximum values of European options and American options.
- Illustrate how the lower bounds of European calls and puts are determined by constructing portfolio combinations that prevent arbitrage and calculate an option's lower bound.
- Determine the lowest prices of European and American calls and puts based on the rules for lower bounds.
- Illustrate how a portfolio (combination) of options establishes the relationship between options that differ only by exercise price.
- Determine how option prices are affected by differences in the time to expiration.

- Illustrate how put-call parity for European options is established by comparing the payoffs on a fiduciary call and a protective put and use the result to create synthetic instruments. Argue why an investor would want to do so.
- Illustrate how violations of put-call parity for European options can be exploited and how those violations are eliminated.
- Compare American options with European options in terms of the lower bounds on option prices and the possibility of early exercise.
- Assess how cash flows on the underlying asset affect put-call parity and the lower bounds on option prices.
- Identify the directional effect of an interest rate change on an option's price.
- Determine an option price and illustrate how an arbitrage opportunity can be exploited in a one-period binomial model.
- Determine an option price in a two-stage binomial model.
- Calculate prices of options on bonds and interest rate options in one- and two-period binomial models.
- Determine how the binomial model value converges as time periods are added.
- Identify and assess the assumptions underlying the Black-Scholes-Merton model.
- Calculate the value of a European option using the Black-Scholes-Merton model.
- Determine how an option price, as represented by the Black-Scholes-Merton model, is affected by each of the input vales (the Greeks).
- Illustrate and interpret the concept of an option's delta and how it is used in dynamic hedging.
- Determine the gamma effect on an option's price and delta.
- Determine how cash flows on the underlying asset affect an option's price.
- Identify and illustrate the two methods of estimating the volatility of the underlying.
- Illustrate how put-call parity for options on forwards (or futures) is established.
- Identify the similarities in American options on forwards and futures, and differentiate them from European options.
- Calculate the value of a European option on forwards (or futures) using the Black model.
- Calculate the value of a European interest rate option using the Black model.
- Evaluate the role of options markets in financial systems and society.

(Adapted from the Learning Outcomes as given on pages 159 and 160 of the prescribed book.)

KEY CONCEPTS

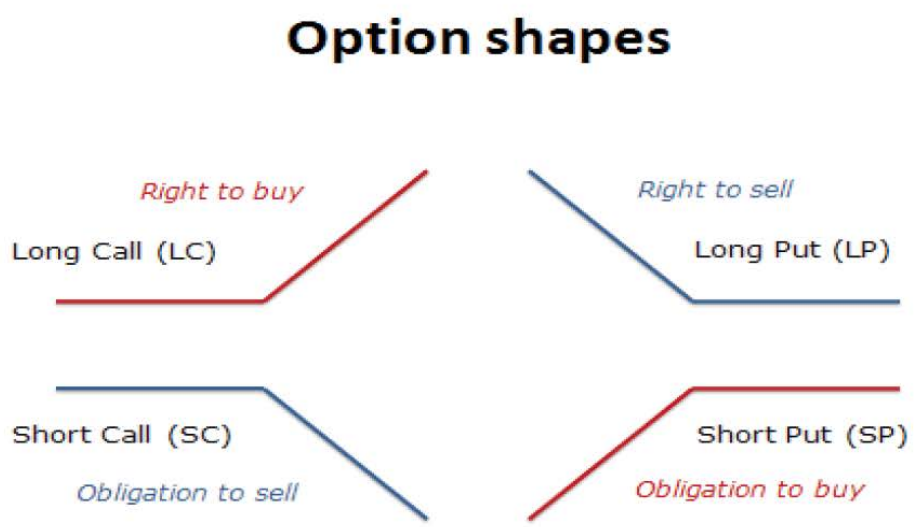
option contracts	call	put
European option	American option	OTC markets
at-the-money	out-of-the-money	in-the-money
options exchanges	stock options	index options
bond options	interest rate options	zero-cost collar
interest rate cap	interest rate floor	caplet
floorlet	interest rate collar	commodity options
currency option	options on futures	fiduciary call
protective put	put-call parity	synthetic put
binomial model	synthetic call	Greeks
hedge ratio	risk-neutral probabilities	risk-neutral valuation
arbitrage opportunity	Black-Scholes Merton	rho
delta	gamma	implied volatility
theta	vega	synthetic forward contract
options on forwards	delta hedge	Black model

OVERVIEW

Options are more versatile than futures, not only providing leverage and insurance (protection), but also allowing for different combinations and strategies (covered call, protective put, straddle and butterfly/bull/bear spreads), depending upon the investor's market expectations and intentions. Possible drawbacks are the more complex calculations and intricate behaviour of option prices. Options contracts can be divided into call options and put options. ***Call options*** grant the holder (long position) the opportunity to buy the underlying security at a price below the current market price, provided that the market price exceeds the call strike before or at expiration (specified contingency). ***Put options***, in contrast, grant the holder (long position) the opportunity to sell the underlying security at a price above the current market price, provided that the put strike exceeds the market price before or at expiration (specified contingency).

The shapes of the different options are illustrated in diagram 6.

Diagram 6 Shapes of option contracts



Read the Introduction on pages 160 and 161 of the prescribed book.

STUDY

Study chapter 4 of the prescribed book. Read through the chapter at least twice before attempting to achieve the learning outcomes and answer the assessment questions.

ACTIVITY and FEEDBACK

Work through Practice Problems 1 to 12 of the prescribed book.

ASSESSMENT

Work through the problems found at the end of chapter 4 of the prescribed book. This should be done without looking at the solutions on pages 250 to 267. The problems can serve as a test on the contents of the study unit.

SUMMARY

Refer to the Key Points as presented on pages 239 to 242 of the prescribed book.

CHECK-LIST

- ☐ Did you read the chapter in full to get an overall impression of the content?
- ☐ Did you complete the activity?
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5 Learning unit 5

Study unit 5 Swap markets and contracts

CONTENTS

LEARNING OUTCOMES and ASSESSMENT CRITERIA

Once you have worked through this study unit, you should be able to do the following:

- Identify the characteristics of swap contracts.
- Demonstrate how swaps are terminated.
- Identify the types of currency swaps.
- Calculate the payments on a currency swap.
- Classify a plain vanilla interest rate swap.
- Calculate the payments on an interest rate swap.
- Identify the types of equity swaps.
- Calculate the payments on an equity swap.
- Distinguish between the pricing and valuation of swaps.
- Illustrate the equivalence of swaps to combinations of other instruments.
- Determine how interest rate swaps are equivalent to a series of off-market forward rate agreements (FRAs).
- Determine how a plain vanilla swap is equivalent to a combination of an interest rate call and an interest rate put.
- Determine the fixed rate on a plain vanilla interest rate swap and the market value of the swap during its life.

- Determine the fixed rate, if applicable, and the foreign notional principal for a given domestic notional principal on a currency swap, and determine the market values of each of the different types of currency swaps during their lives.
- Determine the fixed rate, if applicable, on an equity swap and the market values of the different types of equity swaps during their lives.
- Identify and interpret the characteristics of swaptions, including the difference between payer and receiver swaptions.
- Determine why swaptions exist and identify their applications.
- Illustrate how the payoffs of an interest rate swaption are like those of an option on a coupon-bearing bond.
- Calculate the value of an interest rate swaption on the expiration day.
- Contrast the different ways in which the market value of a swaption at expiration can be received.
- Evaluate forward swaps and distinguish between forward swaps and swaptions.
- Determine how credit risk arises in a swap and distinguish between current credit risk and potential credit risk.
- Identify and assess at what point in a swap's life credit risk is the greatest.
- Interpret the swap spread and what it represents.
- Illustrate how swap credit risk is reduced by both netting and marking to market.
- Evaluate the role that swaps play in the financial system.

(Adapted from the Learning Outcomes as given on pages 269 and 270 of the prescribed book.)

KEY CONCEPTS

swap contracts	currency swaps	interest rate swaps
equity swaps	notional principal	LIBOR
floating rates	commodity swaps	fixed rates
constant maturity swap	FRA	return on equity
diff swaps	amortizing swaps	basis swap
floored swap	arrears swap	overnight index swap
receiver swaption	swaption	capped swap
net payment stream	swaption payoffs	payer swaption
potential credit risk	credit risk	pure cash settlement
interest rate exposure	swap spread	current credit risk

OVERVIEW

Swaps are defined as an agreement between two parties to exchange a series of future payments and are the youngest of the derivative instruments.

Read the Introduction on pages 270 to 272 of the prescribed book.

In this study unit we shall be looking at the characteristics and termination of swap contracts, the different type of swaps including ccurrency swaps (section 3.1 in the textbook), interest rate swaps (section 3.2 in the textbook) and equity swaps (section 3.3. in the textbook). Most of chapter 5 is dedicated to the valuation and pricing of the different types of swap contracts and then lastly the variation of swaps, swaptions and risks and the role of swap contracts.

STUDY

Study chapter 5 of the prescribed book. Read through the chapter at least twice before attempting to achieve the learning outcomes and answer the assessment questions.

ACTIVITY and FEEDBACK

Work through Practice Problems 1 to 8 in the prescribed book.

ASSESSMENT

Work through the problems found at the end of chapter 5 of the prescribed book. This should be done without looking at the solutions on pages 326 to 339. The problems can serve as a test on the contents of the study unit.

SUMMARY

Refer to the Key Points as presented on pages 315 to 318 of the prescribed book.

CHECK-LIST

- ☐ Did you read the chapter in full to get an overall impression of the content?
- ☐ Did you complete the activity?
- ☐ Did you complete the assessment?
- ☐ Did you contact your lecturer with specific questions on problem areas?
- ☐ Have you studied (learned, understood and practised) the content of this chapter?
- ☐ Have you achieved the stated learning outcomes?
- ☐ Would you be able to satisfy the stated assessment criteria?

6 Additional slides

References

These slides were created by
Ms E Botha and
Mr G Marozva
Both are employed at Unisa and are academic staff within the Department Finance, Risk management and Banking

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References (Continued)

The prescribed textbook for this module is:
Analysis of Derivatives for the CFA program ~ By Don Chance (2003)

The prescribed textbook was used to create the following slides.

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INVESTMENTS: DERIVATIVES

CHAPTER 1

FORWARD MARKETS AND CONTRACTS

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- Read
- Basic concepts and terminology

What is a:
- Forward contract
- Futures contract
- Swap
- Option
  - Call option
  - Put option

Forward commitment

Holder has a right but does not impose an obligation

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Chapter 1 – Introduction cont...

Forward Contracts	Contingent Claim
No premium paid at inception	Premium Paid at inception

Question : *What is the advantage of a contingent claims over forward commitments ?*

Answer: Permit gain while protecting against losses..... Why is it so?

Chapter 1 – Introduction cont...

If a risk free rate of interest is 7% and an investor enters into a transaction that has no risk, what would be the rate of return the investor should earn in the absence of the risk

A. 0%
B. between 0% and 7%
C. 7%
D. Less than 7%

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Chapter 1 – Introduction cont...

The spot price of Gold is R930 per ounce and the risk free-rate of interest is 5% per annum. Calculate the equilibrium 6-month forward price per ounce of gold.

930×(1+(0.05/2))=R953.25

Why divide by 2... (6-months i.e. half a year)

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INVESTMENTS: DERIVATIVES

CHAPTER 2

FORWARD MARKETS AND CONTRACTS

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Definition

A **forward contract is an agreement between** two parties in which one party, the buyer, agrees to buy from the other party, the seller, an underlying asset at a future date at a price established today. The contract is customised and each party is subject to the possibility that the other party will default.

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- Forwards
  - Equity forwards
  - Bond/Fixed-income forwards
  - Interest rate forwards (FRAs)
  - Currency forwards

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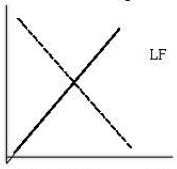
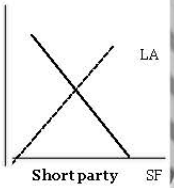
Forwards	Futures
Over the counter	Futures exchange
Private	Public
Customized	Standardized
Default risk	Default free
Not marked to market	**Marked to market**
Held until expiration	Offset possible
Not liquid	Liquid
Unregulated	Regulated

UNISA

Differentiate between the positions held by the long and short parties to a forward contract





- Party that agrees to buy the asset has a long forward position
- Party that agrees to sell the asset has a short forward position

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Pricing and valuation of forward contracts

Are pricing and valuation not the same thing?

- The price is agreed on the initiation date (Forward price or forward rate) i.e. pricing means to determining the forward price or forward rate.
- Valuation, however, means to determine the amount of money that one would need to pay or would expect to receive to engage in the transaction

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Pricing and valuation of forward contracts cont...

F(o,T)- The forward contract price initiated at time **o** and expiring at time **T**

Vo(o,T) – the value of a forward contract initiated at time **o** and expiring at time **T**

Vt(o,T) – the value of a forward contract at the point in time during the life of a contract such as ***t***

Vr(0,T)- Value at expiration

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Pricing and valuation of forward contracts cont...

$$F_T = S_0(1+r)^T$$

Buy asset at So Sell forward contract at F(o,T) Outlay: So	Hold asset and lose interest on out lay	Deliver asset Receive F(o,T)

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Pricing and valuation of forward contracts cont...

$$V_0 = S_0 - \frac{F_T}{(1+r)^T}$$

If **Vo ≠ 0** arbitrage would the prevail

The forward price that eliminates arbitrage:

$$F_T = S_0(1+r)^T$$

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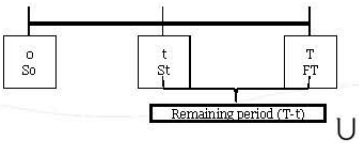
Pricing and valuation of forward contracts cont...

By definition an asset's value is the present value of future cash flows thus,

$$V_t = S_t - \frac{F_T}{(1+r)^{T-t}}$$



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Pricing and valuation of forward contracts cont...

$$F_T = (S_0 - PV(D))(1+r)^T$$

$$PV(D) = \sum \frac{Di}{(1+r)^{T-ti}}$$

When dividends are paid continuously

$$F_T = S_0 e^{-\delta t} \times e^{r_c t}$$

$$r_c = \text{Ln}(1+r)$$

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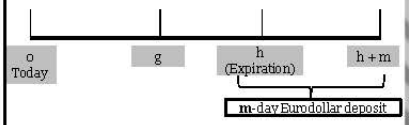
FRAs



h - is the day on which the FRA expires

m – is the number of days on the particular Eurodollar deposit i.e. The underlying asset

h + m is the number of days until the maturity date of the Eurodollar instrument on which the FRA rate is based

g – is the date during the life of the FRA at which we want to determine the value of the FRA

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FRAs Pricing and Valuation

$$FRA_{0,h,m} = \left[\frac{1+L_{0(h+m)}\left(\frac{h+m}{360}\right)}{1+L_{0(h)}\left(\frac{h}{360}\right)} - 1\right]\left(\frac{360}{m}\right)$$

$$V_g(0,h,m) = \frac{1}{1+L_g(h-g)\left(\frac{h-g}{360}\right)} - \frac{1+FRA_{0,h,m}\left(\frac{m}{360}\right)}{1+L_g(h+m-g)\left(\frac{h+m-g}{360}\right)}$$

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Calculate and interpret the payment at expiration of a FRA and identify each of the component terms

$$FRA_{payoff} = NP\left(\frac{(U_{rate} - FRA_{rate}) \times \left(\frac{U_{days}}{360}\right)}{1 + U_{rate} \times \left(\frac{U_{days}}{360}\right)}\right)$$

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- ESKOM P/L is expecting to receive a cash inflow of R20, 000,000.00 in 90 days. Short term interest rates are expected to fall during the next 90 days. In order to hedge against this risk, the company decides to use an FRA that expires in 90 days and is based on 90day LIBOR. The FRA is quoted at 6%. At expiration LIBOR is 5%. Indicate whether the company should take a long or short position to hedge interest rate risk. Using the appropriate terminology, identify the type of FRA used here. Calculate the gain or loss to ESKOM P/L as a consequence of entering the FRA.

$$R20{,}000{,}000.00 \times \left(\frac{(0.05 - 0.06) \times \left(\frac{90}{360}\right)}{1 + 0.05 \times \left(\frac{90}{360}\right)}\right) = -49{,}382.72$$

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• Identify the characteristics of currency forwards

- Exchange of currencies
- Exchange rate specified
- Manage foreign exchange risk

– Domestic risk-free rate
– Foreign risk free rate
– Interest rate parity (IRP)
– Covered interest arbitrage

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Determine the price of a forward contract

- Initial or delivery price

$F_T = S0(1+r)^T = K$

- Forward price during period

$F_T = St(1+r)^{(T-t)}$

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Determine the value of a forward contract at initiation, during the life of the contract, and at expiration

$$Vo = So - \left(\frac{K}{(1+r)^T}\right) = 0$$

$$Vt = St - \left(\frac{K}{(1+r)^{T-t}}\right) = 0$$

alternatively

$$Vt = \frac{FT - K}{(1+r)^{T-t}} = 0$$

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Calculate the price and value of a forward contract on a currency

- **Price – currency forward**

Discrete interest

$$f_0(T) = S_0\left(\frac{(1+r_d)^T}{(1+r_f)^T}\right)$$

Continuous interest

$$f_0(T) = S_0 e^{(r_d{}^c - r_f{}^c)T}$$

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Value – currency forward

Discrete interest

$$Vt = \frac{St}{(1+r_f)^{T-t}} - \frac{FT}{(1+r_d)^{T-t}}$$

$$V_t = St\left[e^{-r_f^c(T-t)}\right] - FT\left[e^{-r_d^c(T-t)}\right]$$

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- **Covered interest arbitrage**

$$(1+rd)^T = (1+rf)^T\left(\frac{F}{S}\right)$$

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INVESTMENTS: DERIVATIVES

CHAPTER 3

FUTURES MARKETS AND CONTRACTS

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Definition

A **forward contract** is an agreement between two parties in which one party, the buyer, agrees to buy from the other party, the seller, an underlying asset at a future date at a price established today. The contract is customized and each party is subject to the possibility that the other party will default.

A **futures contract** is a variation of a forward contract that has essentially the same basic definition, but some clearly distinguishable additional features, the most important being that it is not a private and customized transaction. It is a public, standardized transaction that takes place on a futures exchange.

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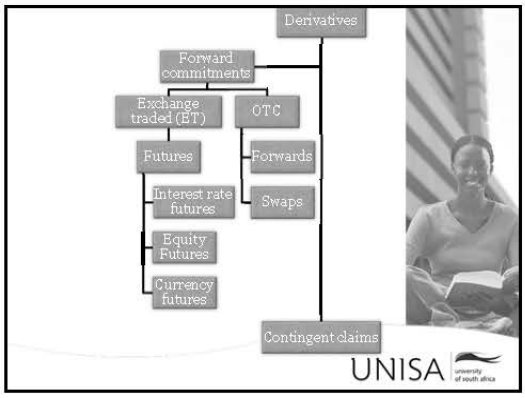
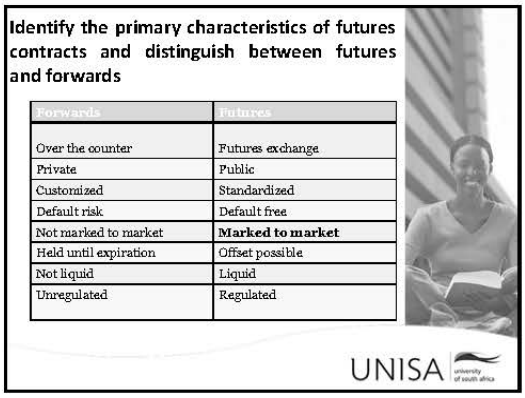

Derivatives
Forward commitments
Exchange traded (ET)
OTC
Futures
Forwards
Interest rate futures
Swaps
Equity Futures
Currency futures
Contingent claims
UNISA


Identify the primary characteristics of futures contracts and distinguish between futures and forwards
Forwards
Futures
Over the counter
Futures exchange
Private
Public
Customized
Standardized
Default risk
Default free
Not marked to market
Marked to market
Held until expiration
Offset possible
Not liquid
Liquid
Unregulated
Regulated
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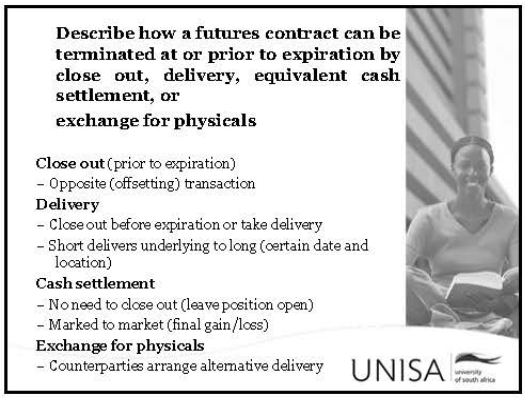
Describe how a futures contract can be terminated at or prior to expiration by close out, delivery, equivalent cash settlement, or exchange for physicals
Close out (prior to expiration)
– Opposite (offsetting) transaction
Delivery
– Close out before expiration or take delivery
– Short delivers underlying to long (certain date and location)
Cash settlement
– No need to close out (leave position open)
– Marked to market (final gain/loss)
Exchange for physicals
– Counterparties arrange alternative delivery
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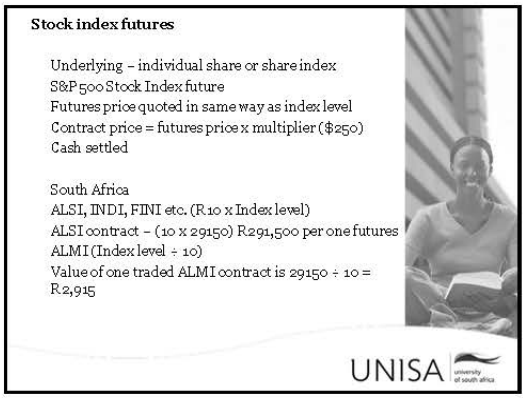
Stock index futures
Underlying – individual share or share index
S&P 500 Stock Index future
Futures price quoted in same way as index level
Contract price = futures price x multiplier ($250)
Cash settled
South Africa
ALSI, INDI, FINI etc. (R10 x Index level)
ALSI contract – (10 x 29150) R291,500 per one futures
ALMI (Index level ÷ 10)
Value of one traded ALMI contract is 29150 ÷ 10 = R2,915
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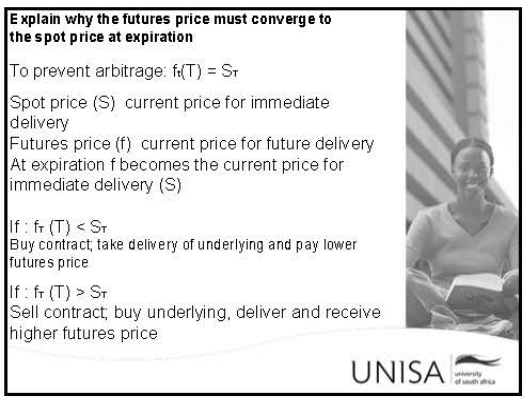
Explain why the futures price must converge to the spot price at expiration
To prevent arbitrage: $f_t(T) = S_T$
Spot price (S) current price for immediate delivery
Futures price (f) current price for future delivery
At expiration f becomes the current price for immediate delivery (S)
If : $f_T(T) < S_T$
Buy contract; take delivery of underlying and pay lower futures price
If : $f_T(T) > S_T$
Sell contract; buy underlying, deliver and receive higher futures price
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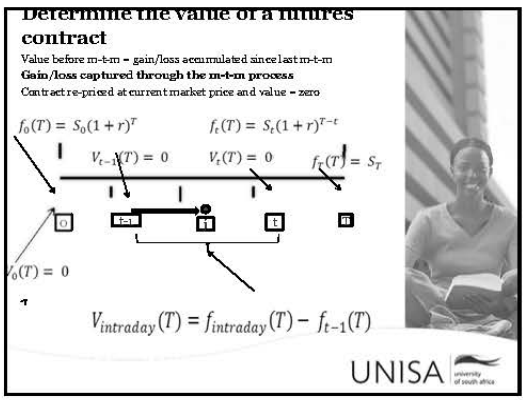
Determine the value of a futures contract
Value before m-t-m = gain/loss accumulated since last m-t-m
Gain/loss captured through the m-t-m process
Contract re-priced at current market price and value = zero
$f_0(T) = S_0(1+r)^T$
$f_t(T) = S_t(1+r)^{T-t}$
$V_{t-1}(T) = 0$
$V_t(T) = 0$
$f_T(T) = S_T$
0
t-1
t
T
$V_0(T) = 0$
$V_{intraday}(T) = f_{intraday}(T) - f_{t-1}(T)$
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Summary: Value of a futures contract

- Like forwards, futures have no value at initiation
- Unlike forwards, futures do not accumulate any value
- Value always zero after adjusting for day's gain or loss (m-t-m)
- Value different from zero only during m-t-m intervals
- Futures value = current price – price at last m-t-m time
- Futures price increase -> value of long position increases
- Value set back to zero by the end-of-day mark to market

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Contrast forward and futures prices

- Futures – settle daily and essentially free of default risk
- Forwards – settle at expiration and subject to default risk

Interest rates positively correlated with futures prices

- Long positions prefer futures to forwards – futures prices higher
- Gains generated when rates increase – invest at higher rates
- Incur losses when rates decrease – cover losses at lower rates
- For example: Gold futures

Interest rates negatively correlated with futures prices

- Prefer not to mark to market – forward prices higher
- For example: Interest rate futures
- Correlation low/close to zero – difference between F and f small
- Assumption: Forward and futures prices are the same
- Ignore effects of marking a futures contract to market

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No-arbitrage futures prices

Cash-and-carry arbitrage: $f_0(T) = S_0(1+r)^T$

Today:
Sell futures contract
Borrow money
Buy underlying

At expiration:
Deliver asset and receive futures price
Repay loan plus interest

Reverse C&C arbitrage:

Today:
- Buy futures contract
- Sell/short underlying
- Invest proceeds

At expiration:
Collect loan plus interest
Pay futures price and take delivery

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Identify the different types of monetary and non-monetary benefits and costs associated with holding the underlying, and how they affect the futures price

Costs associated with storing or holding the underlying
- Increase the no-arbitrage futures price

Financial assets
- No storage costs

- **Monetary benefit to holding asset**
  - Earn dividends, coupons or interest
  - Decrease the no-arbitrage futures price
- **Non-monetary benefit to holding asset**
  - Asset in short supply
  - Convenience yield (decrease price)

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- **No cost or benefit to holding the asset**

$$f_0(T) = S_0(1+r)^T$$

- **Net cost or benefit to holding an asset**

$$f_0(T) = S_0(1+r)^T + \text{FV CB}$$

- Cost-of-carry model
- CB -> cost minus benefit (negative or positive value)

- Costs exceed benefits (net cost) – future value added
- Benefits exceed costs (net benefit) – FV subtracted

- Financial assets
- High(er) cash flows -> lower futures

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- **Forward price**

$$F_T = [S_0 - PV(D)](1+r)^T$$

$$F_T = [S_0(1+r)^T] - FV(D)$$

- **Futures price**

$$f_0(T) = S_0(1+r)^T + FV(CB)$$

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Contrast backwardation and contango

- **Backwardation**
  - Futures price **below the spot price**
  - Significant benefit to holding asset
  - Net benefit (negative cost of carry)
- **Contango**
  - Futures price **above spot price**
  - Little/no benefit to holding asset
  - Net cost (positive cost of carry)

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Treasury bond futures

$$f_0(T) = \frac{B_0(1+r)^T - FV(C)}{CF}$$

$$f_0(T) = \frac{[B_0 - PV(C)](1+r)^T}{CF}$$

Example:

Calculate the no-arbitrage futures price of a 1.2 year futures contract on a 7% T-bond with exactly 10 years to maturity and a price of \$1,040. The annual risk-free rate is 5%. Assume the cheapest to deliver bond has a conversion factor of 1.13.

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Answer:

The semi-annual coupon is \$35. A bondholder will receive two coupons during the contract term – i.e., a payment 0.5 years and 1 year from now.

$$FV(C) = 35(1.05)^{(1.2-0.5)} + 35(1.05)^{(1.2-1)} = \$71.56$$

$$PV(C) = \frac{35}{(1.05)^{0.5}} + \frac{35}{(1.05)^1} = \$67.49$$

$$f_0(T) = \frac{1040(1.05)^{1.2} - 71.56}{1.13} = \$912.52$$

$$f_0(T) = \frac{[1040 - 67.49](1.05)^{1.2}}{1.13} = \$912.52$$

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Stock futures

$$F_T = [S_0(1+r)^T] - FV(D)$$

Example:

Calculate the no-arbitrage price for a 120-day future on a stock currently priced at \$30 and expected to pay a \$0.40 dividend in 15 days and in 105 days. The annual risk-free rate is 5%.

$$FV(D) = 0.40(1.05)^{\frac{105}{365}} + 0.40(1.05)^{\frac{15}{365}} = \$0.8065$$

$$f_0(T) = 30(1.05)^{\frac{120}{365}} - 0.8065 = \$29.68$$

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Stock index futures

$$f_0(T) = S_0 e^{(r^c - \delta^c)T}$$

Example:

The current level of the Nasdaq Index is 1,780. The continuous dividend is 1.1% and the continuously compounded risk-free rate is 3.7%. Calculate the no-arbitrage futures price of an 87-day futures contract on this index.

$$f_0(T) = 1780e^{\left[(0.037-0.011)\left(\frac{87}{365}\right)\right]} = \$1,791.07$$

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Currency futures

$$f_0(T) = S_0\left(\frac{(1+r_d)^T}{(1+r_f)^T}\right)$$

$$f_0(T) = S_0 e^{(r_d{}^c - r_f{}^c)T}$$

Example:

The risk-free rates are 5% in U.S. Dollars (\$) and 6.5% in British pounds (£). The current spot exchange rate is \$1.7301/£. Calculate the no-arbitrage \$ price of a 6-month futures contract.

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Currency futures

Answer:

$r_d{}^c = \ln(1.05) = 0.0488 \quad r_f{}^c = \ln(1.065) = 0.0630$

$f_0(T) = 1.7301\left(\frac{(1.05)^{0.5}}{(1.05)^{0.5}}\right) = \$1.7179$

$f_0(T) = 1.7301e^{(0.0488-0.0630)(0.5)} = \$1.7179$

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INVESTMENTS: DERIVATIVES

CHAPTER 4

OPTION MARKETS AND CONTRACTS

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- Identify the basic elements and characteristics of opti on contracts
- Call options grant the holder (long position) the opportunity to buy the underlying security at a price below the current market price, provided that the market price exceeds the call strike before or at expiration (specified contingency).
- Put options grant the holder (long position) the opportunity to sell the underlying security at a price above the current market price, provided that the put strike exceeds the market price before or at expiration (specified contingency).
- The option seller (short position) in both instances receives a payment (premium) compelling performance at the discretion of the holder.

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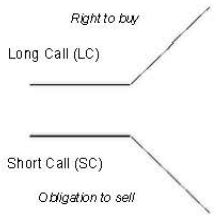
Option shapes



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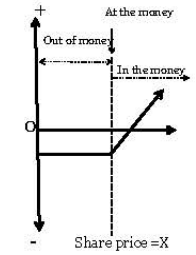
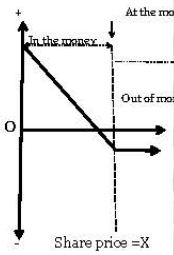
Call option



Put option



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Identify the different varieties of options in terms of the types of instruments underlying them

- Financial options
  Equity options (individual or stock index), bond options, interest rate options, currency options
- Options on futures
  Call options – long position in futures upon exercise
  Put options – short position in futures upon exercise
- Commodity options
  Right to either buy or sell a fixed quantity of physical
  asset at a (fixed) strike price

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JSE Equity Options: Why Trade Options?

- Time to decide
- Protect the value of your shares (put)
- Leverage: small initial requirement can lead to high exposure and profit potential.
- Option holder's losses limited to premium
- Take advantage of unique opportunities:
- Profit in volatile markets (Straddle and Strangle)
- Can be combined with shares already owned to protect the value of your shares at very little cost (Fence)
- Limiting both profit and loss (Collar)
- Flexibility: Tailor-made to your needs

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Notation and variables

Variable	Notation	State	Call option value	Put option value
Spot price	S	Increase	Increase	Decrease
Strike price	X	Higher	Lower	Higher
Volatility	σ	Higher	Higher	Higher
Time to maturity	t	Longer	Higher	Uncertain
Interest rates	r	Higher	Higher	Lower
Call option	C or c		max(o;S – X)	
Put option	P or p			max(o; X – S)

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Explain putcall parity for European options and relate arbitrage an d the construction of synthetic instruments

Fiduciary call – buying a call and investing PV(Payoff is X (otm) or X + (S-X) = S (itm)
Protective put – buying a put and holding asset
Payoff is S (otm) or (X-S) + S = X (itm)

Therefore:
When call is itm put is otm > payoff is S , -
When call is otm, put is itm -> payoff is X

S+ p = c+ PV(X)

S =c +PV(X) -p
P= c +PV(X)- S
C=S+ p -PV(X)
PV(X)= S+ p -c

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Put call parity arbitrage

S + p = c+ PV(X)

c - p = S-PV(X)

c - p >S-PV(X)
Sell call; buy put; buy spot; borrow PV(X)

c - p < S-PV(X)
Buy call; sell put; sell spot; invest PV(X)

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Determine the minimum and maximum values of
European options

The lower bound for any option is zero (otm option)

	Upper bound	Lower bound
Call options	$c \leq S$	$c \geq S - PV(X)$
Put options	$p \leq PV(X)$	$p \geq PV(X) - S$

$$[S - X(1+r)^{-t}] \leq c \leq S$$

$$[X(1+r)^{-t} - S] \leq p \leq X(1+r)^{-t}$$

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Chapter 5 - Swaps

- Equity SWAPs
- Interest rate SWAPs
- Equity and interest rate SWAPs

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Equity Swaps

Consider an equity swap in which the asset manager receives the return of the Russel 2000 Index in return for paying the return on the DJIA. At the inception of the equity swap, the Russel 2000 is at 520.12 and the DJIA is at 9867.33. Calculate the market value of the swap a few months later when the Russel 2000 is at 554.29 and the DJIA is at 9975.54. The notional principal of the swap is $15 million.

$$\text{Russel} = \frac{554.29}{520.12} = 1.0657 \qquad \text{DJIA} = \frac{9,975.54}{9,867.33} = 1.0110$$

$$V_{pay\_DJIA} = \$15,000,000(1.0657 - 1.0110) = \$820,500$$

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Interest rate swaps

- Determining the swap rate = **pricing** of swap
- As rates change over time, the PV of floating payments will either exceed or be less than the PV of fixed payments
  - Difference = **value** of swap
- Market value = difference between bonds
  - Fixed bond minus floating bond
  - Domestic bond minus foreign bond
- PV (receive) minus PV (pay)

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Interest rate swaps….

Consider a two-year interest rate swap with semi-annual payments. Assume a notional principal of $50 million.

Calculate the semi-annual fixed payment and the annualized fixed rate on the swap if the current term structure of LIBOR interest rates is as follows:

$L_0(180) = 0.0688$
$L_0(360) = 0.0700$
$L_0(540) = 0.0715$
$L_0(720) = 0.0723$

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$$B_0(180) = \frac{1}{1+0.0688(180/360)} = 0.9667$$

$$B_0(360) = \frac{1}{1+0.0700(360/360)} = 0.9346$$

$$B_0(540) = \frac{1}{1+0.0715(540/360)} = 0.9031$$

$$B_0(720) = \frac{1}{1+0.0723(720/360)} = 0.8737$$

$$FS(0,4,180) = \frac{1-0.8737}{0.9667+0.9346+0.9031+0.8737} = 0.0343$$

$$\text{Fixed payment} = 0.0343 \times \$50,000,000 = \$1,715,000$$

$$\text{Annualized fixed rate} = 3.43\%(360/180) = 6.86\%$$

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Calculate the market value of the swap 120 days later from the point of view of the party paying the floating rate and receiving the fixed rate, and from the point of view of the party paying the fixed rate and receiving the floating rate if the term structure 120 days later is as follows:

- $L_{120}(60) = 0.0620$
- $L_{120}(240) = 0.0631$
- $L_{120}(420) = 0.0649$
- $L_{120}(600) = 0.0687$

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$$B_{120}(180) = \frac{1}{1+0.0620(60/360)} = 0.9898$$

$$B_{120}(360) = \frac{1}{1+0.0631(240/360)} = 0.9596$$

$$B_{120}(540) = \frac{1}{1+0.0649(420/360)} = 0.9296$$

$$B_{120}(720) = \frac{1}{1+0.0687(600/360)} = 0.8973$$

$$\text{Fixed} = 0.0343(0.9898+0.9596+0.9296+0.8973)+1(0.8973) = 1.0268$$

$$1^{st}\text{ Floating payment} = \left[1+(0.0688)(180/360)\right] = 1.0344$$

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Ans cont...

Discounted with the 60 day present value factor of 0.9898:

$$\text{Float} = 1.0344 \times 0.9898 = 1.0239$$

$$V_{pay\_float} = \$50{,}000{,}000(1.0268 - 1.0239) = \$145{,}000$$

$$V_{pay\_fixed} = \$50{,}000{,}000(1.0239 - 1.0268) = -\$145{,}000$$

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Equity and interest rate swap

Assume an asset manager enters into a one-year equity swap in which he will receive the return on the Nasdaq 100 Index in return for paying a floating interest rate. The swap calls for quarterly payments. The Nasdaq 100 is at 1651.72 at the beginning of the swap. Ninety days later, the rate $L_{90}(90)$ is 0.0665. Calculate the market value of the swap 100 days from the beginning of the swap if the Nasdaq 100 is at 1695.27, the notional principal of the swap is \$50 million, and the term structure is:

$L_{100}(80) = 0.0654$

$L_{100}(170) = 0.0558$

$L_{100}(260) = 0.0507$

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$$B_{100}(80) = \frac{1}{1 + 0.0654(80/360)} = 0.9857$$

Next floating payment $= \left[1 + (0.0665)(90/360)\right] = 1.0168$

Discounted with the 80 day present value factor of 0.9857:

$$\text{Float} = 1.0168 \times 0.9857 = 1.0023$$

$$\text{Equity} = \frac{1{,}695.27}{1{,}651.72} = 1.0264$$

$$V_{pay\_float} = \$50{,}000{,}000(1.0264 - 1.0023) = \$1{,}205{,}000$$

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BEST WISHES FOR YOUR EXAMINATION

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7 Understanding the standard normal table

Cumulative Probabilities for a Standard Normal Distribution

$P(X \leq x) = N(x)$ for $x \geq 0$ or $1 - N(-x)$ for $x < 0$

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.00	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.10	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.20	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.30	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.40	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.50	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.60	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.70	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.80	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.90	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.00	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8
1.10	0.8643	0.8665				29	0.8749	0.8770	0.8790	0.8810	0.8
1.20	0.8849	0.8869				25	0.8944	0.8962	0.8980	0.8997	0.9
1.30	0.9032	0.9049				99	0.9115	0.9131	0.9147	0.9162	0.9
1.40	0.9192	0.9207				51	0.9265	0.9279	0.9292	0.9306	0.9
1.50	0.9332	0.9345				82	0.9394	0.9406	0.9418	0.9429	0.9441
1.60	0.9452	0.9463				95	0.9505	0.9515	0.9525	0.9535	0.9545
1.70	0.9554	0.9564				91	0.9599	0.9608	0.9616	0.9625	0.9633
1.80	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.90	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.00	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.10	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.20	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.30	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.40	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.50	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.60	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.70	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.80	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.90	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.00	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

D2 is calculated as 0.45. Thus go down to 0.4 and then across to 0.05 (0.45) to get 0.6736

D1 is calculated as 0.59. So you go down to 0.5 and then across to 0.09

8 Formula sheet

PRICING AND VALUATION OF FORWARD CONTRACTS

Discrete interest (r)

$$(1+r)^T$$

Continuous interest (r^c)

$$e^{r^c T}$$

Conversion ($r \leftrightarrow r^c$)

$$r^c = \ln(1+r)$$

$$r = e^{r^c} - 1$$

Discount interest

$$\text{Amount}\left[1 - r\left(d/360\right)\right]$$

Add-on interest

$$\text{Amount}\left[1 + r\left(d/360\right)\right]$$

Forward rate agreement (FRA)

Price/rate

$$FRA_{rate} = \left[\frac{1+L_0\left(\frac{h+m}{360}\right)}{1+L_0\left(\frac{h}{360}\right)} - 1\right]\left(\frac{360}{m}\right)$$

Value

$$V_g = \left[\frac{1}{1+L_g\left(\frac{h-g}{360}\right)}\right] - \left[\frac{1+FRA_{rate}\left(\frac{m}{360}\right)}{1+L_g\left(\frac{h+m-g}{360}\right)}\right]$$

Payoff

$$FRA_{payoff} = NP\left[\frac{\left(U_{rate} - FRA_{rate}\right)\left(\frac{U_{days}}{360}\right)}{1+U_{rate}\left(\frac{U_{days}}{360}\right)}\right]$$

Forward contract – no cash flows

Price

$$F_T = S_0(1+r)^T$$

Value

$$V_0 = S_0 - \frac{F_T}{(1+r)^T} \qquad V_t = S_t - \frac{F_T}{(1+r)^{T-t}}$$

Forward contract – equity

Price and value – discrete compounding

$$F_T = \left[S_0 - PV(D)\right](1+r)^T$$

$$V_t = S_t - PV(D) - \left[\frac{F_T}{(1+r)^{T-t}}\right]$$

Price and value – continuous compounding

$$F_T = S_0 e^{\left(r^c - \delta^c\right)T}$$

$$V_t = S_t e^{-\delta(T-t)} - F_T e^{-r(T-t)}$$

Forward contract – fixed income

Price and value

$$F_T = \left[B_0 - PV(C)\right](1+r)^T$$

$$V_t = B_t - PV(C) - \left[\frac{F_T}{(1+r)^{T-t}}\right]$$

Forward contract – currency

Price and value – discrete compounding

$$F_T = S_0\left[\frac{(1+r_d)^{d/365}}{(1+r_f)^{d/365}}\right]$$

$$V_t = \left[\frac{S_t}{(1+r_f)^{T-t}}\right] - \left[\frac{F_T}{(1+r_d)^{T-t}}\right]$$

Price and value – continuous compounding

$$F_T = S_0 e^{\left(r_d^c - r_f^c\right)(d/365)}$$

$$V_t = S_t e^{-r_f^c(T-t)} - F_T e^{-r_d^c(T-t)}$$

Interest rate parity (IRP)

$$(1+r_d)^{d/365} = (1+r_f)^{d/365}\left(\frac{F}{S}\right)$$

PRICING OF FUTURES CONTRACTS

Futures price – no cost or benefit

$$f_0(T) = S_0(1+r)^T$$

Futures price – net cost or benefit

$$f_0(T) = S_0(1+r)^T + FV(CB)$$

Futures price – stock

$$f_0(T) = S_0(1+r)^T - FV(D)$$

Futures price – stock index

$$f_0(T) = S_0 e^{(r^c - \delta^c)T}$$

Futures price – Treasury bill

$$f_0(T) = \frac{B_0(1+r)^T - FV(C)}{\text{Conversion Factor}}$$

Futures price – currency

Discrete compounding

$$f_o(T) = S_0\left[\frac{(1+r_d)^{d/365}}{(1+r_f)^{d/365}}\right]$$

Continuous compounding

$$f_0(T) = S_0 e^{(r_d^c - r_f^c)(d/365)}$$

PRICING AND VALUATION OF SWAPS

Net fixed payment

$$NFP = (\text{swap rate} - \text{LIBOR})\left(\frac{\text{days}}{360}\right)NP$$

Swap fixed rate

$$C = \left(\frac{1 - Z_4}{Z_1 + Z_2 + Z_3 + Z_4}\right)$$

Discount rate

$$Z_{day} = \frac{1}{1 + \left(R_{day} \times \frac{\text{days}}{360}\right)}$$

Market value of interest rate swap

$$MV_{IRS} = V_{\text{floating–rate bond}} - V_{\text{fixed–rate bond}}$$

Market value of currency swap

$$MV_{CS} = V_{\text{domestic bond}} - V_{\text{foreign bond}}$$

Return on equity

$$\text{Return} = \left(\frac{\text{Ending value}}{\text{Beginning value}}\right)$$

Yield on equity

$$\text{Yield} = \left(\frac{\text{Ending value}}{\text{Beginning value}}\right) - 1$$

Payment on equity position

$$PMT = \text{Yield} \times NP$$

Market value of equity swap

$$MV_{ES} = NP(\text{Return}_X - \text{Return}_Y)$$

Swaption payoffs

$$\text{Payoff}_{\text{Swaption–payer}} = (SFR - X)\left(\frac{\text{days}}{360}\right)NP$$

$$\text{Payoff}_{\text{Swaption–receiver}} = (X - SFR)\left(\frac{\text{days}}{360}\right)NP$$

PRICING OF OPTION CONTRACTS

Intrinsic values

$$c = \max\left[0; (S - X)\right]$$

$$p = \max\left[0; (X - S)\right]$$

Bounds – European options

No cash flows – upper and lower

$$\left[S - X(1+r)^{-t}\right] \le c \le S$$

$$\left[X(1+r)^{-t} - S\right] \le p \le X(1+r)^{-t}$$

Cash flows – lower bounds

$$c \ge \left[S - PV(CF)\right] - PV(X)$$

$$p \ge PV(X) - \left[S - PV(X)\right]$$

Bounds – American options

$$\left[S - X(1+r)^{-t}\right] \le C \le S$$

$$(X - S) \le P \le X$$

Put-call parity – European options

No cash flows

$$S + p = c + X(1+r)^{-t}$$

Cash flows

$$\left[S - PV(CF)\right] + p = c + PV(X)$$

Futures contracts

$$F_T(1+r)^{-t} + p = c + X(1+r)^{-t}$$

Put-call parity – American options

$$S - X \le C - P \le S - X(1+r)^{-t}$$

Binomial model

$$p = \frac{(1+r) - d}{u - d}$$

$$f = \frac{\left[(p)(f^{+}) + (1-p)(f^{-})\right]}{(1+r)}$$

$$f = \frac{\left[(p)^2(f^{++}) + 2(p)(1-p)(f^{+-}) + (1-p)^2(f^{--})\right]}{(1+r)^2}$$

Black-Scholes Merton model

Black-Scholes model

$$d_1 = \frac{\ln(S/X) + \left[r^c + (\sigma^2/2)\right]T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

$$c = SN(d_1) - Xe^{-r^cT}N(d_2)$$

$$p = Xe^{-r^cT}N(-d_2) - SN(-d_1)$$

Merton's model

$$d_1 = \frac{\ln(S/X) + \left[(r^c - \delta) + (\sigma^2/2)\right]T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

$$c = Se^{-\delta T}N(d_1) - Xe^{-r^cT}N(d_2)$$

$$p = Xe^{-r^cT}N(-d_2) - Se^{-\delta T}N(-d_1)$$

[$r^c = r_d^c$ and $\delta = r_f^c$ when pricing currency options]

Black's model

$$d_1 = \frac{\ln(F/X) + (\sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

$$c = e^{-r^cT}\left[FN(d_1) - XN(d_2)\right]$$

$$p = e^{-r^cT}\left[XN(-d_2) - FN(-d_1)\right]$$

Delta

$$\text{Delta} = \frac{f_1 - f_0}{S_1 - S_0} = N(d_1)$$

Interest rate options

$$IR_{call} = NP\left(U_{rate} - X_{rate}\right)\left(\frac{d}{360}\right)$$

$$IR_{put} = NP\left(X_{rate} - U_{rate}\right)\left(\frac{d}{360}\right)$$

9 Derivatives podcasts

The podcast links can be copied into a web browser that is enabled to access podcasts. This document does not provide a preview of the podcasts. Only the link to each podcast is provided for the different learning units.

Learning unit 1: Podcast links

- What are derivatives?:
  https://www.youtube.com/watch?v=Wjlw7ZpZVK4

Learning unit 2: Podcast links

- Forward contract introduction:
  https://www.youtube.com/watch?v=H9UEZdAnnt8
- Upper bound on forward settlement price:
  https://www.youtube.com/watch?v=FcRxdq7KGOQ
- Lower bound on forward settlement price:
  https://www.youtube.com/watch?v=_uDb0ZdGb8w

Learning unit 3: Podcast links

- Futures introduction:
  https://www.youtube.com/watch?v=3g6P0lRXotI
- Motivation for the futures exchange:
  https://www.youtube.com/watch?v=iyrOGvbSS2U
- Futures margin mechanics:
  https://www.youtube.com/watch?v=1t42z5U2uoc
- Arbitraging futures contract:
  https://www.youtube.com/watch?v=0jk6uLZ1Tdc

Learning unit 4: Podcast links

- Call payoff diagram:
  https://www.youtube.com/watch?v=MZQxeQYQCUg
- Put payoff diagram:
  https://www.youtube.com/watch?v=VST_U297pH0
- Put-call parity:
  https://www.youtube.com/watch?v=pr-u4LCFYEY
- Introduction to the Black-Sholes formula:
  https://www.youtube.com/watch?v=pr-u4LCFYEY

Learning unit 5: Podcast links

- Interest rate swap 1:
  https://www.youtube.com/watch?v=PLjyj1FJqig