① Question: use the pumping lemma with length and show that the following language is not context free:

$$\{a^n b^{n+1} c^{n-1} \mid n \geq 1\}$$

Introduction

* The first step is to assume that the language $L = \{a^n b^{n+1} c^{n-1} \mid n \geq 1\}$ is context free. This means that there exist a CFG in CNF with, say, p live productions which generates the language.

Because we assume that the language is context free, we may apply the pumping lemma with length.

According to the pumping lemma with length any word $w \in L$ with more than 2^p characters can be broken up into five parts. i.e the word can be written as $w = uvxyz$, with

$$\begin{cases} - \text{length}(vxy) \leq 2^p \\ - \text{length}(x) > 0 \\ - \text{length}(v) + \text{length}(y) > 0 \end{cases}$$

and where all words of the form $uv^n xy^n z$ with $n \geq 1$ are also in the language.

* Now we must choose a suitable word from L which is long enough. The word $w = a^{2^p} b^{2^p+1} c^{2^p-1}$ is a good choice. It is in L and it has more than 2^p characters.

Let us examine the different ways in which the word $a^{2^p} b^{2^p+1} c^{2^p-1}$ can be broken up to five parts

② Remember that vxy cannot have more than 2^p characters this means that there are only a few possible ways in which vxy may occur within the word:
(i) Consisting of letters of one type only. i.e a's from a^{2^p} group, or b's from b^{2^p+1} group or c's from c^{2^p-1} group.
(ii) Consisting of a's followed by b's i.e straddling the a^{2^p} group and b^{2^p+1} group or
(iii) Consisting of b's followed by c's, i.e straddling the b^{2^p+1} group and the c^{2^p-1} group.

Body of the proof

Case I

Suppose vxy consists entirely of a's, i.e consists only of characters from a^{2^p} group. According to pumping lemma with length the word $uvvxyyz$ is also in the language L. This ~~pumped~~ pumped word will have more than 2^p a's in ~~from~~ of $b^{2^p+1}c^{2^p-1}$, therefore the required pattern will be disturbed. Such word is not in L. Similarly if vxy consists entirely of b's (b^{2^p+1} group), according to pumping lemma with length the word $uvvxyyz$ is also in L. This pumped will start with 2^p a's followed by more than 2^{p+1} b's followed by c^{2^p-1}. Such word is not in L. The same argument will hold if vxy consists entirely of c's, i.e consist only of characters from the c^{2^p-1} group, because the pumped word will start with $a^{2^p}b^{2^p+1}$ but will then have more than 2^p-1 c's. In All ~~three~~ three cases the pattern will be lost when the word is pumped

③ The pumped word is not on the required form. therefore the pumped word is not in L.

Case II.

Suppose vxy consists of part of the a^{2^p} group and part of b^{2^p+1} group. According to the pumping lemma with length the word $uvvxyyz$ is also in the language L. This pumped word will have more than 2^p a's and/or more than 2^p+1 and/or more than one ab substring, followed by c^{2^p-1} and such a word is not in L.

Case III

Similarly, suppose vxy consists of part of the b^{2^p+1} group and part of the c^{2^p-1} group. According to the pumping lemma with length the word $uvvxyyz$ must also be in the language L, but this word will start with a^{2^p}, now followed by more than 2^p+1 b's and/or more than 2^p-1 c's and/or more than one bc substring. Such word is not in L.

We have now seen that all possible choices of vxy **Conclusion**
lead to the fact that the word uv^2xy^2z cannot
be in the language L. This means that the language
does not conform to what the pumping lemma with
length states about context free languages. Therefore
the initial assumption that L is context free must
be discarded. We conclude that the given language is
not context-free. Remark: If it were context-free according
to pumping lemma with length there should be at least one choice of
vxy for which a pumped word belongs to the language