Tutorial Letter 101/3/2018

Formal Logic 3

COS3761

Semesters 1 and 2

School of Computing

This tutorial letter contains important information about your module.

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CONTENTS

1 INTRODUCTION AND WELCOME

Dear Student

Welcome to Formal Logic 3 (COS3761). We hope that you will find the module interesting and stimulating. You are most welcome to contact your lecturers for any academic queries regarding the module – see sections 3.1 and 3.2 below. (See section 3.3 for information regarding other kinds of queries.)

This tutorial letter contains general information about COS3761: we discuss the tutorial matter, student support, the syllabus, the requirements for examination admission, the semester mark and how to submit assignments. In addition, this tut letter contains Assignments 1, 2 and 3, first for the first semester and then for the second semester as well.

Some quick facts about the three assignments:

- You must submit at least one assignment to get examination admission.
- The three assignments contribute towards your semester mark, which counts 20% of your final mark.

2 PURPOSE AND OUTCOMES OF THE MODULE

2.1 PURPOSE

The purpose of studying logic is to refine one's natural ability to reason and argue. Logic is concerned with training the mind to think clearly. The aim of logic is to obtain clarity in the definition and arrangement of our ideas and other mental images, consistency in our judgements, and validity in our processes of inference. Logic is about representing knowledge in a precise language so that a computer can reason about it, i.e. so that an algorithm can be defined to make valid deductions from the knowledge. Logic is applied extensively in the fields of Artificial Intelligence, Computer Science and Philosophy, and this module aims to provide a solid foundation for these studies.

2.2 OUTCOMES

At the end of this module you should have obtained strategies for thinking effectively, know about the common errors in reasoning which should be avoided and have developed effective techniques for evaluating arguments. You should know some of the logic languages used to represent knowledge, and understand some of the computer algorithms used to reason about the knowledge represented in such languages.

Abbreviated syllabus:

- propositional logic – declarative sentences, natural deduction, semantics, normal forms
- predicate logic – syntax, natural deduction, semantics, undecidability
- modal logics – syntax, semantics, logic engineering, natural deduction, multi-agent systems

3 LECTURER AND CONTACT DETAILS

3.1 LECTURERS

The names and contact details of the lecturers for COS3761 are specified in Tutorial Letter COSALLF/301/4/2018

. Students may contact lecturers by mail, e-mail or telephone. *We recommend the use of e-mail*.

Students may make appointments to see a lecturer, but this has to be done well in advance. Students should mention their student numbers in all communications with the lecturers.

3.2 DEPARTMENT

If you have not yet received the contact details of your lecturer and would like to speak to him or her, you may contact the secretary of the School of Computing. Remember to mention your student number.

This is for academic queries only. Please do not contact the School about missing tutorial matter, cancellation of a module, payments, enquiries about the registration of assignments, and so on, but rather the relevant department as indicated in the brochure *my Studies @ Unisa*.

3.3 UNIVERSITY

The brochure *my Studies @ Unisa* (that you should have received with your tutorial matter) contains information about computer laboratories, the library, myUnisa, assistance with study skills and so on. It also contains contact details of several Unisa departments, for example Examinations, Assignments, Despatch, Finances and Student Administration. Remember to mention your student number when contacting the University.

4 MODULE RELATED RESOURCES

4.1 TUTORIAL MATTER

The prescribed book (see section 4.2) is essential study material, as are the tutorial letters that you will receive during the course of the semester.

Tutorial Letter 101 (this tutorial letter) is the most important document in terms of specifying the scope of this module, and what you need to do complete it successfully.

Additional notes are available in Additional Resources on myUnisa. They are meant to help you to master the material in the prescribed book. They explain some of the concepts in more detail, and give additional examples, etc. They are intended to be used together with the prescribed book.

The 200 series tutorial letters (Tutorial Letters 201, 202 and 203) will discuss the assignments. These will be made available on myUnisa after the respective due dates of the assignments.

Only Tut Letter 101 will be printed and sent to you. All other tutorial matter (besides the prescribed book) will only be available in electronic form on myUnisa.

One of the requirements for study in the School of Computing is to have regular access to myUnisa because you are expected to use it for your studies. When you attempt to use this facility for the first time, you have to register. Go to my.unisa.ac.za and click on “Claim UNISA login”. Then follow the instructions on the screen. You will be given a password for future use.

To download study material from myUnisa, log in to myUnisa using your student number and password and choose COS3761. Tut Letter 101 is available in Official Study Material (in the menu on the left), and Tut Letters 201, 202 and 203 will be available in Additional Resources. All other tutorial matter will be available in Learning Units or in Additional Resources.

4.2 PRESCRIBED BOOK

The prescribed book for this course is

Huth, M & Ryan, M. 2005. *Logic in computer science: modelling and reasoning about systems*. 2nd edition. United Kingdom: Cambridge University Press.

We cover chapters 1, 2 and 5 of the prescribed book in this module. Not all the sections of these chapters are prescribed, though. The following list specifies which sections of the prescribed chapters are for examination purposes, and which you may read for interest's sake:

Sections 1.1, 1.2, 1.3, 1.4, 1.5.1 and 1.5.3 Sections 1.5.2 and 1.6	Examination purposes Read only
Sections 2.1, 2.2, 2.3, 2.4 and 2,5 Theorem 2.22 and Section 2.6 Section 2.7	Examination purposes Read only Leave out
Sections 5.1, 5.2, 5.3, 5.4, 5.5.1 and 5.5.2 Sections 5.5.3 and 5.5.4	Examination purposes Read only

You have to purchase a copy of the prescribed book yourself at any of the official booksellers mentioned in the brochure *my Studies @ Unisa.* If you have any difficulties obtaining a copy of the prescribed book from these bookshops, please contact the Unisa Prescribed Book Section as soon as possible.

4.3 LEARNING UNITS ON MYUNISA

Three learning units are provided on the COS3761 website on myUnisa, one for each assignment. They are intended to give guidance on how to work through the tutorial matter for each assignment. They also provide some hints on how to tackle and answer the assignments.

4.4 MO001 TUTORIAL LETTER

There is an additional tutorial letter MO001 available in Additional Resources on the myUnisa website for COS3761. It is a collection of many of the other resources available on the COS3761 website in one document. For example, it contains the three learning units, as well as the list of errata in the prescribed book, the additional notes for the three prescribed chapters, and solutions to selected exercises.

The reason why these resources are duplicated and collected together in this single tutorial letter is to allow students who do not have regular access to the internet to use this single document in place of the individual resources. Some students (who do have regular access to the internet) prefer to work from a limited number of documents, rather than from multiple individual ones. MO001 attempts to address this preference.

5 STUDENT SUPPORT SERVICES FOR THE MODULE

Important information appears in the brochure *my Studies @ Unisa*. The Student Services Bureau of Unisa provides support for students in general academic matters, such as selecting appropriate modules, developing study skills, adapting to distance education and general difficulties with studies.

Although we, the COS3761 lecturers, do not take active part in the discussion forums on myUnisa, we would like to encourage you to join your fellow students there.

You may also organise your own study group. However, we expect every member of a study group to write and submit the assignments on his or her own. Thus, discuss problems and find solutions together as a group, but then *do the assignment yourself and submit your own work*. It is dishonest to pass off the work of somebody else as your own. Plagiarism is the act of taking the words, ideas and thoughts of others and passing them off as your own. It is a form of theft which involves a number of dishonest academic activities. The *Disciplinary Code for Students* (2004) is given to all students at registration. You are advised to study the Code, especially sections 2.1.13 and 2.1.14 (2004:3–4). Kindly read the University's *Policy on Copyright Infringement and Plagiarism* as well.

5.1 STUDENTS WHO HAVE LIMITED ACCESS TO THE INTERNET

Although all modules in the School of Computing require students to have access to the internet, we realise that there are some students who only have limited access.

To try to make it easier for such students to study COS3761, we have made the following arrangements:

- All COS3761 tutorial matter available on myUnisa, except the solutions to assignments in Tutorial Letters 201, 202 and 203, are collected together into a single document MO001. (See section 4.4.) As far as we know, this tutorial letter will be printed and sent to all students. In any case, it is available for download in Additional Resources on myUnisa. Such students also have to download Tutorial Letters 201, 202 and 203 when they are available.
- Students can submit assignments by post instead of myUnisa. (See section 7.4.1.) Unfortunately, such assignments will not be returned by post because marked assignments have to be downloaded from myUnisa.

6 MODULE SPECIFIC STUDY PLAN

Use the brochure *my Studies @ Unisa* for general time management and planning skills.

The due dates for assignments are given in section 7.3 below. The learning units for COS3761 on myUnisa give directions for working though the relevant sections of the prescribed book and the other resources, and what to do in preparation for the assignments and the exam.

7 ASSESSMENT

7.1 ASSESSMENT PLAN

The marks that you obtain for Assignments 1, 2 and 3 form the semester mark for COS3761. The semester mark forms 20% and the examination 80% of the final mark for the module. The weights of the COS3761 assignments are indicated in the table below.

Assignment	Weight
1	30%
2	40%
3	30%

Example: Suppose a student gets 60% for Assignment 1, 45% for Assignment 2 and doesn't submit Assignment 3. In order to calculate the contribution to the semester mark, the mark obtained for each assignment is multiplied by the weight. This then forms part of the 20% that the semester mark contributes to the final mark. Therefore:

Assignment	Marks obtained	Weight	Contribution to semester mark	
1	60%	30%	60/100 x 30/100 x 20	3.6
2	45%	40%	45/100 x 40/100 x 20	3.6
3	0%	30%	0/100 x 30/100 x 20	0.0
Total				7.2

In this example, the student has a semester mark of 7.0 out of a possible 20. Note that the semester mark does not form part of the final mark of a supplementary examination.

7.2 ASSIGNMENT NUMBERS

Every assignment has a general assignment number and a unique assignment number.

Semester	General assignment number	Unique assignment number	Due date
1	1	**765814**	21 February 2018
	2	**873176**	28 March 2018
	3	**809076**	25 April 2018
2	1	**745876**	08 August 2018
	2	**855673**	12 September 2018
	3	**753314**	05 October 2018

7.3 DUE DATES

The due dates of assignments are given in the table in section 7.2 above. Please note, no extension of the due dates is possible.

7.4 SUBMISSION OF ASSIGNMENTS

Doing the assignments is extremely important for mastering the study material. We strongly advise you to complete and submit all three assignments.

The three assignments for both the first and second semesters are given in section 8 of this tutorial letter. The tutorial matter that should be studied for each assignment appears at the start of each assignment. When submitting your assignments, follow the guidelines given in this tutorial letter, in Tutorial Letter COSALLF/301/4/2016 and in the brochure *my Studies @ Unisa*.

- ***Assignment 1*** is a *multiple-choice assignment* and should be submitted electronically through myUnisa. No extension will be granted for this assignment. In other words, it must be submitted by the due date.

- ***Assignment 2*** is a *written assignment* and should be submitted electronically as a PDF file through myUnisa. The semester system does not allow for the late submission of this assignment.
- ***Assignment 3*** is a *multiple-choice assignment* and should be submitted electronically through myUnisa. No extension will be granted for this assignment.

If myUnisa is offline when you want to submit an assignment, you need not contact us because we will be aware of it. Simply submit it as soon as myUnisa is available again.

It is a good idea to check whether your assignment has been registered on the system after you have submitted it. If it hasn't, there was a problem with capturing your answers, or with uploading your assignment file. You must either attempt to submit the assignment again, or contact the lecturers to address the problem.

For detailed information on assignments and their submission, please refer to the brochure *my Studies @ Unisa*, which you received with your study package.

To submit an assignment via myUnisa:
- Go to myUnisa.
- Log in with your student number and password.
- Select the module.
- Click on "Assignments" in the menu on the left-hand side of the screen.
- Click on the assignment number you wish to submit.
- Follow the instructions.

7.5 SUBMISSION BY POST

As stated in section 5.1, we are aware that there are students who do not have regular access to the internet but who want to submit their assignments. Such students can follow this procedure:

Multiple-choice assignments:
- Fill in your answers to the questions on a mark-reading sheet.
- Submit it as explained in the brochure *my Studies @ Unisa*.

Written assignments:
- Type a document with your answers to the questions, print it out and staple it in an assignment cover.
- Submit the assignment as explained in the brochure *my Studies @ Unisa*.

Please note that if you submit any of your assignments in one of the above ways, it will take time for your assignment to reach Unisa. You must ensure that your assignment is posted in time to be received and recorded on the assignment database by the due date.

Also, you will not receive your marked assignment back in printed form. Unfortunately all feedback on assignments is electronic. For example, if you submit a written document by post, the first thing the Assignment Department will do when they receive your assignment will be to scan it. This electronic version will be sent to us for marking, and will be marked electronically like all other assignments submitted via myUnisa. It will also be returned via myUnisa. If you don't have access to the internet, you will have to get someone to check your myLife account for the notice that your assignment has been marked, or check for you on myUnisa. They will have to download your marked assignment for you.

8 ASSIGNMENTS:

Section 8.1

THE ASSIGNMENTS OF THE FIRST SEMESTER 2018

ASSIGNMENT 1 FIRST SEMESTER

(MULTIPLE CHOICE)

SUBMISSION: On multiple choice form or electronically through myUnisa

Please note that Assignment 1 has to be submitted in order to gain examination admission. It will be to your own advantage to check whether the assignment has been registered on the system after a few days.

If you want to submit the assignment electronically and myUnisa is off-line during that time, you need not contact us, because we will be aware of it. Simply submit it as soon as myUnisa is available again.

DUE DATE	**21 February 2018**
EXTENSION	**No extension will be granted for Assignment 1**.
TUTORIAL MATTER	Textbook chapter 1
WEIGHT OF CONTRIBUTION TO SEMESTER MARK	**30%**
UNIQUE NUMBER	**765814**
QUESTIONS	20 questions. 5 options each. Choose one option in every question.

We recommend

- that you write out a formal proof for (at least) questions 7 to 11 before choosing an option and
- if you have access to a computer, that you use the Fitch software (supplied with the textbook of the second-level Formal Logic module COS2661) to help you to choose the correct option in questions dealing with formal proofs.

Propositional logic symbol	Declarative sentence associated with the symbol
p	Susan saves
q	Susan buys a house
r	The house has seven bathrooms
s	The house is used as a guesthouse
t	Peter likes the house

Table 1

QUESTION 1

Using the symbols and their intended meaning given in table 1, which of the options below is a correct propositional logic translation of the following English sentence?

Susan buys a house only if the house does not have seven bathrooms.

Option 1: $\neg r \rightarrow q$
Option 2: $\neg r \rightarrow \neg q$
Option 3: $q \rightarrow \neg r$
Option 4: $\neg q \rightarrow \neg r$
Option 5: None of the above options is a correct translation.

QUESTION 2

Using the symbols and their intended meaning given in table 1, which of the options below is a correct propositional logic translation of the following English sentence?

Unless Peter does not like the house, Susan saves and buys a house used as guest house.

Option 1: $t \leftrightarrow (p \land q \land s)$
Option 2: $\neg t \leftrightarrow (p \land q \land s)$
Option 3: $\neg t \rightarrow (p \land q \land s)$
Option 4: $t \rightarrow (p \land q \land s)$
Option 5: None of the above options is a correct translation.

QUESTION 3

Using the symbols and their intended meaning given in table 1, which of the options below is a correct propositional logic translation of the following English sentence?

Peter likes the house if and only if it has seven bathrooms and is not used as guesthouse.

Option 1: $t \rightarrow (r \land \neg s)$
Option 2: $(r \land \neg s) \rightarrow t$
Option 3: $t \leftrightarrow (r \land \neg s)$
Option 4: $r \leftrightarrow (t \land \neg s)$
Option 5: None of the above options is a correct translation.

QUESTION 4

Using the symbols and their intended meaning given in table 1, which of the options below is a correct English translation of the following propositional logic sentence?

$s \rightarrow r$

Option 1: The house is used as a guesthouse if it has seven bathrooms.
Option 2: The house is used as a guesthouse only if it has seven bathrooms.
Option 3: The house is used as a guesthouse and it has seven bathrooms.
Option 4: The house does not have seven bathrooms or is used as a guesthouse.
Option 5: None of the above options is a correct translation.

QUESTION 5

Using the symbols and their intended meaning given in table 1, which of the options below is a correct English translation of the following propositional logic sentence?

$\neg p \rightarrow (\neg q \wedge r)$

Option 1: If Susan does not save, she does not buy a house that does not have seven bathrooms.
Option 2: Unless Susan saves, she does not buy a house with seven bathrooms.
Option 3: If Susan saves, she buys a house with seven bathrooms.
Option 4: Susan does not buy a house with seven bathrooms if she saves.
Option 5: None of the above options is a correct translation.

QUESTION 6

Using the symbols and their intended meaning given in table 1, which of the options below is a correct English translation of the following propositional logic sentence?

$s \leftrightarrow (\neg q \wedge t)$

Option 1: The house is used as guesthouse if Susan does not buy it but Peter likes it.
Option 2: If the house is used as guesthouse, Susan does not buy it but Peter likes it.
Option 3: The house is used as guesthouse if and only if Susan does not buy it but Peter likes it.
Option 4: The house is used as guesthouse or Susan does not buy it but Peter likes it.
Option 5: None of the above options is a correct translation.

QUESTION 7

Suppose you have to formally prove the validity of the following sequent using the basic natural deduction rules:

$$p \rightarrow q \wedge r \vdash (p \rightarrow q) \wedge (p \rightarrow r)$$

Write out a proof on some rough paper and then choose the option below that gives a correct strategy.

Option 1:

- Start with the premise $p \rightarrow q \wedge r$.
- Have a subproof starting with the assumption of p.
- Inside the subproof, $p \rightarrow q$ should be derived using the $\rightarrow$ elimination rule, the $\wedge$ elimination rule and the $\rightarrow$ introduction rule, and $p \rightarrow r$ should be derived using the same three rules.
- The $\wedge$ introduction rule should be used in the last step to derive $(p \rightarrow q) \wedge (p \rightarrow r)$.

Option 2:

- Start with the premise $p \rightarrow q \wedge r$.
- Assume p.

- Derive $p \rightarrow q$ using the $\rightarrow$ elimination rule, the $\wedge$ elimination rule and the $\rightarrow$ introduction rule, and derive $p \rightarrow r$ using the same three rules.
- Use the $\wedge$ introduction rule in the last step to derive $(p \rightarrow q) \wedge (p \rightarrow r)$.

Option 3:

- Start with the premise $p \rightarrow q \wedge r$.
- Have two subproofs, both starting with the assumption of p.
- The first subproof should end on $p \rightarrow q$ and the second subproof should end on $p \rightarrow r$.
- The $\rightarrow$ elimination rule, the $\wedge$ elimination rule and the $\rightarrow$ introduction rule should be used in both subproofs.
- The $\wedge$ introduction rule should be used in the last step to derive $(p \rightarrow q) \wedge (p \rightarrow r)$.

Option 4:

- Start with the premise $p \rightarrow q \wedge r$.
- Have two subproofs, both starting with the assumption of p.
- The first subproof should end on q and the second subproof should end on r.
- The $\rightarrow$ elimination rule and the $\wedge$ elimination rule should be used in both subproofs.
- The $\rightarrow$ introduction rule should be used after each subproof to introduce $p \rightarrow q$ and $p \rightarrow r$, respectively.
- The $\wedge$ introduction rule should be used in the last step to derive $(p \rightarrow q) \wedge (p \rightarrow r)$.

Option 5: None of the above options describes a correct strategy.

QUESTION 8

Suppose you have to formally prove the validity of the following sequent using the basic natural deduction rules:

$$(p \rightarrow r) \wedge (q \rightarrow r) \vdash (p \wedge q) \rightarrow r$$

Which of the options below is a correct proof?

Option 1:

1	$(p \rightarrow r) \wedge (q \rightarrow r)$	premise
2	$p \rightarrow r$	$\wedge e_1$ 1
3	$p \wedge q$	assumption
4	p	$\wedge e_1$ 3
5	r	$\rightarrow e$ 2, 4
6	$(p \wedge q) \rightarrow r$	$\rightarrow i$ 3 - 5

Option 2:

1	$(p \rightarrow r) \wedge (q \rightarrow r)$	premise
2	$p \wedge q$	assumption
3	p	$\wedge e_1$ 2
4	r	$\rightarrow e$ 1, 3
6	$(p \wedge q) \rightarrow r$	$\rightarrow i$ 2 - 4

***Option 3*:**

1	$(p \to r) \land (q \to r)$	premise
2	$p \to r$	$\land e_1$ 1
3	$p \land q$	assumption
4	p	$\land e_1$ 3
5	r	$\to e$ 1
6	$(p \land q) \to r$	$\to i$ 3 - 5

***Option 4*:**

1	$(p \to r) \land (q \to r)$	premise
2	$p \to r$	$\land e_1$ 1
3	$p \land q$	assumption
4	p	$\land e_1$ 3
5	r	$\to e$ 1
6	$(p \land q) \to r$	$\to i$ 3 - 5

***Option 5*:** None of the options above is a correct proof.

QUESTION 9

We have to prove the validity of the following sequent using the rules of natural deduction:

$\vdash \neg p \to (p \to (p \to q))$

The following proof is given but two lines are omitted. Which of the options below gives the correct propositional logic sentence and the associated correct rule in both lines?

1		
2	p	assumption
3	p	assumption
4	$\bot$	$\neg e$ 1, 3
5	q	$\bot e$ 4
6	$p \to q$	$\to i$ 3 - 5
7		
8	$\neg p \to (p \to (p \to q))$	$\to i$ 1 – 7

***Option 1*:**

1	$\neg p$	premise
7	$p \to (p \to q)$	$\to i$ 2 – 6

***Option 2*:**

1	$\neg p$	assumption
7	$p \to (p \to q)$	$\to i$ 2 - 6

***Option 3*:**

1	p	assumption

7 $p \to (p \to q)$ $\to$i 2 - 6

Option 4:

1 $\neg p$ assumption

7 $\neg p \to (p \to (p \to q))$ $\to$i 2 - 6

Option 5: None of the options above gives the correct propositional logic sentence and associated rule for both lines.

QUESTION 10

We have to prove the following sequent using the basic natural deduction rules:

$$p \to (q \to r),\ p,\ \neg r \vdash \neg q$$

In the proof below the rules that are used in three of the lines are omitted. Which option below gives the correct rule in each line?

1	$p \to (q \to r)$	premise
2	p	premise
3	$\neg r$	premise
4	$q \to r$	
5	q	assumption
6	r	$\to$e 4, 5
7	$\bot$	
8	$\neg q$	

(Lines 5–7 are enclosed in a box.)

Option 1:

4 $\to$i 1, 2
7 $\neg$e 3, 6
8 $\neg$i 5 - 7

Option 2:

4 $\to$e 1, 2
7 $\neg$e 3, 6
8 $\neg$i 5 - 7

Option 2:

4 $\to$i 1, 2
7 $\neg$e 3, 6
8 $\neg$i 5

Option 2:

4 $\to$e 1, 2
7 $\neg$e 3, 6
8 $\neg$i 5

Option 5: None of the options above gives correct rules for all three lines.

Question 11

We have to prove the following sequent using the basic natural deduction rules:

$$(p \wedge q) \to r,\ r \to s,\ q \wedge \neg s \vdash \neg p$$

In the proof below the sentences in three of the lines are omitted. Which option below gives the correct sentence in each line?

1	$(p \wedge q) \to r$	premise
2	$r \to s$	premise
3	$q \wedge \neg s$	premise
4		assumption
5	q	$\wedge e_1$ 3
6		$\wedge i$ 4, 5
7	r	$\to e$ 1, 6
8	s	$\to e$ 2, 7
9	$\neg s$	$\wedge e_2$ 3
10		$\neg e$ 8, 9
11	$\neg p$	$\neg i$ 4 - 10

Option 1:

4 $p \vee \neg s$

6 $(p \vee \neg s) \wedge q$

10 $p \vee s$

Option 2:

4 $q \to r$

6 $p \wedge r$

10 s

Option 3:

4 p

6 $p \wedge s$

10 $\bot$

Option 4:

4 p

6 $p \wedge q$

10 $\bot$

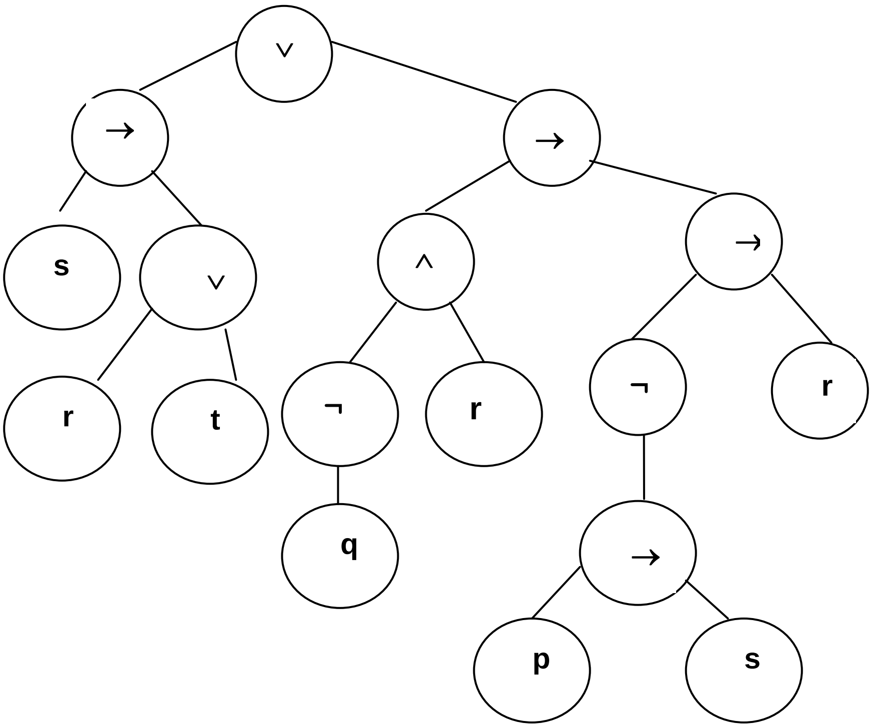
Option 5: None of the options above gives the correct sentence in every line.

QUESTION 12

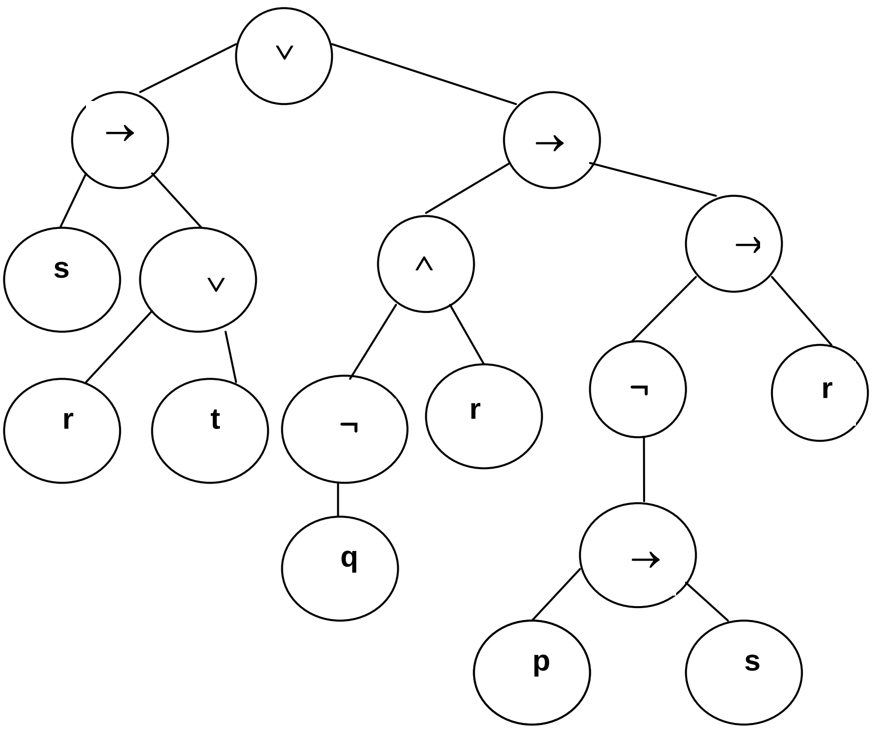
Given the following propositional logic sentence, which of the options below is the corresponding parse tree?

$(((s \to (r \vee t)) \vee ((\neg q) \wedge r)) \to ((\neg(p \to s)) \to r))$

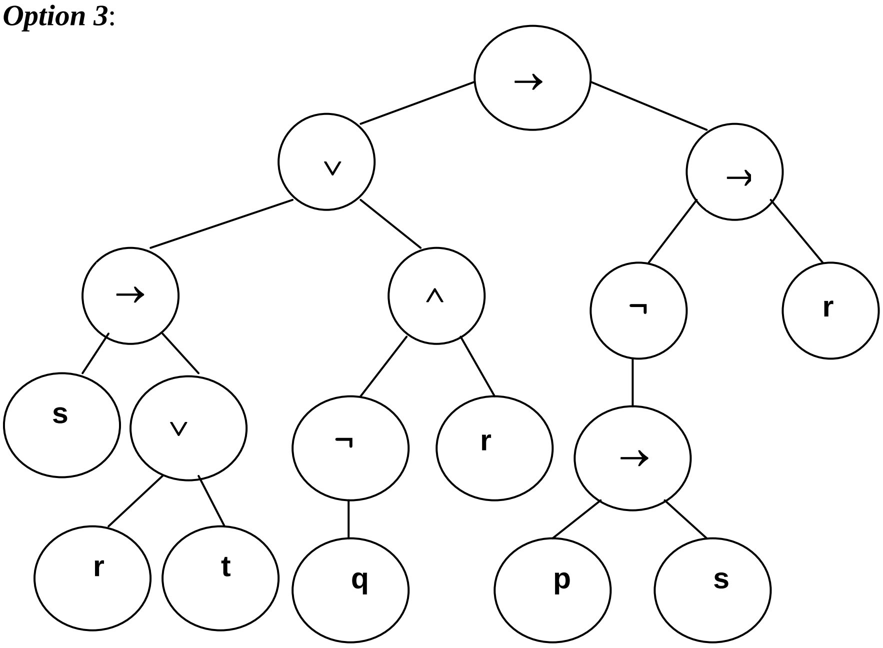
Option 1:



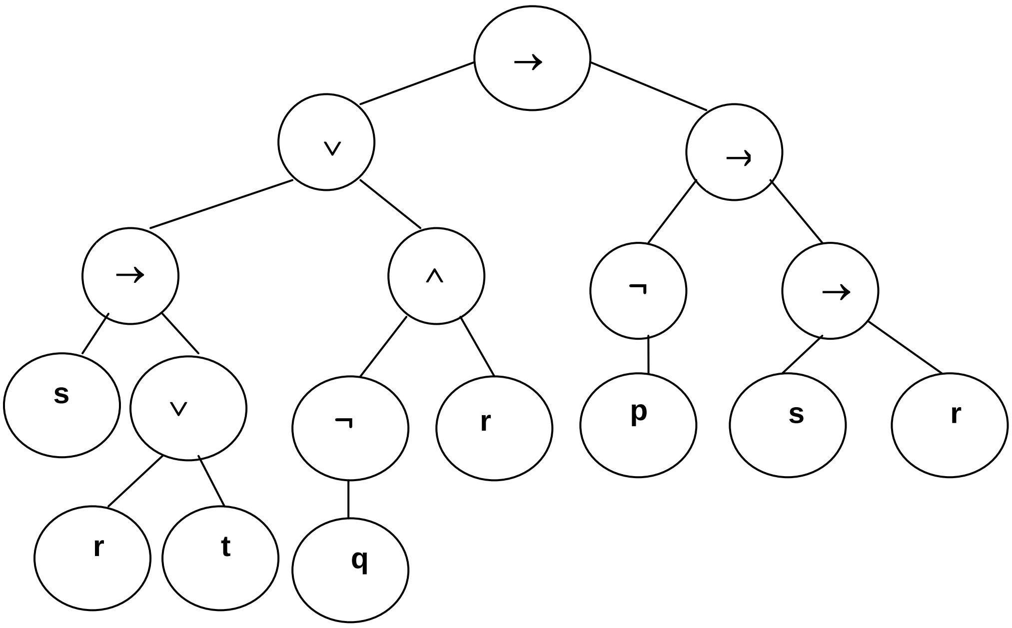
Option 2:



Option 3:



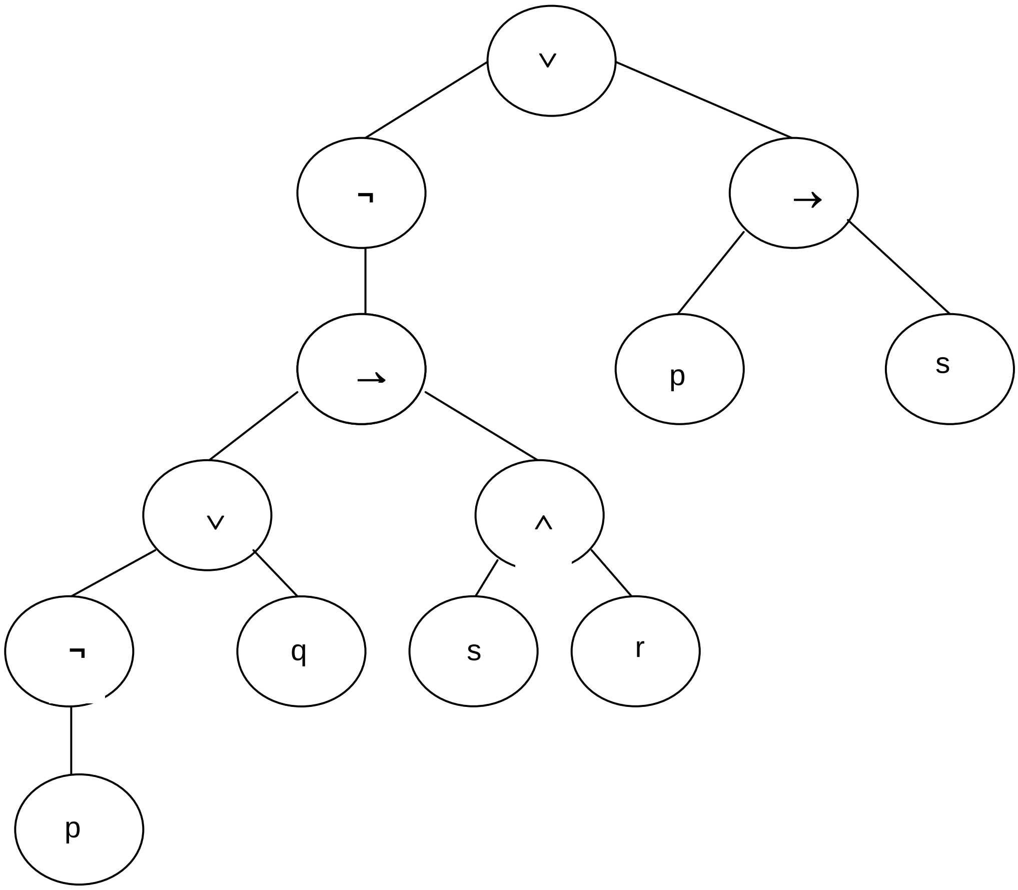
Option 4:



Option 5: None of the options above gives the correct parse trees for the given sentence.

Question 13

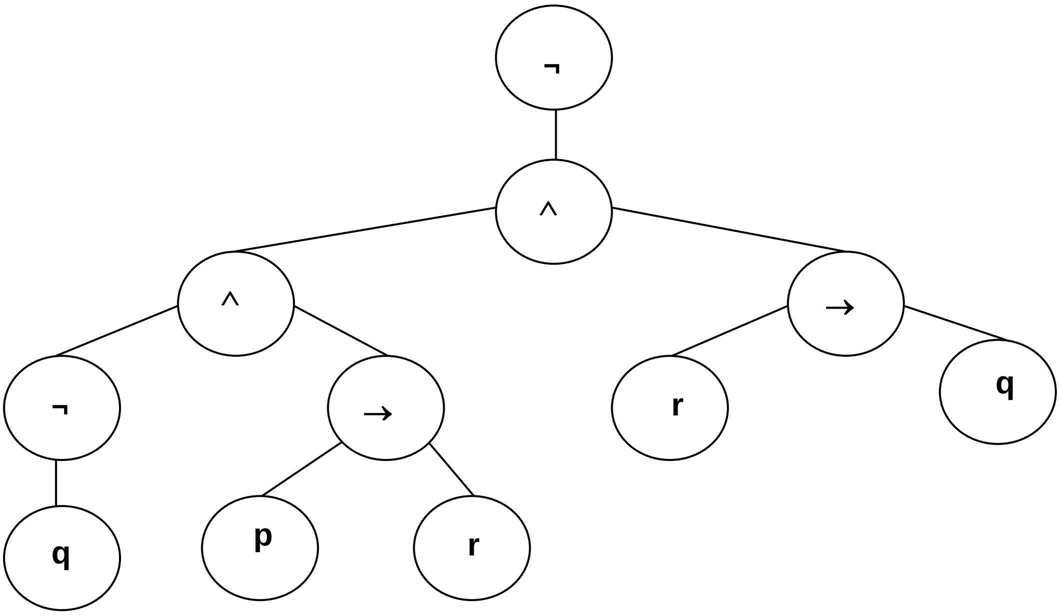
Given the following parse tree, which option below gives the associated propositional logic sentence?



Option 1: ¬ ((¬p ∨ q) → (s ∧ r)) ∨ (p →s)
Option 2: ¬ (¬p ∨ q) → (s ∧ r) ∨ (p →s)
Option 3: ¬ (¬p ∨ q) → s ∧ (r ∨ (p →s))
Option 4: ¬ (¬p ∨ (q → s) ∧ r) ∨ (p →s)
Option 5: None of the options above gives the correct associated sentence.

QUESTION 14

Given the following parse tree, which option below gives the associated propositional logic sentence?



Option 1: ¬ (¬ (q ∧ (p → r)) ∧ (r → q))
Option 2: ¬ (¬ q ∧ (p → r)) ∧ (r → q)
Option 3: ¬ ((¬ (q ∧ (p → r)) ∧ (r → q)))
Option 4: ¬ ((¬ q ∧ (p → r)) ∧ (r → q))
Option 5: None of the options above gives the correct associated sentence.

QUESTION 15

Consider the following sequent and then choose the correct option below.

(p ∧ q) → s, ¬ s ⊢ p ∨ ¬ q

Option 1: The sequent is valid and can be formally proved using natural deduction rules.
Option 2: The sequent is valid because the relation (p ∧ q) → s, ¬ s ⊨ p ∨ ¬ q holds as shown by the following valuation: p = F, q = F, s = F.
Option 3: The sequent is not valid because the relation (p ∧ q) → s, ¬ s ⊨ p ∨ ¬ q does not hold as shown by the following valuation: p = F, q = T, s = F.
Option 4: The sequent is not valid because the relation (p ∧ q) → s, ¬ s ⊨ p ∨ ¬ q does not hold as shown by the following valuation: p = F, q = T, s = T
Option 5: None of the options above is correct.

QUESTION 16

Consider the following sequent and then choose the correct option below.

$\neg r \rightarrow (p \vee q),\; r \wedge \neg q \vdash p \rightarrow q$

Option 1: The sequent is valid and can be formally proved using natural deduction rules.

Option 2: The sequent is valid as shown by the following valuation:
$p = F,\; q = F,\; r = T$

Option 3: The sequent is not valid as shown by the following valuation:
$p = F,\; q = F,\; r = T$

Option 4: The sequent is not valid as shown by the following valuation:
$p = T,\; q = F,\; r = T$

Option 5: None of the options above is correct.

QUESTION 17

Draw the truth tables of the following three propositional logic sentences and then choose the correct option below.

$\neg r \rightarrow p$

$\neg r \wedge q$

$p \vee q \rightarrow r$

Option 1: The sentence $p \vee q \rightarrow r$ is not semantically entailed by $\neg r \rightarrow p$ and $\neg r \wedge q$, because the final columns of the three truth tables are not identical.

Option 2: The sentence $p \vee q \rightarrow r$ is not semantically entailed by $\neg r \rightarrow p$ and $\neg r \wedge q$. It is clear from the line where $p = T,\; q = T,\; r = F$.

Option 3: The sentences $\neg r \rightarrow p$ and $\neg r \wedge q$ are semantically equivalent, because their final columns agree for two evaluations.

Option 4: $\neg r \rightarrow p,\; \neg r \wedge q \vdash p \vee q \rightarrow r$

Option 5: None of the options above is correct.

QUESTION 18

Consider the following:

$A, B \vDash C$

where A, B and C are propositional logic sentences. Choose the correct option below.

Option 1: $A, B \vDash C$ means that A, B and C are semantically equivalent.

Option 2: $A, B \vDash C$ means that A, B and C are logically equivalent.

Option 3: $A, B \vDash C$ means that C will be true if both A and B are true.

Option 4: $A, B \vDash C$ means that C will only be true if both A and B are true.

Option 5: None of the options above is correct.

QUESTION 19

Suppose the HORN algorithm is used to determine whether the following propositional logic sentence is satisfiable or not:

$$(p \wedge q \wedge w \rightarrow \bot) \wedge (t \rightarrow \bot) \wedge (r \rightarrow p) \wedge (T \rightarrow r) \wedge (T \rightarrow q) \wedge (r \wedge u \rightarrow w) \wedge (u \rightarrow s)$$

After the first step has been executed, we have the following (underlining is used to indicate marking):

$$(p \wedge q \wedge w \rightarrow \bot) \wedge (t \rightarrow \bot) \wedge (r \rightarrow p) \wedge (\underline{T} \rightarrow r) \wedge (\underline{T} \rightarrow q) \wedge (r \wedge u \rightarrow w) \wedge (u \rightarrow s)$$

Which of the options below gives the situation after the next step has been completed?

Option 1: $(\underline{p} \wedge q \wedge w \rightarrow \bot) \wedge (t \rightarrow \bot) \wedge (r \rightarrow \underline{p}) \wedge (\underline{T} \rightarrow r) \wedge (\underline{T} \rightarrow q) \wedge (r \wedge u \rightarrow w) \wedge (u \rightarrow s)$

Option 2: $(p \wedge \underline{q} \wedge w \rightarrow \bot) \wedge (t \rightarrow \bot) \wedge (r \rightarrow p) \wedge (\underline{T} \rightarrow \underline{r}) \wedge (\underline{T} \rightarrow \underline{q}) \wedge (\underline{r} \wedge u \rightarrow w) \wedge (u \rightarrow s)$

Option 3: $(p \wedge \underline{q} \wedge w \rightarrow \bot) \wedge (t \rightarrow \bot) \wedge (\underline{r} \rightarrow \underline{p}) \wedge (\underline{T} \rightarrow \underline{r}) \wedge (\underline{T} \rightarrow \underline{q}) \wedge (\underline{r} \wedge u \rightarrow w) \wedge (u \rightarrow s)$

Option 4: $(p \wedge \underline{q} \wedge w \rightarrow \bot) \wedge (t \rightarrow \bot) \wedge (\underline{r} \rightarrow \underline{p}) \wedge (\underline{T} \rightarrow \underline{r}) \wedge (\underline{T} \rightarrow \underline{q}) \wedge (\underline{r} \wedge u \rightarrow \underline{w}) \wedge (u \rightarrow s)$
Option 5: None of the options above is correct.

QUESTION 20

Dealing with propositional logic, which of the options below is correct?

Option 1: A model is a specific type of valuation.
Option 2: A sequent is valid if all valuations make the premises and the conclusion true.
Option 3: A formula is semantically entailed by other formulas if at least one valuation makes all the formulas true.
Option 4: If we can show that $A, B \models C$, there exists a proof of $A, B \vdash C$.
Option 5: More than one of the options above are correct.

ASSIGNMENT 2 FIRST SEMESTER

SUBMISSION: Printouts or electronically through myUnisa (as one .pdf file)

It will be to your own advantage to check whether the assignment has been registered on the system after a few days.

If you want to submit the assignment electronically and myUnisa is off-line during that time, you need not contact us, because we will be aware of it. Simply submit it as soon as myUnisa is available again.

DUE DATE	**28 March 2018**
EXTENSION	No extension of the due date
TUTORIAL MATTER	Textbook: All previous material and chapter 2
WEIGHT OF CONTRIBUTION TO SEMESTER MARK	**40%**
unique number	**873176**

Predicate symbols	
J(x)	x is a serious jogger
R(x)	x is a race
T(x, y)	x takes part in race y
E(x, y)	x is equal to y
W(x, y)	x wins race y
Constants	
a	Aggie
b	Ben
c	Civvy race

Table 2

QUESTION 1 [18]

Use the predicate and constant symbols and their intended meanings given in table 2 to translate the English sentences given below into predicate logic:

Question 1.1

One can only be a serious jogger if one takes part in some race.

Question 1.2

Aggie will take part in the Civvy race but she will only win if Ben also participates.

Question 1.3

There is a race where Aggie, Ben and at least one other jogger participates.

Question 1.4

Neither Aggie nor Ben wins the Civvy race.

Question 1.5

The winners of all races take part in the Civvy race.

Question 1.6

There is a race which the winner of the Civvy race does not win.

QUESTION 2 **[10]**

Use the predicate and constant symbols and their intended meanings given in table 2 and translate the following sentences of predicate logic into English or Afrikaans:

Question 2.1

$\forall x\ \forall y\ ((R(y) \land W(x, y)) \rightarrow \neg E(a, x))$

Question 2.2

$\forall x\ (R(x) \rightarrow (T(a, x) \land T(b, x))$

Question 2.3

$\forall x\ ((R(x) \land T(a, x)) \rightarrow \neg W(b, x))$

Question 2.4

$\exists x\ (R(x) \land \forall y\ (J(y) \rightarrow \neg W(y, x)))$

Question 2.5

$\exists x\ \exists y\ (W(a, x) \land W(b, y) \land \neg E(x, y))$

QUESTION 3 **[7]**

Let

- P and Q be two predicate symbols, each with two arguments,

- f a function symbol with one argument and
- c a constant.

For each of the following, state whether it is a term or a well-formed formula (wff) or neither. If it is not a term or a wff, state the reason.

3.1 $\exists c\ P(c, x)$
3.2 $\exists y\ \forall x\ (P(x, y) \rightarrow Q(x, c))$
3.3 $\forall x\ \forall y\ Q(x, P(y, x))$
3.4 $f(f(x))$
3.5 $f(P(c, x))$
3.6 $Q(c)$
3.7 $P(f(x), c) \wedge Q(c, x)$

QUESTION 4 **[10]**

Let φ be the formula

$$\exists x\ [Q(x, y, z) \rightarrow \exists y\ (P(y, z) \vee P(z, x))]$$

where P is a predicate symbol with two arguments and Q is a predicate symbol with three arguments.

Question 4.1 **(4)**

Draw the parse tree of the formula and indicate the free and bound variables.

Question 4.2 **(6)**

Suppose f is a function symbol with one argument. For each of the following substitutions, state whether it will create a problem. If there is no problem, write down the substituted formula. If there will be a problem, state how you would solve it and then write down the substituted formula.

4.2.1 $\varphi[f(y) / z]$
4.2.2 $\varphi[f(z) / y]$
4.2.3 $\varphi[f(x) / y]$

QUESTION 5 **[4]**

Show that the following set of formulas is consistent. (Do it by constructing a model where both formulas are true.)

$$\forall x\ \forall y\ \forall z\ (S(x, y) \wedge S(y, z) \rightarrow S(x, z))$$
$$\forall x\ \neg S(x, x)$$

QUESTION 6 **[6]**

Given the sentence

$$\forall x\ \forall y\ (R(x, y) \vee R(y, x) \rightarrow \neg R(x, x))$$

where R is a predicate with two arguments, construct two models: one model where the sentence is true and another model where the sentence is false.

QUESTION 7 **[4]**

Given the sentence

$$\forall x\ \forall y\ [(R(x, y) \wedge \neg R(y, x)) \vee R(y, y)],$$

does the model *M* below satisfy it? Explain your answer.

A = {a, b, c, d}
R^M = {(a, a), (a, b), (a, c), (a, d), (b, a)}

QUESTION 8 **[6]**

Show that the validity of the following sequents cannot be proved by finding for each of them a model where all formulas to the left of ├ evaluate to T but the formula to the right of ├ evaluates to F.

Question 8.1

$\forall x\,(R(x) \lor Q(x)) \vdash \forall x\,R(x) \lor \forall x\,Q(x)$

Question 8.2

$\exists x\,(\neg R(x) \land Q(x)) \vdash \forall x\,(R(x) \to Q(x))$

QUESTION 9 **[35]**

Using the rules of natural deduction, prove the validity of the following sequents in predicate logic. In all cases, number your steps, indicate which rule you are using and indicate subproofs clearly.

Question 9.1 **(6)**

$\forall x\,(P(x) \land Q(x)) \vdash \forall x\,P(x) \land \forall x\,Q(x)$

Question 9.2 **(5)**

$\neg\exists x\,P(x) \vdash \forall x \neg P(x)$

Question 9.3 **(6)**

$\exists x\,(\neg P(x) \land \neg Q(x)) \vdash \exists x\,(\neg [P(x) \land Q(x)])$

Question 9.4 **(10)**

$\forall x\,\forall y\,\forall z\,(S(x, y) \land S(y, z) \to S(x, z)),\ \forall x\,\neg S(x, x) \vdash \forall x\,\forall y\,(S(x, y) \to \neg S(y, x))$

Question 9.5 **(8)**

$\exists x\,(\neg P(x) \lor Q(x)) \vdash \exists x\,[\neg (P(x) \land \neg Q(x))]$

ASSIGNMENT 3 FIRST SEMESTER

DUE DATE	**25 April 2018**
EXTENSION	Not applicable
TUTORIAL MATTER	No extension of the due date
WEIGHT OF CONTRIBUTION TO SEMESTER MARK	**30%**
UNIQUE NUMBER	**809076**
QUESTIONS	20 questions. 5 options each. Choose one option in every question

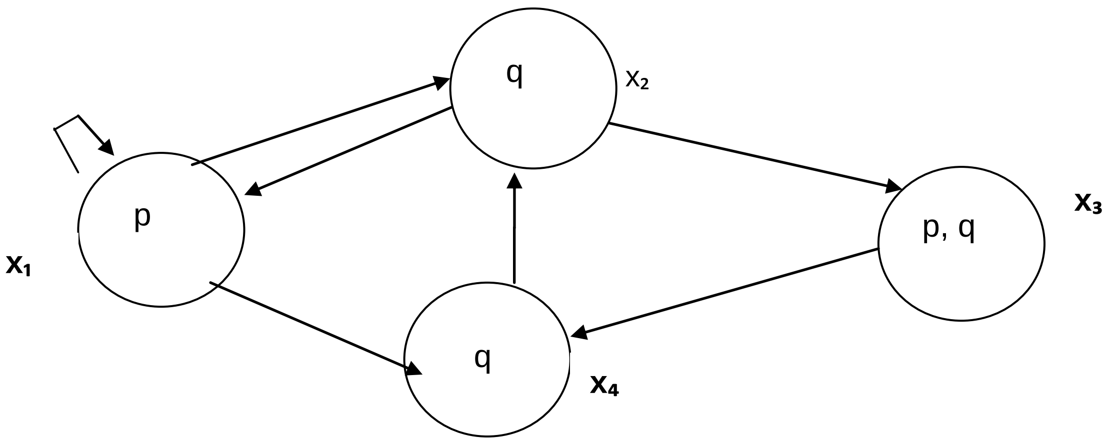


Figure 1: Kripke model used in Questions 1, 2, 3, 4 and 5

QUESTION 1

In which world of the Kripke model in Figure 1 is the formula $\Diamond p \wedge \Box q$ true?

Option 1: world x_1

Option 2: world x_2

Option 3: world x_3,

Option 4: Option 1 and Option 3 is true.

Option 5: The formula is not true in any world of the Kripke model.

QUESTION 2

Which of the following does not hold in the Kripke model in Figure 1?

Option 1: $x_1 \Vdash \Diamond \Diamond p$

.*Option 2*: $x_2 \Vdash \Box p$

.*Option 3*: $x_3 \Vdash \Box p \wedge \Box q$

.*Option 4*: $x_4 \Vdash \Box\Box p$

Option 5: None of the above options are true

QUESTION 3

Which of the following holds in the Kripke model given in Figure 1?

Option 1: $x_1 \Vdash \Box p$

.*Option 2*: $x_2 \Vdash \Diamond (p \vee q)$

.*Option 3*: $x_3 \Vdash \Diamond p \wedge \Box \neg q$

Option 4: $x_4 \Vdash \Box (p \wedge q)$

.*Option 5*: None of the options above holds in the given Kripke model in Figure 1.

QUESTION 4

Which of the following formulas is true in the Kripke model given in Figure 1?

Option 1: $\Diamond$ p

Option 2: $\Box$ q

.*Option 3*: $\Box \Diamond$ q

.*Option 4*: $\Box$ p

Option 5: None of the options above is true in the Kripke model in Figure 1.

QUESTION 5

Which of the following formulas is false in the Kripke model given in Figure 1?

Option 1: $p \vee q$

Option 2: $\Box \Diamond p$

Option 3: $\Box (p \vee q)$

Option 4: $p \vee \Diamond q$

Option 5: None of the above options are false in the Kripke model in Figure 1.

QUESTION 6

If we interpret $\Box \phi$ as "It ought to be that ϕ", which of the following formulas correctly expresses the English sentence

It ought to be that if I will get a gold medal then I it is permitted that I will get a gold medal.

where p stands for the declarative sentence "I will get a gold medal"?

Option 1: $\Box (p \rightarrow \neg \Box \neg p)$

Option 2: $\Box(p \rightarrow \neg \Diamond p)$

Option 3: $\Box p \rightarrow \Diamond \neg p$

Option 4: $\Box p \rightarrow \Box p$

Option 5: It is impossible to translate this sentence into a formula of modal logic with the required interpretation.

QUESTION 7

If we interpret $\Box \phi$ as "It is necessarily true that ϕ", why should the formula scheme $\Box \phi \rightarrow \Box \Box \phi$ hold in this modality?

Option 1: Because for all formulas ϕ, it is necessarily true that if ϕ then ϕ.

Option 2: Because for all formulas ϕ, if ϕ is necessarily true, then it is necessary that it is necessarily true.

Option 3: Because for all formulas ϕ, if ϕ is not possibly true, then it is true.

Option 4: Because for all formulas ϕ, ϕ is necessarily true if it is true.

Option 5: $\Box\,\phi \rightarrow \Box\,\Box\,\phi$ should not hold in this modality

QUESTION 8

If we interpret $\Box\,\phi$ as "After any execution of program P, ϕ holds", why should the formula scheme $\Box\,\phi \rightarrow \Diamond\,\phi$ not hold in this modality?

Option 1: Because it is not the case that if ϕ holds after every execution P, then ϕ does not hold after some execution of P.

Option 2: Because for a program P that never executes correctly, there is no execution of P after which ϕ holds.

Option 3: Because even if there is some execution of P after which ϕ does not hold, it doesn't mean that ϕ does not hold after any execution of P.

Option 4: Because there may be a program P such that even though ϕ holds after every execution P, ϕ does not hold after some execution.

Option 5: $\Box\,\phi \rightarrow \Diamond\,\phi$ should hold in this modality, because if ϕ holds after every execution of P, it should hold after at least one execution of P.

QUESTION 9

If we interpret $\Box\,\phi$ as "Always in the future (where the future does not include the present) it will be true that ϕ", which of the following formulas should be valid?

Option 1: $\Box\,p \rightarrow p$

Option 2: $\Box\,p \rightarrow \Box\,\Box\,p$

Option 3: $\Box\,p \rightarrow \Diamond\,p$

Option 4: $\Box\,p \vee \Box\,\neg\,p$

Option 5: All of these formulas should hold in this modality.

QUESTION 10

If we interpret $\Box \phi$ as "Agent A believes ϕ", what is the English translation of the formula $\Box p \rightarrow \Box \Diamond q$?

Option 1: If agent A believes p then agent B believes not q.

Option 2: If agent A believes p then he believes that agent B does not believe q.

Option 3: If agent A believes p then he believes that he does not believe q.

Option 4: Agent A believes p and he believes that agent B believes not q.

Option 5: Agent A believes p but he doesn't believe q.

QUESTION 11

If we interpret $\Box \phi$ as "Agent A believes ϕ", what formula will be correctly translated to English as

If agent A believes p then he believes not q

Option 1: $\Box p \rightarrow \Box \neg q$

Option 2: $\Box (p \rightarrow \neg q)$

Option 3: $\Box p \rightarrow \Diamond q$

Option 4: $\Box p \rightarrow \Diamond \neg q$

Option 5: $\Diamond (\neg p \vee \neg q)$

QUESTION 12

If we interpret K_i as agent I knows ϕ, the formula scheme $\neg\phi \rightarrow K_1 \neg K_1 \phi$ *means*

Option 1: If ϕ is true then agent 1 knows that he does not know ϕ

Option 2: If ϕ is false then agent 1 knows that he does not know ϕ

Option 3: If ϕ is true then agent 1 knows that he knows ϕ

Option 4: If ϕ is false then agent 1 knows that he knows ϕ

Option 5: None of the above is correct

The following natural deduction proof (without reasons) is referred to in Questions 13, 14 and 15:

1	$\neg \Box \neg (p \to q)$	premise
2	$\Box p$	
3	$\Box \neg q$	
4	$p \to q$	assumption
5	p	$\Box$e 2
6	q	$\to$e 4, 5
7	$\neg q$	$\Box$e 3
8	$\bot$	$\neg$e 6, 7
9	$\neg (p \to q)$	$\neg$i 4 - 8
10	$\Box \neg (p \to q)$	$\Box$i 4 - 9
11	$\bot$	$\neg$e 10, 1
12	$\neg \Box \neg q$	$\neg$i 3 - 11
13	$\Box p \to \neg \Box \neg$	$\to$i 2 – 12

QUESTION 13

How many times are □ elimination and introduction rules used in the above proof?

Option 1: None

Option 2: □ elimination and □ introduction once are both used only once.

Option 3: □ elimination is used only once but □ introduction twice.

Option 4: □ elimination is used twice but □ introduction only once.

Option 5: □ elimination and □ introduction are both used twice.

QUESTION 14

What is the correct reason for steps 1, 2 and 3 of the above proof?

Option 1: 1 premise
2 assumption
3 assumption

Option 2: 1 premise
2 ¬e 1
3 ¬i 2

Option 3: 1 assumption
2 ¬e 1
3 □e 4

Option 4: 1 assumption
2 □i 2
3 assumption

Option 5: 1 premise
2 □i 1
3 ¬i 2

QUESTION 15

What sequent is proved by the above proof?

Option 1: $\Diamond \top \vdash \Box p \rightarrow \Diamond p$

Option 2: $\vdash \Box p \rightarrow \Diamond p$

Option 3: $\neg \Box \bot \vdash \Box p \rightarrow \neg \Box \neg q$

Option 4: $\vdash \neg \Box \neg (p \rightarrow q) \rightarrow (\Box p \rightarrow \neg \Box \neg q)$

Option 5: It's impossible to say without the reasons.

The following incomplete natural deduction proof is referred to in Questions 16 and 17:

1	$\Box p$	assumption
2	$\Box \neg p$	assumption
3	$\neg \Box \neg p$	
4	$\bot$	
5	$\neg \Box \neg \Box p$	$\neg$ 2-4
6	$\Box p \rightarrow \neg \Box \neg \Box p$	$\rightarrow$i 1-5

QUESTION 16

What formulas and their reasons are missing in steps 3 and 4 of the above proof?

Option 1: 3 $\neg \Box p$ axiom T 2
4 $\bot$ $\neg$e 1, 3

Option 2: 3 $\neg \Box p$ axiom T 2
4 $\bot$ $\neg$e 1, 2

Option 3: 3 $\neg \Box p$ axiom T 2
4 $\bot$ $\neg$e 2, 3

Option 4: 3 $\neg \Box p$ axiom T 1
4 $\bot$ $\neg$e 1, 3

Option 5: None of the above.

QUESTION 17

What rule is used in line 6?

Option 1: $\rightarrow$ i

Option 2: $\neg$e

Option 3: $\rightarrow$ e

Option 4: $\neg$ i

Option 5: KT45

QUESTION 18

What proof strategy would you use to prove the following sequent:

$\Box (p \wedge q) \vdash_{KT4} \Box \Box p \wedge \Box \Box q$

Option 1: Open a solid box and start with $\Box (p \wedge q)$ as an assumption

Use axiom T to remove the $\Box$ to get $p \wedge q$.

Use $\wedge$ elimination twice to obtain the separate atomic formulas.

Use axiom 4 twice, i.e. once on each atomic formula, to add a $\Box$ to each.

Use axiom 4 twice, i.e. once on $\Box$ p and once on $\Box$ q, to get $\Box \Box$ p and $\Box \Box$ q.

Combine $\Box \Box$ p and $\Box \Box$ q using $\wedge$ introduction.

Close the solid box to get the result.

Option 2: Start with $\Box (p \wedge q)$ as a premise.

Use axiom T to remove the $\Box$ to get $p \wedge q$.

Open a dashed box and use $\wedge$ elimination twice to obtain the separate atomic formulas.

Use axiom 4 twice, i.e. once on each atomic formula, to add a $\Box$ to each.

Close the dashed box and use $\Box$ introduction twice, i.e. once on $\Box$ p and once on $\Box$ q,

to get $\Box \Box$ p and $\Box \Box$ q.

Combine $\Box \Box$ p and $\Box \Box$ q using $\wedge$ introduction.

Option 3: Start with $\Box (p \wedge q)$ as a premise.

Open a dashed box and use $\Box$ elimination to get $p \wedge q$.

- Use $\wedge$ elimination twice to obtain the separate atomic formulas.
- Close the dashed box and use $\Box$ introduction twice, i.e. once on each atomic formula.
- Use axiom 4 twice, once on $\Box$ p and once on $\Box$ q, to get $\Box \Box$ p and $\Box \Box$ q.
- Combine $\Box \Box$ p and $\Box \Box$ q using $\wedge$ introduction.

Option 4:

- Open a solid box and start with $\Box$ (p $\wedge$ q) as an assumption.
- Open a dashed box and use $\Box$ elimination to get p $\wedge$ q.
- Use $\wedge$ elimination twice to obtain the separate atomic formulas.
- Use axiom 4 twice, i.e. once on each atomic formula, to add a $\Box$ to each.
- Close the dashed box and use $\Box$ introduction twice, i.e. once on $\Box$ p and once on $\Box$ q, to get $\Box \Box$ p and $\Box \Box$ q.
- Close the solid box to get the result.

Option 5: This is not a valid sequent in KT4.

QUESTION 19

If we interpret $K_i \phi$ as "Agent i knows ϕ", what is the English translation of the formula $\neg K_1 K_2 p \rightarrow q$?

Option 1: Agent 1 knows that agent 2 doesn't know that p implies q.

Option 2: Agent 1 doesn't know that agent 2 knows that p implies q.

Option 3: If agent 1 knows that agent 2 doesn't know p, then q.

Option 4: If agent 1 doesn't know that agent 2 knows p, then q.

Option 5: Agent 1 knows that if agent 2 doesn't know p, then q.

QUESTION 20

If we interpret $K_i \phi$ as "Agent i knows ϕ", what formula of modal logic is correctly translated to English as

If agent 1 does not know not p then agent 2 doesn't know q.

Option 1: $K_1 p \rightarrow K_2 \neg q$

Option 2: $\neg (K_1 p \wedge K_2 q)$

Option 3: $K_1 (p \rightarrow \neg K_2 q)$

Option 4: $K_1 \neg K_2 (p \rightarrow q)$

Option 5: $\neg K_1 \neg p \rightarrow \neg K_2 q$

Section 8.2

ASSIGNMENT 1 SECOND SEMESTER

(MULTIPLE CHOICE)

SUBMISSION: On multiple choice form or electronically through myUnisa

Please note that Assignment 1 has to be submitted in order to gain examination admission. It will be to your own advantage to check whether the assignment has been registered on the system after a few days.

If you want to submit the assignment electronically and myUnisa is off-line during that time, you need not contact us, because we will be aware of it. Simply submit it as soon as myUnisa is available again.

DUE DATE	**8 August 2018**
EXTENSION	**No extension will be granted for Assignment 1**.
TUTORIAL MATTER	Textbook chapter 1
WEIGHT OF CONTRIBUTION TO SEMESTER MARK	30%
UNIQUE NUMBER	**745876**
QUESTIONS	20 questions. 5 options each. Choose one option in every question.

We recommend

- that you write out a formal proof for (at least) questions 7 to 11 before choosing an option and
- if you have access to a computer, that you use the Fitch software (supplied with the textbook of the second-level Formal Logic module COS2661) to help you to choose the correct option in questions dealing with formal proofs.

Propositional logic symbol	Declarative sentence associated with the symbol
p	Susan saves
q	Susan buys a house
r	The house has seven bathrooms
s	The house is used as a guesthouse
t	Peter likes the house

Table 3

Question 1

Using the symbols and their intended meaning given in table 3, which of the options below is a correct propositional logic translation of the following English sentence?

The house is used as a guesthouse only if it has seven bathrooms.

Option 1: $r \rightarrow s$
Option 2: $s \rightarrow r$
Option 3: $s \leftrightarrow r$
Option 4: $\neg s \vee \neg r$
Option 5: None of the above options is a correct translation.

Question 2

Using the symbols and their intended meaning given in table 3, which of the options below is a correct propositional logic translation of the following English sentence?

Unless Susan saves, she does not buy a house with seven bathrooms.

Option 1: $\neg q \rightarrow (\neg p \wedge r)$
Option 2: $(\neg q \wedge r) \rightarrow \neg p$
Option 3: $\neg p \leftrightarrow (\neg q \wedge r)$
Option 4: $\neg p \rightarrow (\neg q \wedge r)$
Option 5: None of the above options is a correct translation.

Question 3

Using the symbols and their intended meaning given in table 3, which of the options below is a correct propositional logic translation of the following English sentence?

The house is used as guesthouse if and only if Susan does not buy it but Peter likes it.

Option 1: $(s \wedge t) \leftrightarrow \neg q$
Option 2: $s \leftrightarrow (\neg q \wedge t)$
Option 3: $s \leftrightarrow (\neg q \vee t)$
Option 4: $(s \rightarrow \neg q) \wedge (t \rightarrow s)$
Option 5: None of the above options is a correct translation.

Question 4

Using the symbols and their intended meaning given in table 3, which of the options below is a correct English translation of the following propositional logic sentence?

$q \rightarrow \neg r$

Option 1: Susan buys a house with seven bathrooms.
Option 2: Susan buys a house if the house has seven bathrooms.
Option 3: Susan buys a house only if the house does not have seven bathrooms.
Option 4: If the house does not have seven bathrooms, Susan buys it.
Option 5: None of the above options is a correct translation.

Question 5

Using the symbols and their intended meaning given in table 3, which of the options below is a correct English translation of the following propositional logic sentence?

$t \rightarrow (p \wedge q \wedge s)$

Option 1: Unless Pete does not like the house, Susan saves and buys a house used as guesthouse.
Option 2: If Pete likes the house, Susan saves and buys it provided that it is used as a guesthouse.
Option 3: If Pete does not like the house, Susan does not save and buy a house used as guesthouse
Option 4: If Susan saves and buys a house used as guesthouse, Pete likes the house.
Option 5: None of the above options is a correct translation.

Question 6

Using the symbols and their intended meaning given in table 3, which of the options below is a correct English translation of the following propositional logic sentence?

$t \leftrightarrow (r \wedge \neg s)$

Option 1: Any house liked by Pete has seven bathrooms and is not used as guesthouse.
Option 2: Pete likes the house if and only it has seven bathrooms or is not used as guesthouse.
Option 3: If the house has seven bathrooms and is not used as guesthouse, Pete likes it.
Option 4: Pete likes the house if it has seven bathrooms and is not used as guesthouse.
Option 5: None of the above options is a correct translation.

Question 7

Suppose you have to formally prove the validity of the following sequent using the basic natural deduction rules:

$$(p \wedge q) \vee (p \wedge r) \vdash p \wedge (q \vee r)$$

Write out a proof on some rough paper and then choose the option below that gives a correct strategy.

Option 1:

- Start with the premise $(p \wedge q) \vee (p \wedge r)$.
- Have a subproof starting with the assumption of p.

- Have two subsubproofs: the first starting with the assumption of q and ending on $q \vee r$, using the $\vee$ introduction rule, and the second starting with the assumption of r and ending on $q \vee r$, using the $\vee$ introduction rule.
- The subproof should end on $p \wedge (q \vee r)$ using the $\wedge$ introduction rule.
- The $\vee$ elimination rule should be used in the last step to derive $p \wedge (q \vee r)$.

Option 2:

- Start with the premise $(p \wedge q) \vee (p \wedge r)$.
- Have two subproofs, one starting with the assumption of p and the other with the assumption of $q \vee r$.
- Both subproofs should end on $p \wedge (q \vee r)$, using the $\wedge$ introduction rule in two cases and the $\vee$ introduction rule in one case.
- The $\vee$ elimination rule should be used in the last step to derive $p \wedge (q \vee r)$.

Option 3:

- Start with the premise $(p \wedge q) \vee (p \wedge r)$.
- Have two subproofs, one starting with the assumption of $p \wedge q$ and the other with the assumption of $p \wedge r$.
- Both subproofs should end on $p \wedge (q \vee r)$.
- The $\wedge$ elimination rule, the $\vee$ introduction rule and the $\wedge$ introduction rule should be used in both subproofs.
- The $\vee$ elimination rule should be used in the last step to derive $p \wedge (q \vee r)$.

Option 4:

- Start with the premise $(p \wedge q) \vee (p \wedge r)$.
- Have two subproofs, one starting with the assumption of $p \wedge q$ and the other with the assumption of $p \wedge r$.
- Both subproofs should end on $p \wedge (q \vee r)$.
- The $\wedge$ elimination rule, the $\vee$ elimination rule and the $\wedge$ introduction rule should be used in both subproofs.
- The $\vee$ elimination rule should again be used in the last step to derive $p \wedge (q \vee r)$.

Option 5: None of the above options describes a correct strategy.

Question 8

Suppose you have to formally prove the validity of the following sequent using the basic natural deduction rules:

$$(p \wedge q) \rightarrow r \vdash p \rightarrow (q \rightarrow r)$$

Which of the options below is a correct proof?

Option 1:

1	$(p \wedge q) \rightarrow r$	
2	p	assumption
3	$p \wedge q$	$\wedge$i 2
4	r	$\rightarrow$e 1, 3
5	q	$\wedge$e 3
6	$q \rightarrow r$	$\rightarrow$i 5 - 6
7	$p \rightarrow (q \rightarrow r)$	$\rightarrow$i 2 - 6

Option 2:

1	$(p \wedge q) \rightarrow r$	
2	p	$\wedge e_1$ 1
3	$p \wedge q$	$\wedge$i 2
4	r	$\rightarrow$e 1, 3
5	$p \rightarrow (q \rightarrow r)$	$\rightarrow$i 2 – 4

Option 3:

1	$(p \wedge q) \rightarrow r$	
2	p	assumption
3	q	assumption
4	$q \rightarrow r$	$\wedge$e 1
5	$p \rightarrow (q \rightarrow r)$	$\rightarrow$ i 2 – 4

Option 4:

1	$(p \wedge q) \rightarrow r$	
2	p	assumption
3	q	assumption
4	$p \wedge q$	$\wedge i_2$ 2, 3
5	r	$\rightarrow$ e 1, 4
6	$q \rightarrow r$	$\rightarrow$ i 3 – 5
7	$p \rightarrow (q \rightarrow r)$	$\rightarrow$ i 2 – 6

Option 5: None of the options above is a correct proof.

Question 9

We have to prove the validity of the following sequent using the rules of natural deduction:

$p \rightarrow q,\ r \rightarrow s \vdash (p \vee r) \rightarrow (q \vee s)$

The following proof is given but two lines are omitted. Which of the options below gives the correct propositional logic sentence and the associated correct rule in both lines?

1	$p \rightarrow q$	premise
2	$r \rightarrow s$	premise
3	$p \vee r$	assumption
4	p	assumption
5	q	$\rightarrow$e 1, 4
6		
7	r	assumption
8	s	$\rightarrow$e 2, 7
9	$q \vee s$	$\vee i_2$ 8
10		
11	$(p \vee r) \rightarrow (q \vee s)$	$\rightarrow$i 3 – 10

Option 1:

6	$q \vee s$	$\vee i_1$ 5
10	$q \vee s$	$\vee$e 3, 4 – 6, 7 - 9

Option 2:

6	$q \vee s$	$\vee i_1$ 5
10	$(p \vee r) \rightarrow (q \vee s)$	$\rightarrow$i 3 – 9

Option 3:

6	$(p \vee r) \rightarrow (q \vee s)$	$\rightarrow$i 3 – 5
10	$q \vee s$	$\vee$e 3, 4 – 6, 7 - 9

Option 4:

6	$(p \vee r) \rightarrow q$	$\rightarrow$i 3 – 5
10	$q \vee s$	$\vee$e 3, 4 – 6, 7 - 9

Option 5: None of the options above gives the correct propositional logic sentence and associated rule for both lines.

Question 10

We have to prove the following sequent using the basic natural deduction rules:

$(p \vee (q \rightarrow p)) \wedge q \vdash p$

In the proof below the rules that are used in three of the lines are omitted. Which option below gives the correct rule in each line?

1	$(p \vee (q \rightarrow p)) \wedge q$	premise
2	$p \vee (q \rightarrow p)$	$\wedge e_1$ 1
3	q	

4	p	assumption
5	p	copy 4
6	$q \to p$	
7	p	$\to$e 3, 6
8	p	

Option 1:

2 assumption
6 assumption
8 $\vee$e 2, 4 – 5, 6 - 7

Option 2:

3 $\wedge e_2$ 1
6 $\to$i 3 - 5
8 $\vee$e 2, 4 – 5, 6 - 7

Option 3:

3 $\wedge e_2$ 1
6 assumption
8 $\vee$e 2, 4 – 5, 6 - 7

Option 4:

3 $\wedge e_2$ 1
6 assumption
8 $\vee$e 2, 4 - 7

Option 5: None of the options above gives correct rules for all three lines.

Question 11

We have to prove the following sequent using the basic natural deduction rules:

$$\vdash (\neg p \vee q) \to (p \to q)$$

In the proof below the sentences in three of the lines are omitted. Which option below gives the correct sentence in each line?

1	$\neg p \vee q$	assumption
2		assumption
3	$\neg p$	assumption
4	$\bot$	$\neg$e 2, 3
5	q	$\bot$e 4
6		assumption
7	q	copy 6
8	q	$\vee$e 1, 3 – 5, 6 - 7
9		$\rightarrow$i 2 - 8
10	$(\neg p \vee q) \rightarrow (p \rightarrow q)$	$\rightarrow$i 1 – 9

Option 1:

2 $p \rightarrow q$

6 q

9 $(\neg p \vee q) \rightarrow (p \rightarrow q)$

Option 2:

2 p

6 q

9 $p \rightarrow q$

Option 3:

2 p

6 p

9 $p \rightarrow q$

Option 4:

2 p

6 q

9 $(\neg p \vee q) \rightarrow q$

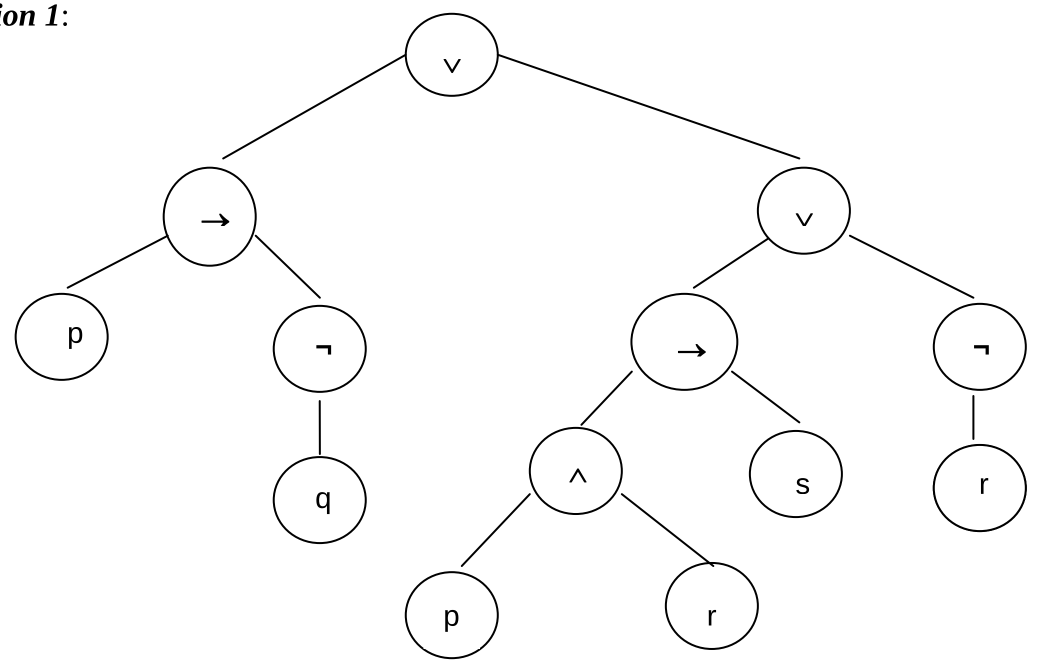
Option 5: None of the options above gives the correct sentence in every line.

Question 12

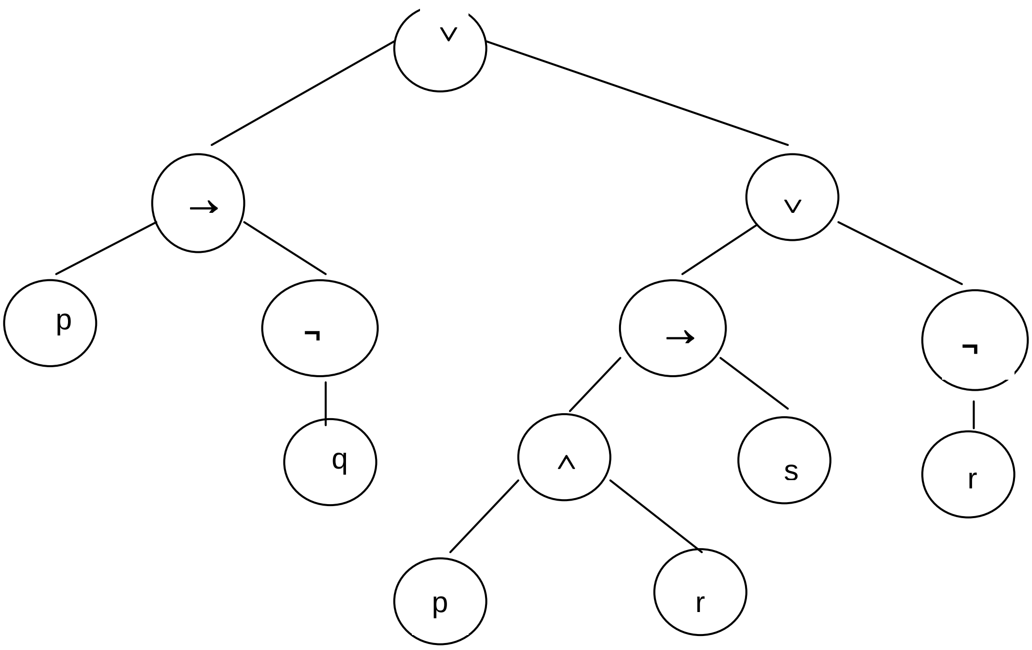
Given the following propositional logic sentence, which of the options below is the corresponding parse tree?

$((p \to \neg q) \vee (p \wedge r) \to s) \vee \neg r$

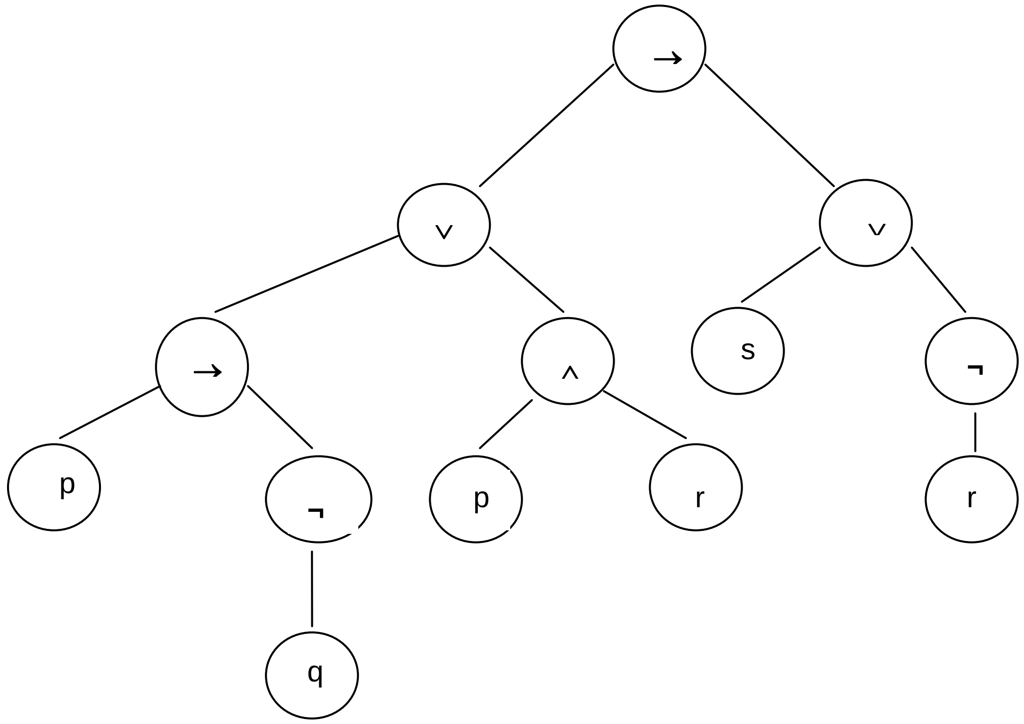
Option 1:



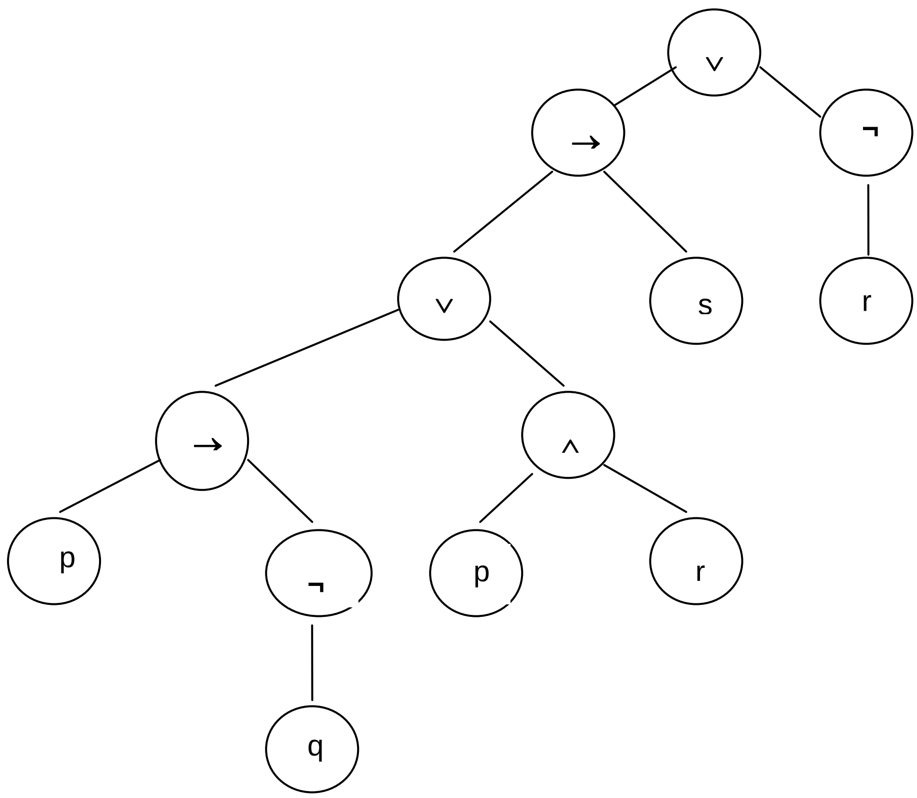
Option 2:



Option 3:



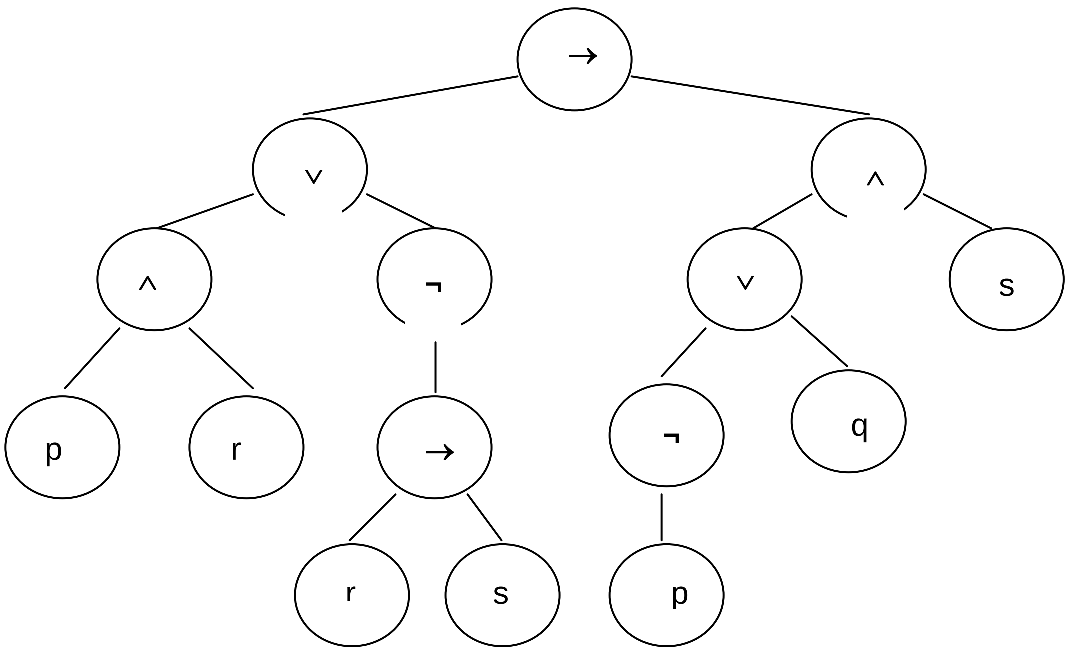
Option 4:



Option 5: None of the options above gives the correct parse tree for the given sentence.

Question 13

Given the following parse tree, which option below gives the associated propositional logic sentence?



Option 1: $(p \wedge r) \vee \neg ((r \rightarrow s) \rightarrow (\neg p \vee q)) \wedge s$

Option 2: $(p \wedge r) \vee (\neg (r \rightarrow s) \rightarrow (\neg p \vee q)) \wedge s$

Option 3: $((p \wedge r) \vee \neg (r \rightarrow s)) \rightarrow (\neg p \vee q) \wedge s$

Option 4: $(p \wedge r) \vee \neg (r \rightarrow s) \rightarrow \neg p \vee (q \wedge s)$

Option 5: None of the options above gives the correct associated sentence.

QUESTION 14

Which of the following sequents are valid:

Option 1: $p \rightarrow q, \neg p \vdash \neg q$

Option2: $(p \rightarrow q \wedge r, q \vdash r$

Option 3: $(p \rightarrow q \wedge r, \neg q \vdash p$

Option 4 : $\neg q, p \leftrightarrow q \vdash \neg p$

Option 5 : Option 2 and option 4 are valid sequents.

Question 15

Consider the following sequent and then choose the correct option below.

$p \rightarrow (q \vee r) \vdash (p \rightarrow q) \wedge (p \rightarrow r)$

Option 1: The sequent is valid and can be formally proved using natural deduction rules.

Option 2: The sequent is valid as shown by the following valuation: p = F, q = F, r = T

Option 3: The sequent is not valid as shown by the following valuation: p = F, q = F, r = F

Option 4: The sequent is not valid as shown by the following valuation: p = T, q = F, r = T

Option 5: None of the options above is correct

Question 16

Consider the following sequent and then choose the correct option below.

$$p \rightarrow (\neg q \vee r), \neg r \vdash \neg q \rightarrow \neg p$$

Option 1: The sequent is valid and can be formally proved using natural deduction rules.

Option 2: The sequent is valid as shown by the following valuation:
p = F, q = F, r = F

Option 3: The sequent is not valid as shown by the following valuation:
p = T, q = F, r = F

Option 4: The sequent is not valid as shown by the following valuation:
p = F, q = F, r = F

Option 5: None of the options above is correct.

Question 17

Draw the truth tables of the following three propositional logic sentences and then choose the correct option below.

$\neg r \rightarrow (p \vee q)$
$r \wedge \neg q$
$r \rightarrow q$

Option 1: The sentence $r \rightarrow q$ is not semantically entailed by $\neg r \rightarrow (p \vee q)$ and $r \wedge \neg q$, because the final columns of the three truth tables are not identical.

Option 2: The sentence $r \rightarrow q$ is not semantically entailed by $\neg r \rightarrow (p \vee q)$ and $r \wedge \neg q$.
It is clear from the line where p = T, q = F, r = T and the line where p = F, q = F, r = T.

Option 3: The sentences $\neg r \rightarrow (p \vee q)$ and $r \wedge \neg q$ are semantically equivalent, because their final columns agree for three evaluations.

Option 4: $\neg r \rightarrow (p \vee q),\ r \wedge \neg q \vdash r \rightarrow q$

Option 5: None of the options above is correct.

Question 18

Consider the following:

$$A, B \vDash C$$

where A, B and C are propositional logic sentences. Choose the correct option below.

Option 1: $A, B \vDash C$ means that A, B and C are semantically equivalent.

Option 2: $A, B \vDash C$ means that A, B and C are logically equivalent.

Option 3: A, B $\models$ C means that C will be true if both A and B are true.
Option 4: A, B $\models$ C means that C will only be true if both A and B are true.
Option 5: None of the options above is correct.

Question 19

Suppose the HORN algorithm is used to determine whether the following propositional logic sentence is satisfiable or not:

$$(p \wedge q \wedge w \rightarrow \bot) \wedge (t \rightarrow \bot) \wedge (r \rightarrow p) \wedge (T \rightarrow r) \wedge (T \rightarrow q) \wedge (u \rightarrow s) \wedge (T \rightarrow u)$$

After the first step has been executed, we have the following (underlining is used to indicate marking):

$$(p \wedge q \wedge w \rightarrow \bot) \wedge (t \rightarrow \bot) \wedge (r \rightarrow p) \wedge (\underline{T} \rightarrow r) \wedge (\underline{T} \rightarrow q) \wedge (u \rightarrow s) \wedge (\underline{T} \rightarrow u)$$

Which of the options below gives the situation after the next step has been completed?

Option 1: $(\underline{p} \wedge q \wedge w \rightarrow \bot) \wedge (t \rightarrow \bot) \wedge (r \rightarrow \underline{p}) \wedge (\underline{T} \rightarrow r) \wedge (\underline{T} \rightarrow q) \wedge (u \rightarrow s) \wedge (\underline{T} \rightarrow u)$
Option 2: $(p \wedge q \wedge w \rightarrow \bot) \wedge (t \rightarrow \bot) \wedge (r \rightarrow p) \wedge (\underline{T} \rightarrow \underline{r}) \wedge (\underline{T} \rightarrow \underline{q}) \wedge (u \rightarrow s) \wedge (\underline{T} \rightarrow \underline{u})$
Option 3: $(p \wedge \underline{q} \wedge w \rightarrow \bot) \wedge (t \rightarrow \bot) \wedge (\underline{r} \rightarrow p) \wedge (\underline{T} \rightarrow \underline{r}) \wedge (\underline{T} \rightarrow \underline{q}) \wedge (\underline{u} \rightarrow s) \wedge (\underline{T} \rightarrow \underline{u})$
Option 4: $(p \wedge \underline{q} \wedge w \rightarrow \underline{\bot}) \wedge (t \rightarrow \bot) \wedge (\underline{r} \rightarrow \underline{p}) \wedge (\underline{T} \rightarrow \underline{r}) \wedge (\underline{T} \rightarrow \underline{q}) \wedge (\underline{u} \rightarrow \underline{s}) \wedge (\underline{T} \rightarrow \underline{u})$
Option 5: None of the options above is correct.

Question 20

Dealing with propositional logic, which of the options below is correct?

Option 1: A model is a specific type of valuation.
Option 2: A sequent is valid if all valuations make the premises and the conclusion true.
Option 3: A formula is semantically entailed by other formulas if at least one valuation makes all the formulas true.
Option 4: If we can show that A, B $\models$ C, there exists a proof of A, B $\vdash$ C.
Option 5: More than one of the options above are correct.

ASSIGNMENT 2 SECOND SEMESTER

SUBMISSION: Printouts or electronically through myUnisa (as one .pdf file)

DUE DATE	**12 September 2018**
EXTENSION	No extension of the due date
Tutorial matter	Textbook chapter 2
WEIGHT OF CONTRIBUTION TO SEMESTER MARK	40%
UNIQUE NUMBER	**855673**

Predicate symbols	
A(x)	x is an artist
P(x)	x is a painting
M(x, y)	x painted y
L(x, y)	x likes painting y
R(x)	x is rich
Constants	
b	Busi
h	Hebo
v	Vincent

Table 4

QUESTION 1 [18]

Use the predicate and constant symbols and their intended meanings given in table 4 to translate the English sentences given below into predicate logic:

Question 1.1

No artist is rich.

Question 1.2

Hebo likes all paintings not painted by Busi.

Question 1.3

Busi likes a specific painting painted by either Vincent or Hebo.

Question 1.4

Vincent only likes paintings made by himself.

Question 1.5

Busi likes a painting if and only if Vincent also likes it.

Question 1.6

Either Busi or Hebo is an artist but not both.

QUESTION 2 [10]

Use the predicate and constant symbols and their intended meanings given in table 4 and translate the following sentences of predicate logic into English or Afrikaans:

Question 2.1

$\forall x\,(P(x) \rightarrow \exists y\,(A(y) \wedge M(y, x)))$

Question 2.2

$\exists x\,(A(x) \wedge R(x) \wedge \forall y\,(P(y) \rightarrow \neg M(x, y)))$

Question 2.3

$\forall x\,((P(x) \wedge M(v, x)) \rightarrow L(v, x))$

Question 2.4

$\forall x\,(P(x) \rightarrow \exists y\,L(y, x))$

Question 2.5

$\exists x\,\forall y\,((P(y) \wedge \neg M(x, y)) \rightarrow L(x, y))$

QUESTION 3 [7]

Let

- P and Q be two predicate symbols, each with two arguments,
- f a function symbol with one argument and
- c a constant.

For each of the following, state whether it is a term or a well-formed formula (wff) or neither. If it is not a term or a wff, state the reason.

3.1 $Q(c)$
3.2 $\forall x\,(P(x, x) \wedge \exists c\,P(c, x))$
3.3 $f(c) \rightarrow \forall x\,\forall y\,P(x, y)$
3.4 $f(Q(c, x))$
3.5 $\exists x\,\exists y\,P(x, Q(x, y))$
3.6 $f(f(x))$
3.7 $P(f(x), c)$

QUESTION 4 [10]

Let φ be the formula

$$\forall x\,[(\exists y\,Q(x, y, z) \vee \exists z\,P(y, z)) \rightarrow P(y, x)]$$

where P is a predicate symbol with two arguments and Q is a predicate symbol with three arguments.

Question 4.1 **(4)**

Draw the parse tree of the formula and indicate the free and bound variables.

QUESTION 4.2 **(6)**

Suppose f is a function symbol with one argument. For each of the following substitutions, state whether it will create a problem. If there is no problem, write down the substituted formula. If there will be a problem, state how you would solve it and then write down the substituted formula.

4.2.1 $\varphi[f(z) / z]$
4.2.2 $\varphi[f(z) / y]$
4.2.3 $\varphi[f(x) / y]$

QUESTION 5 **[4]**

Show that the following set of formulas is consistent. (Do it by constructing a model where both formulas are true.)

$\forall x\, (S(x) \rightarrow Q(x))$
$\exists x\, (Q(x) \land \neg S(x))$

QUESTION 6 **[6]**

Given the sentence
$\forall x\, \forall y\, \forall z\, (R(x, y) \land R(y, z) \rightarrow R(x, z))$
where R is a predicate with two arguments, construct two models: one model where the sentence is true and another model where the sentence is false.

QUESTION 7 **[4]**

Given the sentence
$\forall x\, \exists y\, (R(x, y) \land R(y, y))$,
does the model *M* below satisfy it? Explain your answer.

$A = \{a, b, c, d\}$
$R^M = \{(a, a), (b, a), (c, a), (d, b), (b, b)\}$

QUESTION 8 **[6]**

Show that the validity of the following sequents cannot be proved by finding for each of them a model where all formulas to the left of ├ evaluate to T but the formula to the right of ├ evaluates to F.

Question 8.1

$\forall x\, \exists y\, S(x, y) \vdash \exists y\, \forall x\, S(x, y)$

Question 8.2

$\exists x\ (\neg R(x) \vee \neg Q(x)) \vdash \forall x\ (R(x) \vee Q(x))$

QUESTION 9 **[35]**

Using the rules of natural deduction, prove the validity of the following sequents in predicate logic. In all cases, number your steps, indicate which rule you are using and indicate subproofs clearly.

Question 9.1 **(5)**

$\forall x\ (P(x) \rightarrow \neg Q(x)) \vdash \neg\ [\exists x\ (P(x) \wedge Q(x))]$

Question 9.2 **(4)**

$\forall x\ (P(x) \wedge Q(x)) \vdash \forall x\ (P(x) \rightarrow Q(x))$

Question 9.3 **(6)**

$\forall x\ \forall y\ (Q(y) \rightarrow F(x)) \vdash \exists y\ Q(y) \rightarrow \forall x\ F(x)$

Question 9.4 **(10)**

$\forall x\ (P(x) \vee Q(x)),\ \exists x\ \neg Q(x),\ \forall x\ (R(x) \rightarrow \neg P(x)) \vdash \exists x\ \neg R(x)$

Question 9.5 **(10)**

$\forall x\ (P(x) \rightarrow (Q(x) \vee R(x))),\ \neg \exists x\ (P(x) \wedge R(x)) \vdash \forall x\ (P(x) \rightarrow Q(x))$

ASSIGNMENT 3 SECOND SEMESTER

Submission: The assignment should NOT be submitted.

You should assess it yourself when you have completed it. The model solution is given in the appendix at the end of this tutorial letter.
Note that there will be examination questions set on this part of the study material also.

DUE DATE	**05 October 2018**
EXTENSION	Not applicable
TUTORIAL MATTER	Chapter 5
WEIGHT OF CONTRIBUTION TO SEMESTER MARK	30%
UNIQUE NUMBER	**753314**

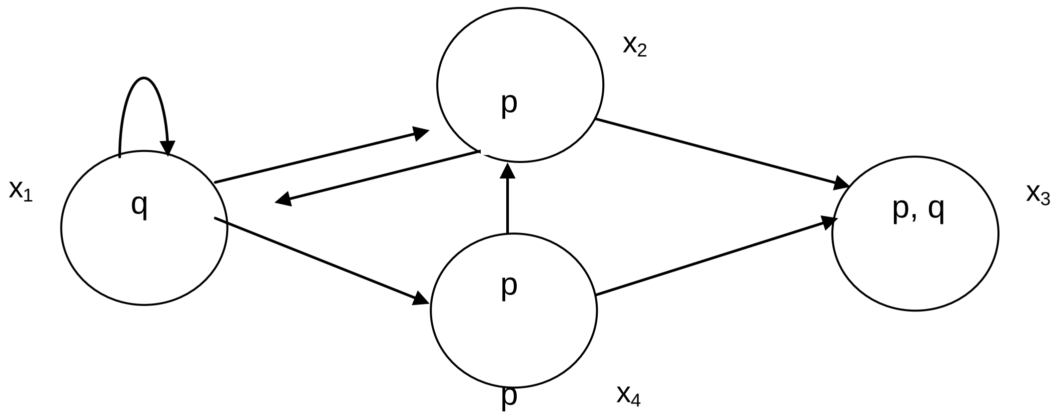


Figure 1: Kripke model used in Questions 1, 2, 3, 4 and 5

QUESTION 1

In which world of the Kripke model in Figure 1 is the formula $\Box p \wedge \Diamond \Box q$ true?

Option 1: x_1
Option 2: x_2
Option 3: x_3
Option 4: x_4
Option 5: The formula is not true in any world of the Kripke model.

QUESTION 2

Which of the following holds in the Kripke model given in Figure 1?

Option 1: $x_1 \Vdash \Box p$

Option 2: $x_2 \Vdash \Box (p \wedge q)$

Option 3: $x_3 \Vdash \Box p \wedge \Diamond q$

Option 4: $x_3 \Vdash \Box \Box \neg p$

Option 5: $x_4 \Vdash p \rightarrow q$

QUESTION 3

Which of the following does not hold in the Kripke model given in Figure 1?

Option 1: $x_1 \Vdash \Diamond q$

Option 2: $x_1 \Vdash \Box (p \vee q)$

Option 3: $x_2 \Vdash \Diamond p \wedge \Box q$

Option 4: $x_3 \Vdash \Box (p \wedge \neg q)$

Option 5: $x_4 \Vdash \Box (p \rightarrow q)$

QUESTION 4

Which of the following formulas is true in the Kripke model given in Figure 1?

Option 1: $\Diamond p$

Option 2: $\Box q$

Option 3: $\Box \Diamond q$

Option 4: $\Box (p \vee q)$

Option 5: $q \rightarrow p$

QUESTION 5

Which of the following formulas is false in the Kripke model given in Figure 1?

Option 1: $p \vee q$

Option 2: $\Box \Diamond p$

Option 3: $\Box (p \vee q)$

Option 4: $p \vee \Diamond q$

Option 5: $\Box p \vee \Diamond q$

QUESTION 6

If we interpret $\Box \phi$ as "It ought to be that ϕ", which of the following formulas correctly expresses the English sentence

It ought to be that if I am happy, I'm allowed to be unhappy.

where p stands for the declarative sentence "I am happy"?

Option 1: $\Box p \to \Diamond \neg p$

Option 2: $\Diamond \neg p \vee \neg \Box p$

Option 3: $\Box (p \to \neg \Box p)$

Option 4: $\Box (p \to \neg \Diamond p)$

Option 5: It is impossible to translate this sentence into a formula of modal logic with the required interpretation.

QUESTION 7

If we interpret $\Box \phi$ as "It is necessarily true that ϕ", why should the formula scheme $\Box \phi \to \phi$ hold in this modality?

Option 1: Because for all formulas ϕ, it is necessarily true that if ϕ then ϕ.

Option 2: Because for all formulas ϕ, if ϕ is necessarily true, then it is true.

Option 3: Because for all formulas ϕ, if ϕ is not possibly true, then it is true.

Option 4: Because for all formulas ϕ, ϕ is necessarily true if it is true.

Option 5: $\Box \phi \to \phi$ should not hold in this modality.

QUESTION 8

If we interpret $\Box \phi$ as "After any execution of program P, ϕ holds", why should the formula scheme $\Box \phi \to \phi$ not hold in this modality?

Option 1: Just because ϕ holds after every execution of P doesn't necessarily mean that ϕ holds before execution of P.

Option 2: Because it is not that case that after any execution of P, if ϕ holds then ϕ holds.

Option 3: Because if ϕ does not hold before execution of P, it doesn't necessarily mean that ϕ holds after any execution of P.

Option 4: Because if ϕ does not hold after every execution of P, it doesn't necessarily mean that ϕ holds before any execution of P.

Option 5: $\Box \phi \to \phi$ should hold in this modality, because if ϕ holds after any execution of P, then ϕ holds.

QUESTION 9

If we interpret $\Box \phi$ as "Always in the future (where the future does not include the present) it will be true that ϕ", which of the following formulas should be valid?

Option 1: $\Box p \to \Box \Box p$

Option 2: $\neg \Box p \vee \neg \Diamond \Diamond \neg p$

Option 3: $\neg \Diamond \neg p \to \Box p$

Option 4: $\neg \Box \Box p \to \Diamond \neg p$

Option 5: All of the above are valid

QUESTION 10

If we interpret $\square\,\phi$ as "Agent A believes ϕ", what is the English translation of the formula $\square\, p \rightarrow \neg \Diamond\, q$?

Option 1: If Agent A believes p, then Agent B does not believe q.
Option 2: Agent A believes that if p, then q is not consistent with Agent A's beliefs.
Option 3: Agent A believes that if p, then Agent B does not believe q.
Option 4: If Agent A believes p, then Agent A believes not q.
Option 5: Agent A believes p if Agent A believes not q.

QUESTION 11

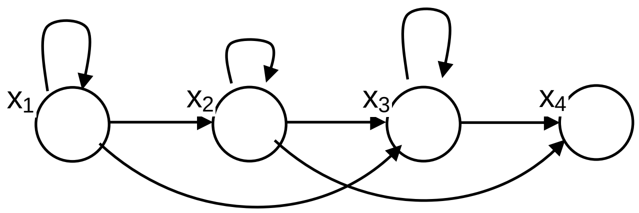
If we interpret $\square\,\phi$ as "Agent A believes ϕ", what formula will be correctly translated to English as

Agent A does not believe p or q.

Option 1: $\Diamond \neg (p \vee q)$
Option 2: $\square \neg (p \vee q)$
Option 3: $\neg \square\, p \vee q$
Option 4: $\Diamond \neg p \vee q$
Option 5: $\square \neg p \wedge \square \neg q$

QUESTION 12

Consider the following Kripke frame:



Which of the following modal logics does this frame conform to?

Option 1: KT
Option 2: KB
Option 3: KD
Option 4: K4
Option 5: KT45

The following natural deduction proof (without reasons) is referred to in Questions 13, 14 and 15:

1	$\Box (p \wedge q)$
2	$p \wedge q$
3	p
4	q
5	$\Box p$
6	$\Box q$
7	$\Box p \wedge \Box q$
8	$\Box (p \wedge q) \rightarrow (\Box p \wedge \Box q)$

QUESTION 13

How many times are □ elimination and introduction rules used in the above proof?

Option 1: None

Option 2: □ elimination and □ introduction are both only used once.

Option 3: □ elimination is used only once but □ introduction twice.

Option 4: □ elimination is used twice but □ introduction only once.

Option 5: □ elimination and □ introduction are both used twice.

QUESTION 14

What is the correct reason for steps 1, 2 and 3 of the above proof?

Option 1:	1	assumption
	2	axiom T in line
	3	$\wedge$e 2
Option 2:	1	premise
	2	□i 1
	3	□e 2
Option 3:	1	premise
	2	assumption
	3	$\wedge$i 2
Option 4:	1	□i 1
	2	axiom T in line 2
	3	$\wedge$i 3,4
Option 5	1	assumption
	2	□e 1
	3	$\wedge$e 2

QUESTION 15

What sequent is proved by the above proof?

Option 1: □ (p ∧ q) ⊢ □ (p ∧ q) → (□ p ∧ □ q)

Option 2: □ (p ∧ q) ⊢ □ p ∧ □ q

Option 3: ⊢ □ (p ∧ q) → (□ p ∧ □ q)

Option 4: ⊢ □ p ∧ □ q

Option 5: It's impossible to say without the reasons.

The following incomplete natural deduction proof is referred to in Questions 16 and 17:

1	□ ¬ □ ¬ □ p	assumption
2		
3	¬ □ p	assumption
4		
5	⊥	¬ e 2,4
6	□ p	PBC 3-5
7		
8	□ ¬ □ ¬ □ p → □ □ p	→i 1-7

QUESTION 16

Rules T, 4 and 5 are used in the missing lines of the above proof. Which rule is used in which line?

Option 1: Rule T is used in line 2, rule 4 is used in line 4 and rule 5 is used in line 7.

Option 2: Rule 4 is used in line 2, rule 5 is used in line 4 and rule T is used in line 7.

Option 3: Rule T is used in line 2, rule 5 is used in line 4 and rule 4 is used in line 7.

Option 4: Rule 5 is used in line 2, rule T is used in line 4 and rule 4 is used in line 7.

Option 5: Rule 4 is used in line 2, rule T is used in line 4 and rule 5 is used in line 7.

QUESTION 17

What formulas belong in the missing lines of the above proof?

Option 1: ¬ □ ¬ □ p in line 2, □ ¬ □ p in line 4 and □ □ p in line 7

Option 2: □ □ ¬ □ ¬ □ p in line 2, □ ¬ □ ¬ □ p in line 4 and □ □ p in line 7

Option 3: ¬ □ ¬ □ p in line 2, ¬ □ ¬ □ p in line 4 and p in line 7

Option 4: □ □ ¬ □ ¬ □ p in line 2, ¬ □ ¬ □ p in line 4 and p in line 7

Option 5: □ ¬ □ p in line 2, ¬ □ ¬ □ p in line 4 and □ ¬ □ ¬ □ p → □ □ p in line 7

QUESTION 18

What proof strategy would you use to prove the following sequent:

$\vdash_{KT4} \Box \Box (p \wedge q) \rightarrow (\Box p \wedge \Box q)$

Option 1:

- Open a solid box and start with $\Box \Box (p \wedge q)$ as an assumption.
- Use axiom T to remove one $\Box$.
- Open a dashed box and use $\Box$ elimination to get $p \wedge q$.
- Use $\wedge$ elimination twice to obtain the separate atomic formulas.
- Close the dashed box and use $\Box$ introduction twice, i.e. once on each of the atomic formulas.
- Combine $\Box$ p and $\Box$ q using $\wedge$ introduction.
- Close the solid box and use $\rightarrow$ introduction on the first and last formulas to get the result.

Option 2:

- Start with $\Box \Box (p \wedge q)$ as a premise.
- Use axiom T twice to remove both $\Box$ to get $p \wedge q$.
- Use $\wedge$ elimination once to obtain the separate atomic formulas.
- Use axiom 4 twice, i.e. once on each atomic formula to add a $\Box$.
- Combine $\Box$ p and $\Box$ q using $\wedge$ introduction.
- Use $\rightarrow$ introduction on the first and last formulas to get the result.

Option 3:

- Start with $\Box \Box (p \wedge q)$ as a premise.
- Open a dashed box and use $\Box$ elimination to get $\Box (p \wedge q)$.
- Open another dashed box and use $\Box$ elimination to get $p \wedge q$.
- Use $\wedge$ elimination twice to obtain the separate atomic formulas.
- Close the first dashed box and use $\Box$ introduction on the first atomic formula.
- Close the second dashed box and use $\Box$ introduction on the second atomic formula.
- Combine $\Box$ p and $\Box$ q using $\wedge$ introduction.
- Use $\rightarrow$ introduction on the first and last formulas to get the result.

Option 4:

- Open a solid box and start with $\Box \Box (p \wedge q)$ as an assumption.
- Open a dashed box and use $\Box$ elimination to get $\Box (p \wedge q)$.
- Use axiom T to remove one $\Box$ to get $p \wedge q$.
- Use $\wedge$ elimination twice to obtain the separate atomic formulas.
- Use axiom 4 twice, i.e. once on each atomic formula to add a $\Box$.
- Close the dashed box and combine $\Box$ p and $\Box$ q using $\wedge$ introduction.
- Close the solid box and use $\rightarrow$ introduction on the first and last formulas to get the result.

Option 5: This is not a valid sequent in KT4.

QUESTION 19

If we interpret $K_i \phi$ as "Agent i knows ϕ", what is the English translation of the formula $K_1 K_2 \neg p \rightarrow q$?

Option 1: Agent 1 knows that agent 2 doesn't know p implies q.

Option 2: Agent 1 knows that agent 2 knows that not p implies q.

Option 3: Agent 1 knows that if agent 2 doesn't know p, then q.

Option 4: Agent 1 knows that if agent 2 knows not p, then q.

Option 5: If agent 1 knows that agent 2 knows not p, then q.

QUESTION 20

If we interpret $K_i \phi$ as "Agent i knows ϕ", what formula of modal logic is correctly translated to English as

Agent 1 knows p but he doesn't know that agent 2 knows q.

Option 1: $K_1 (p \wedge \neg K_2 q)$

Option 2: $K_1 (p \wedge K_2 \neg q)$

Option 3: $K_1 p \wedge \neg K_1 K_2 q$

Option 4: $K_1 p \wedge K_1 \neg K_2 q$

Option 5: $K_1 p \wedge K_1 K_2 \neg q$

9 EXAMINATIONS

Use the brochure *my Studies @ Unisa* for general examination guidelines and examination preparation guidelines.
Make a note of your examination dates and arrange with your employer for leave in good time. The COS3761 examination will be in May or June if you are registered for the first semester and in October or November if you are registered for the second semester. Check for clashes on the examination timetable and should there be any between your modules, discuss them with the Student Administration department.

- To gain admission to the examination, you have to submit at least one assignment by its due date.

Although the examination paper is set in English only, you may answer in either Afrikaans or English.

10 CONCLUSION

We trust that you enjoy your studies in COS3761, and wish you every success!