Tutorial letter 203/2/2017

Formal Logic 3

COS3761

Semester 1

School of Computing

Solutions to assignment 3

BAR CODE

Question	Option
1	1
2	3
3	2
4	3
5	2
6	1
7	2
8	2
9	2
10	3
11	1
12	2
13	4
14	1
15	4
16	1
17	1
18	3
19	4
20	5

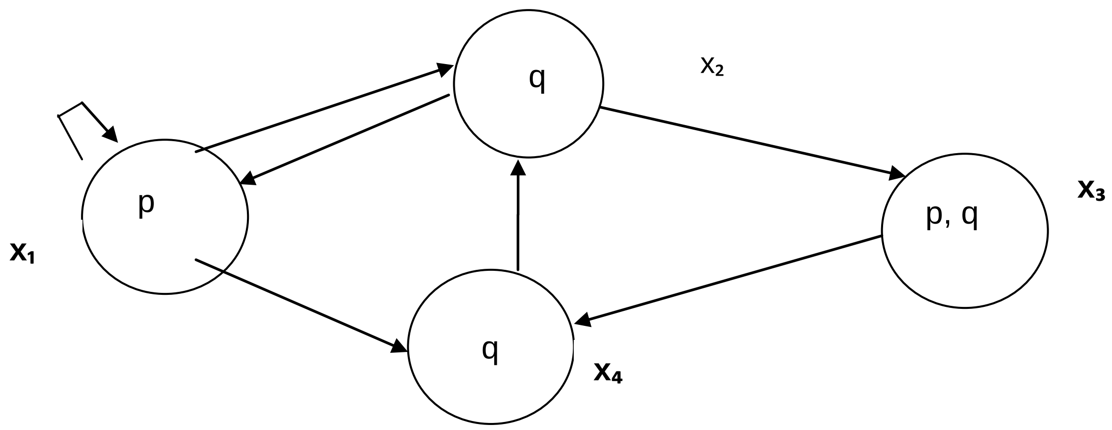

Figure 1: Kripke model used in Questions 1, 2, 3, 4 and 5

QUESTION 1

In which world of the Kripke model in Figure 1 is the formula $\Diamond p \wedge \Box q$ true?

Option 1: world x_1

For $\Diamond p \wedge \Box q$ to be true in world $\mathbf{x_1}$,$\Diamond p$ should be true in world x_1 and $\Box q$ should be true in world x_1.For $\Diamond p$ to be true in world x_1 , p must be true in at least one world accessible from x_1.The worlds accessible from x_1 is x_1**,** x_2 and $\mathbf{x_4}$. where p is true in x_1 . Also q is true in all the worlds accessible from x_1. Therefore the entire formula is true in world x_1...

Option 2: world x_2

For $\Diamond p \wedge \Box q$ to be true in world x_2, ,$\Diamond p$ should be true in world x_2 and $\Box q$ should be true in world x_2.For $\Diamond p$ to be true in world x_2, there should be at least one world accessible from x_2 where p is true .The world accessible from x_2 is x_1 and $\mathbf{x_3}$ where p is true. For $\Box q$ to be true in world x_2, q must be true in all All the worlds accessible from x_2 which are and $\mathbf{x_3}$ but q is false in x_1 .Therefore the formula is not true in **world** x_2

Option 3: world x_3,

For $\Diamond p \wedge \Box q$ to be true in world $\mathbf{x_3}$, $\Diamond p$ should be true in world $\mathbf{x_3}$ and $\Box q$ should be true in world $\mathbf{x_3}$ .For $\Diamond p$ to be true in world $\mathbf{x_3}$,p must be true in at least one world accessible from $\mathbf{x_3}$ The only world accessible from $\mathbf{x_3}$ is $\mathbf{x_4}$. But p is false in world $\mathbf{x_4}$. Therefore the formula is not true in $\mathbf{x_3}$.

Option 4: Option 1 and Option 3 is true.

Option 5: The formula is not true in any world of the Kripke model.

QUESTION 2

Which of the following does not hold in the Kripke model in Figure 1?

Option 1: $\mathbf{x_1} \Vdash \Diamond \Diamond p$

The relation holds. For ◊ ◊ p to be true in $\mathbf{x_1}$, there must be at least one world accessible from $\mathbf{x_1}$, where ◊ p is true. The world accessible from x_1 are x_2 and x_4. ◊ p is true in x_2 since x_3 is accessible from x_2 and p is true in x_3 Therefore the relation holds.

Option 2: $x_2 \Vdash \Box p$

The relation holds. For □ p to be true in world x_2, p must be true in all the worlds accessible from x_2 which are x_3 and $\mathbf{x_1}$. Therefore the relation holds.

Option 3: $x_3 \Vdash \Box p \wedge \Box q$

The relation does not hold. For □ p ∧ □ q to be true in x_3 , both of □ p and □ q must be true in world x_3 . In other words both p and q must be true in all worlds accessible from x_3 **.** The only world accessible from x_3 which is x_4 where p is false. Therefore relation does not hold.

Option 4: $x_4 \Vdash \Box\Box p$

For □□ p to be true in x_4, □ p must be true in all worlds accessible from x_4, namely x_2. For □ p to be true in x_2, namely x_1 and x_3. p is true in x_1 and x_3, so the relation holds.

Option 5: None of the above options are true

QUESTION 3

Which of the following holds in the Kripke model given in Figure 1?

Option 1: $x_1 \Vdash \Box p$

For □ p to be true in world x_1 ,p must be true in all the worlds accessible from x_1 which are world x_2, and world x_4 . But neither the worlds x_2 and x_4 have p in them. Therefore the relation does not hold.

Option 2: $x_2 \Vdash \Diamond(p \vee q)$

For ◊(p ∨ q) to be true in world x_2, p ∨ q must be true in atleast one world accessible from x_2.The worlds accessible from x_2 **are** x_1 **and** x_3 **.** p ∨ q is true in x_1 **.** Therefore the relation holds.

Option 3: $x_3 \Vdash \Diamond p \wedge \Box \neg q$

For ◊ p ∧ □ ¬ q to be true in world x_3, both ◊ p and □ ¬ q must be true in world x_3. But ◊ p is false in $x_3$because p is not true in the world accessible from x_3. namely . x_4.Therefore the relation does not hold.

Option 4: $x_4 \Vdash \Box (p \wedge q)$

For □ (p ∧ q) to be true in world x_4, p ∧ q must be true in all the worlds accessible from x_4 which is x_2.But p is not true in world x_2. .Therefore the relation does not hold.

Option 5: None of the options above holds in the given Kripke model in Figure 1.

QUESTION 4

Which of the following formulas is true in the Kripke model given in Figure 1?

Option 1: ◊ p

The formula is false in the worlds **x_3 because there is no world accessible from x_3 in which p is true.** Therefore the formula does not hold in the Kripke model in Figure1**.**

Option 2: □ q

The formula is false in the world x_2 **because** x_1 **is accessible from** x_2 **and q is not true there.** Therefore the formula does not hold in the Kripke model figure 1.

Option 3: □ ◊ q

The formula holds in the Kripke model Figure1 as it is true in all the worlds in the given Kripke model. Check this yourself.

Option 4: □ p

The formula does not hold as it is false in the worlds x_1**,** x_3 and x_4**.** Check this yourself.

Option 5: None of the options above is true in the Kripke model in Figure 1.

QUESTION 5

Which of the following formulas is false in the Kripke model given in Figure 1?

Option 1: p ∨ q

The formula is true in all the worlds in the Kripke model in Figure 1.

***Option 2*: □ ◊ p**

The formula is false in world x_3. because the worlds accessible from x_3.

Option 3: □ (p ∨ q)

The formula is true in all worlds in the Kripke model in Figure 1.

Option 4: p ∨ ◊ q

In world x_1, p is true and ◊ q is also true. This implies that world accessible from x_1, is world x_2 where q is also true. Therefore the formula is true in x_1.

In world x_2, p is false but the world accessible from x_2 which is x_3 where q is true. Therefore the formula is true in x_2.

In world x_3 ,p is true and ◊ q is also true in world c because the world accessible from x_3 is the world x_4 where q is true. Therefore the formula is true in x_3.

In world x_4, p is false but ◊ q is true because the world accessible from world x_4 is world x_2 where q is true. Therefore the formula is true in x_4..

Option 5: None of the above options are false in the Kripke model in Figure 1.

QUESTION 6

If we interpret □ ϕ as "It ought to be that ϕ ", which of the following formulas correctly expresses the English sentence

It ought to be that if I will get a gold medal then I it is permitted that I will get a gold medal.

where p stands for the declarative sentence "I will get a gold medal"?

Option 1: $\square$ (p $\rightarrow \neg \square \neg$ p)

Option 2: $\square$(p $\rightarrow \neg \Diamond$ p)

Option 3: $\square$ p $\rightarrow \Diamond \neg$ p

Option 4: $\square$ p $\rightarrow \square$ p

Option 5: It is impossible to translate this sentence into a formula of modal logic with the required interpretation.

QUESTION 7

If we interpret $\square\ \phi$ as "It is necessarily true that ϕ", why should the formula scheme $\square\ \phi \rightarrow \square\ \square\ \phi$ hold in this modality?

Option 1: Because for all formulas ϕ, it is necessarily true that if ϕ then ϕ.

Option 2: Because for all formulas ϕ, if ϕ is necessarily true, then it is necessary that it is necessarily true.

Option 3: Because for all formulas ϕ, if ϕ is not possibly true, then it is true.

Option 4: Because for all formulas ϕ, ϕ is necessarily true if it is true.

Option 5: $\square\ \phi \rightarrow \square\ \square\ \phi$ should not hold in this modality

QUESTION 8

If we interpret $\square\ \phi$ as "After any execution of program P, ϕ holds", why should the formula scheme $\square\ \phi \rightarrow \Diamond\ \phi$ not hold in this modality?

Option 1: Because it is not the case that if ϕ holds after every execution P, then ϕ does not hold after some execution of P.

Option 2: Because for a program P that never executes correctly, there is no execution of P after which ϕ holds.

Option 3: Because even if there is some execution of P after which ϕ does not hold, it doesn't mean that ϕ does not hold after any execution of P.

Option 4: Because there may be a program P such that even though ϕ holds after every execution P, ϕ does not hold after some execution.

Option 5: $\square\ \phi \rightarrow \Diamond\ \phi$ should hold in this modality, because if ϕ holds after every execution of P, it should hold after at least one execution of P.

QUESTION 9

If we interpret $\Box \phi$ as "Always in the future (where the future does not include the present) it will be true that ϕ", which of the following formulas should be valid?

Option 1: $\Box p \to p$

Option 2: $\Box p \to \Box \Box p$

Option 3: $\Box p \to \Diamond p$

Option 4: $\Box p \lor \Box \neg p$

Option 5: All of these formulas should hold in this modality.

QUESTION 10

If we interpret $\Box \phi$ as "Agent A believes ϕ", what is the English translation of the formula $\Box p \to \Box \Diamond q$?

Option 1: If agent A believes p then agent B believes not q.

Option 2: If agent A believes p then he believes that agent B does not believe q.

Option 3: If agent A believes p then he believes that he does not believe q.

Option 4: Agent A believes p and he believes that agent B believes not q.

Option 5: Agent A believes p but he doesn't believe q.

QUESTION 11

If we interpret $\Box \phi$ as "Agent A believes ϕ", what formula will be correctly translated to English as

If agent A believes p then he believes not q

Option 1: $\Box p \to \Box \neg q$

Option 2: $\Box (p \to \neg q)$

Option 3: $\Box p \to \Diamond q$

Option 4: $\Box p \to \Diamond \neg q$

Option 5: $\Diamond (\neg p \lor \neg q)$

QUESTION 12

If we interpret K_i as agent I knows ϕ, the formula scheme $\neg\phi \rightarrow K_1 \neg K_1 \phi$ *means*

Option 1: If ϕ is true then agent 1 knows that he does not know ϕ

Option 2: If ϕ is false then agent 1 knows that he does not know ϕ

Option 3: If ϕ is true then agent 1 knows that he knows ϕ

Option 4: If ϕ is false then agent 1 knows that he knows ϕ

Option 5: None of the above is correct

The following natural deduction proof (without reasons) is referred to in Questions 13, 14 and 15:

1	$\neg \Box \neg (p \rightarrow q)$	premise
2	$\Box p$	
3	$\Box \neg q$	
4	$p \rightarrow q$	assumption
5	p	$\Box$ e 2
6	q	$\rightarrow$ e 4, 5
7	$\neg q$	$\Box$ e 3
8	$\bot$	$\neg$ e 6, 7
9	$\neg (p \rightarrow q)$	$\neg$ i 4 - 8
10	$\Box \neg (p \rightarrow q)$	$\Box$ i 4 - 9
11	$\bot$	$\neg$ e 10, 1
12	$\neg \Box \neg q$	$\neg$ i 3 - 11
13	$\Box p \rightarrow \neg \Box \neg$	$\rightarrow$ i 2 – 12

QUESTION 13

How many times are $\Box$ elimination and introduction rules used in the above proof?

Option 1: None

Option 2: □ elimination and □ introduction once are both used only once.

Option 3: □ elimination is used only once but □ introduction twice.

Option 4: □ elimination is used twice but □ introduction only once.

Option 5: □ elimination and □ introduction are both used twice.

QUESTION 14

What is the correct reason for steps 1, 2 and 3 of the above proof?

Option 1:
1 premise
2 assumption
3 assumption

Option 2:
1 premise
2 ¬e 1
3 ¬i 2

Option 3:
1 assumption
2 ¬e 1
3 □e 4

Option 4:
1 assumption
2 □i 2
3 assumption

Option 5:
1 premise
2 □i 1
3 ¬i 2

QUESTION 15

What sequent is proved by the above proof?

Option 1: $\Diamond \top \vdash \Box p \rightarrow \Diamond p$

Option 2: $\vdash \Box p \rightarrow \Diamond p$

Option 3: $\neg \Box \bot \vdash \Box p \rightarrow \neg \Box \neg q$

Option 4: $\vdash \neg \Box \neg (p \rightarrow q) \rightarrow (\Box p \rightarrow \neg \Box \neg q)$

Option 5: It's impossible to say without the reasons.

The following incomplete natural deduction proof is referred to in Questions 16 and 17:

1	$\Box p$	assumption
2	$\Box \neg p$	assumption
3	$\neg \Box p$	
4	$\bot$	
5	$\neg \Box \neg \Box p$	$\neg$ 2-4
6	$\Box p \to \neg \Box \neg \Box p$	

QUESTION 16

What formulas and their reasons are missing in steps 3 and 4 of the above proof?

Option 1: 3 $\neg \Box p$ axiom T 2
4 $\bot$ $\neg$e 1, 3

Option 2: 3 $\neg \Box p$ axiom T 2
4 $\bot$ $\neg$e 1, 2

Option 3: 3 $\neg \Box p$ axiom T 2
4 $\bot$ $\neg$e 2, 3

Option 4: 3 $\neg \Box p$ axiom T 1
4 $\bot$ $\neg$e 1, 3

Option 5: None of the above.

QUESTION 17

What rule is used in line 6?

Option 1: $\to$i

Option 2: $\neg$e

Option 3: $\to$e

Option 4: $\neg$i

Option 5: KT45

QUESTION 18

What proof strategy would you use to prove the following sequent:

$\Box (p \wedge q) \vdash_{KT4} \Box \Box p \wedge \Box \Box q$

Option 1:

- Open a solid box and start with □ (p ∧ q) as an assumption.
- Use axiom T to remove the □ to get p ∧ q.
- Use ∧ elimination twice to obtain the separate atomic formulas.
- Use axiom 4 twice, i.e. once on each atomic formula, to add a □ to each.
- Use axiom 4 twice, i.e. once on □ p and once on □ q, to get □ □ p and □ □ q.
- Combine □ □ p and □ □ q using ∧ introduction.
- Close the solid box to get the result.

Option 2:

- Start with □ (p ∧ q) as a premise.
- Use axiom T to remove the □ to get p ∧ q.
- Open a dashed box and use ∧ elimination twice to obtain the separate atomic formulas.
- Use axiom 4 twice, i.e. once on each atomic formula, to add a □ to each.
- Close the dashed box and use □ introduction twice, i.e. once on □ p and once on □ q, to get □ □ p and □ □ q.
- Combine □ □ p and □ □ q using ∧ introduction.

Option 3:

- Start with □ (p ∧ q) as a premise.
- Open a dashed box and use □ elimination to get p ∧ q.
- Use ∧ elimination twice to obtain the separate atomic formulas.
- Close the dashed box and use □ introduction twice, i.e. once on each atomic formula.
- Use axiom 4 twice, once on □ p and once on □ q, to get □ □ p and □ □ q.
- Combine □ □ p and □ □ q using ∧ introduction.

Option 4:

- Open a solid box and start with □ (p ∧ q) as an assumption.
- Open a dashed box and use □ elimination to get p ∧ q.
- Use ∧ elimination twice to obtain the separate atomic formulas.
- Use axiom 4 twice, i.e. once on each atomic formula, to add a □ to each.
- Close the dashed box and use □ introduction twice, i.e. once on □ p and once on □ q, to get □ □ p and □ □ q.
- Close the solid box to get the result.

Option 5: This is not a valid sequent in KT4.

QUESTION 19

If we interpret $K_i \phi$ as "Agent i knows ϕ", what is the English translation of the formula $\neg K_1 K_2 p \rightarrow q$?

Option 1: Agent 1 knows that agent 2 doesn't know that p implies q.

Option 2: Agent 1 doesn't know that agent 2 knows that p implies q.

Option 3: If agent 1 knows that agent 2 doesn't know p, then q.

Option 4: If agent 1 doesn't know that agent 2 knows p, then q.

Option 5: Agent 1 knows that if agent 2 doesn't know p, then q.

QUESTION 20

If we interpret $K_i \phi$ as "Agent i knows ϕ", what formula of modal logic is correctly translated to English as

If agent 1 does not know not p then agent 2 doesn't know q.

Option 1: $K_1 p \rightarrow K_2 \neg q$

Option 2: $\neg (K_1 p \wedge K_2 q)$

Option 3: $K_1 (p \rightarrow \neg K_2 q)$

Option 4: $K_1 \neg K_2 (p \rightarrow q)$

Option 5: $\neg K_1 \neg p \rightarrow \neg K_2 q$