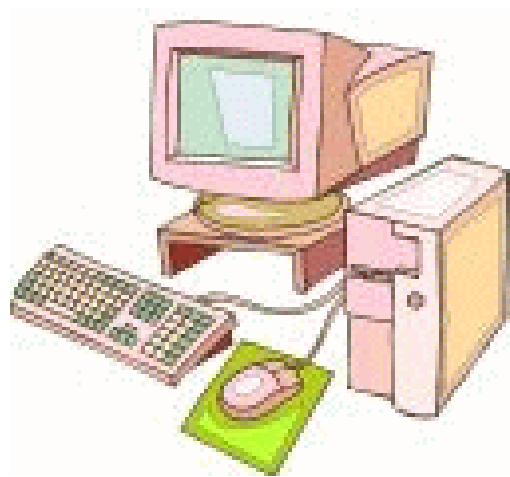


Formal logic 3

**Tutorial letter 201 (second semester)
for
COS3761**

Solutions to Assignment 1



School of Computing

TUTORIAL MATTER

SEMESTER MARK

The first two assignments contribute towards your semester mark for this module. Assignment 1 carries a weight of **30%** and the second assignment a weight of 70%. The semester mark contributes 10% to the final mark for this module. Suppose a student gets 60% for Assignment 1 and 50% for Assignment 2. In order to calculate the semester mark, the mark obtained for the specific assignment is multiplied by the weight. This then forms part of the 10% that the semester mark contributes to the final mark. Therefore:

Assignment	Marks obtained	Weight	Contribution to semester mark	
1	60%	30%	$60/100 \times 30/100 \times 10$	1.80
2	50%	70%	$50/100 \times 70/100 \times 10$	3.50
TOTAL				5.30

Hence, in this example the semester mark is 5.30. Note that the semester mark will not form part of the final mark if the supplementary examination is written.

SOLUTION TO ASSIGNMENT 1

Question	Correct option
1	2
2	4
3	2
4	3
5	1
6	5
7	3
8	4
9	1
10	3
11	2
12	4
13	3
14	5
15	4
16	3
17	2
18	5
19	3
20	1

- The prescribed book for Formal Logic 2 (module COS2661 or COS261C) is Barwise, J. & J. Etchemendy. 1999, 2000, 2002, 2003. *Language, proof and logic*. Stanford: Center for the Study of Language and Information. ISBN: 1-57586-374-X.
- The prescribed book for Formal Logic 3 (module COS3761 or COS361F) is *Logic in Computer Science: Modelling and Reasoning about Systems* by Michael Huth and Mark Ryan. 2004. ISBN: 0 521 54310 X paperback. Second edition.

QUESTION 1 option 2

You should make sure that you know how “only if” is translated into FOL.

QUESTION 2 option 4

Consult page 180 of the prescribed book for Formal Logic 2 for an explanation of the translation of “unless”.

QUESTION 3 option 2

“If and only if” is translated with \leftrightarrow .

QUESTION 4 option 3

If this is not quite clear, consult page 180 of the prescribed book for Formal Logic 2 for an explanation of the translation of “only if”.

QUESTION 5 option 1

You could think of the FOL sentence as starting with $\neg\neg t \rightarrow \dots$. Consult page 180 of the prescribed book for Formal Logic 2 for an explanation of the translation of “unless”.

QUESTION 6 option 5

A correct translation would have been *Pete likes the house if and only it has seven bathrooms and is not used as guesthouse.*

QUESTION 7 option 3

A correct proof is given below. Option 4 is not correct because the \vee elimination rule should not be cited inside the subproofs.

1	$(p \wedge q) \vee (p \wedge r)$	premise
2	$p \wedge q$	assumption
3	p	$\wedge e_1$ 2
4	q	$\wedge e_2$ 2
5	$q \vee r$	$\vee i_1$ 4
6	$p \wedge (q \vee r)$	$\wedge i$ 3, 5
7	$p \wedge r$	assumption
8	p	$\wedge e_1$ 7
9	r	$\wedge e_2$ 7
10	$q \vee r$	$\vee i_2$ 9
11	$p \wedge (q \vee r)$	$\wedge i$ 8, 10
12	$p \wedge (q \vee r)$	$\vee e$ 1, 2-6, 7-11

QUESTION 8 option 4

The third line of option 1 and option 2 is already incorrect because the \wedge introduction rule may only be used when both conjuncts have been derived earlier. That is why the rule also requires two (earlier) line numbers.

The proof given in option 3 correctly contains a sub-sub-proof – needed because the goal has two occurrences of the \rightarrow symbol – but the \wedge elimination rule is incorrectly applied in line 4.

QUESTION 9 option 1

A correct proof is given below.

1	$p \rightarrow q$	premise
2	$r \rightarrow s$	premise
3	$p \vee r$	assumption
4	p	assumption
5	q	$\rightarrow e$ 1, 4
6	$q \vee s$	$\vee i_1$ 5
7	r	assumption
8	s	$\rightarrow e$ 2, 7
9	$q \vee s$	$\vee i_2$ 8
10	$q \vee s$	$\vee e$ 3, 4-6, 7-9
11	$(p \vee r) \rightarrow (q \vee s)$	$\rightarrow i$ 3 – 10

QUESTION 10 option 3

1	$(p \vee (q \rightarrow p)) \wedge q$	premise
2	$p \vee (q \rightarrow p)$	$\wedge e_1$ 1
3	q	$\wedge e_2$ 1
4	p	assumption
5	p	copy 4
6	$q \rightarrow p$	assumption
7	p	$\rightarrow e$ 3, 6
8	p	$\vee e$ 2, 4-5, 6-7

If the answer is not clear, study the basic natural deduction rules again. They are summarised on page 27 of your textbook.

QUESTION 11 option 2

1	$\neg p \vee q$	assumption
2	p	assumption
3	$\neg p$	assumption
4	\perp	$\neg e$ 2, 3
5	q	$\perp e$ 4
6	q	assumption
7	q	copy 6
8	q	$\vee e$ 1, 3-5, 6-7
9	$p \rightarrow q$	$\rightarrow i$ 2-8
10	$(\neg p \vee q) \rightarrow (p \rightarrow q)$	$\rightarrow i$ 1-9

If this is not clear, study the basic natural deduction rules again. They are summarised on page 27 of your textbook (and from page 557 of the textbook of Formal Logic 2). Note that the " $\neg e$ rule" was called the " \perp Intro" rule in the textbook of Formal Logic 2.

QUESTION 12 option 4

Remember that \wedge and \vee bind more tightly than \rightarrow . Also note the brackets.

The main connective of the formula is \vee : $((p \rightarrow \neg q) \vee (p \wedge r) \rightarrow s) \vee \neg r$.

The main connective of the left hand side $((p \rightarrow \neg q) \vee (p \wedge r) \rightarrow s)$ is \rightarrow and the main connective of the left hand side of this formula is \vee : $(p \rightarrow \neg q) \vee (p \wedge r)$.

QUESTION 13 option 3

Make sure that you agree with this. Note again that \wedge and \vee bind more tightly than \rightarrow .

QUESTION 14 option 5

Option 1 is not a valid sequent because for $p=F, q=T$,

Option 2 is a valid sequent for $p=T, q=T, r=T$

Option 3 is not a valid sequent for $p=F, q=T$,

Option 4 is a valid sequent for $p=T, q=F, r=T$

QUESTION 15 option 4

Option 4 is the right answer because conclusion is false with valuation of p, q, r

QUESTION 16 option 3

The sequent would be valid only if the right hand side formula is *true for all valuations that make all the left hand side formulas true*. In this case the valuation indicated by option 3 makes both left hand side formulas true but the right hand side formula false, thus we have a counter-example. Option 4 is incorrect – the valuation given there is not a counter-example.

QUESTION 17 option 2

The combined truth table is

p	q	r	$\neg r$	$p \vee q$	$\neg r \rightarrow (p \vee q)$	$\neg q$	$r \wedge \neg q$	$r \rightarrow q$
T	T	T	F	T	T	F	F	T
T	T	F	T	T	T	F	F	T
T	F	T	F	T	T	T	T	F
T	F	F	T	T	T	T	F	T
F	T	T	F	T	T	F	F	T
F	T	F	T	T	T	F	F	T
F	F	T	F	F	T	T	T	F
F	F	F	T	F	F	T	F	T

See page 46 of your textbook for an explanation of semantic entailment for propositional logic.

Note that option 4 does not involve the entailment relation \models . However, if the sentence $r \rightarrow q$ were semantically entailed by $\neg r \rightarrow (p \vee q)$ and $r \wedge \neg q$, option 4 would have been true.

QUESTION 18 option 5

The sequent $A, B \models C$ means that C will be true if both A and B are true. See page 46 of your textbook for an explanation of the semantic entailment relation \models .

QUESTION 19 option 3

After the first step we have

$$(p \wedge q \wedge w \rightarrow \perp) \wedge (t \rightarrow \perp) \wedge (r \rightarrow p) \wedge (\underline{T} \rightarrow r) \wedge (\underline{T} \rightarrow q) \wedge (u \rightarrow s) \wedge (\underline{T} \rightarrow u)$$

Now we mark all occurrences of

r (because of $\underline{T} \rightarrow r$)

and q (because of $\underline{T} \rightarrow q$)

and u (because of $\underline{T} \rightarrow u$).

Thus

$$(p \wedge \underline{q} \wedge w \rightarrow \perp) \wedge (t \rightarrow \perp) \wedge (r \rightarrow p) \wedge (\underline{T} \rightarrow \underline{r}) \wedge (\underline{T} \rightarrow \underline{q}) \wedge (u \rightarrow s) \wedge (\underline{T} \rightarrow \underline{u})$$

QUESTION 20 option 1

Option 2: Incorrect: a sequent is valid if all valuations that make the premises true, also make the conclusion true.

Option 3: Incorrect: a formula is semantically entailed by other formulas if the formula is true whenever the other formulas are all true.

Option 4: Correct: propositional logic is complete (and sound). See Theorems 1.35 and 1.37 as well as tutorial letter 102.