Memorandum of May/June 2016

Question 1.1

(i) $s \rightarrow (q \wedge \neg t)$ or $\neg (s \wedge (\neg q \vee t))$ or any logically equivalent formula

(ii) It is windy and if the temperature is above 20° C, there is a dust storm.

Question 1.2

(i) $q \rightarrow r \vdash (p \rightarrow q) \rightarrow (p \rightarrow r)$

1	$q \rightarrow r$	premise	
2	$p \rightarrow q$	assumption	✓
3	p	assumption	✓
4	q	$\rightarrow$e 2, 3	✓
5	r	$\rightarrow$e 1, 4	✓
6	$p \rightarrow r$	$\rightarrow$i 3 – 5	✓
7	$(p \rightarrow q) \rightarrow (p \rightarrow r)$	$\rightarrow$i 2 – 6	✓

(ii) $q \rightarrow (p \rightarrow r), \neg r, q \vdash \neg p$

1	$q \rightarrow (p \rightarrow r)$	premise	
2	$\neg r$	premise	✓
3	q	premise	
4	p	assumption	✓
5	$p \rightarrow r$	$\rightarrow$e 3, 1	✓
6	r	$\rightarrow$e 4, 5	✓
7	$\bot$	$\neg$e 6, 2	✓
8	$\neg p$	$\neg$i 4 - 7	✓

Question 1.3

To show that a sequent is not valid, we must find a valuation in which the formulas on the left-hand side are true, but in which the formula on the right-hand side is false.

To make $\neg q \rightarrow \neg p$ false, q must be false and p must be true. To make $\neg r$ true, r must be false. We note that with this valuation, the formula $p \rightarrow (\neg q \vee r)$ is also true. ✓✓

p:	T
q:	F
r:	F

✓✓

Question 1.4

$(\underline{\top} \rightarrow p) \wedge (p \rightarrow r) \wedge (p \wedge \underline{\top} \rightarrow q) \wedge (r \wedge q \rightarrow \bot)$ ✓
$(\underline{\top} \rightarrow \underline{p}) \wedge (\underline{p} \rightarrow r) \wedge (\underline{p} \wedge \underline{\top} \rightarrow q) \wedge (r \wedge q \rightarrow \bot)$ ✓
$(\underline{\top} \rightarrow \underline{p}) \wedge (\underline{p} \rightarrow \underline{r}) \wedge (\underline{p} \wedge \underline{\top} \rightarrow \underline{q}) \wedge (\underline{r} \wedge \underline{q} \rightarrow \bot)$ ✓✓
$(\underline{\top} \rightarrow \underline{p}) \wedge (\underline{p} \rightarrow \underline{r}) \wedge (\underline{p} \wedge \underline{\top} \rightarrow \underline{q}) \wedge (\underline{r} \wedge \underline{q} \rightarrow \underline{\bot})$ ✓

Question 2.1

(i) For all clubs it is the case that there exists a club against which they do not win when they play against it. In other words, no club wins all their matches.

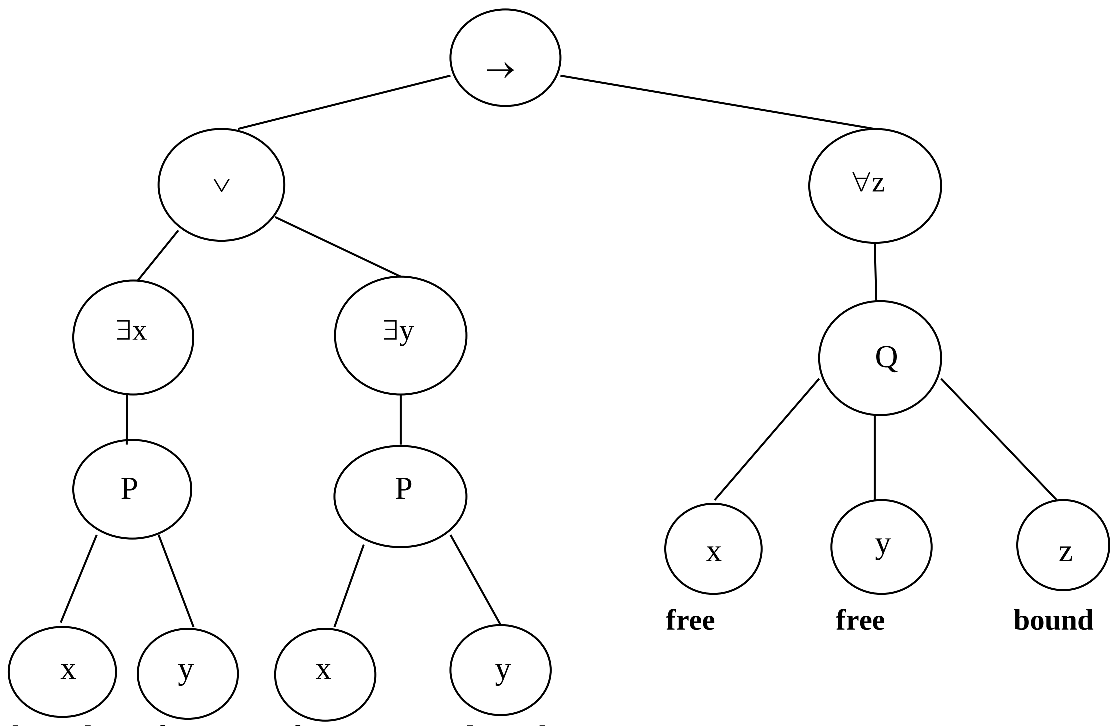
(ii) $\exists x\, (C(x) \wedge P(b(a), x)) \rightarrow \forall x\, (R(x) \rightarrow P(x, l))$

Question 2.2

(i) No A quantifier should not be followed by constant.
(ii) Yes Wff
(iii) No two binary operators may not follow one another.
(iv) No Predicates P and A must have the correct number of arguments.
(v) Yes Wff

Question 2.3

(i)



(ii) **bound** **free** **free** **bound**

(iii) No. The free occurrence of y falls within the scope of $\exists x$.

Question 2.4

1		$\forall x\,(A(x) \vee B(x))$	premise	
2		$\forall x\,(A(x) \rightarrow C(x))$	premise	
3		$\exists x \neg C(x)$	premise	
4	x_0	$\neg C(x_0)$	assumption	$[x_0/x]$
5		$A(x_0) \vee B(x_0)$	$\forall x$ e 1	
6		$A(x_0) \rightarrow C(x_0)$	$\forall x$ e 2	
7		$A(x_0)$	assumption	
8		$C(x_0)$	$\rightarrow$ e 6, 7	
9		$\bot$	$\neg$ e 4, 8	
10		$B(x_0)$	$\bot$ e 9	
11		$B(x_0)$	assumption	
12		$B(x_0)$	copy 11	
13		$B(x_0)$	$\vee$ e 5, 7-10, 11-12	
14		$\exists x\, B(x)$	$\exists x$ i 13	
15		$\exists x\, B(x)$	$\exists x$ e 3, 4-14	

Question 2.5

(i) Consider the following mathematical model M:

Let the universe A of concrete values be the set of integers.

R^M: Interpret R(x, y) as "x > y".

The formula is true in this model, because if one integer is greater than another neither is greater than itself.

Consider the following non-mathematical model N:

As required, let the universe A of concrete values be {a, b, c}.

R^N = {(a, b), (a, c), (b, c), (b, a), (c, a), (c, b)} or any subset thereof. (R^N must just not contain any ordered pairs of identical values.)

The formula is true in this model, because without any ordered pairs of identical values, the consequent of the implication will always be true.

(ii) Consider the following mathematical model M:

Let the universe A of concrete values be the set of integers.

R^M: Interpret R(x, y) as "x = y".

The formula is false in this model because for every pair of integers x and y that make the antecedent true (i.e. x = y), will make the consequent will be false.

Consider the following non-mathematical model N:

As required, let the universe *A* of concrete values be {a, b, c}.

R^N: = {(a, a), (b, b), (c, c)} or any set that has a non-empty intersection with this set. (R^N must therefore contain at least one ordered pair of identical values.)

The formula is false in this model because any pair of values that are equal will make the antecedent true, but the consequent false.

Question 3.1

(i)

(a) Does not hold. For $\Box \Diamond q$ to be true in world x_1, $\Diamond q$ must be true in all worlds accessible from x_1, namely x_2 and x_4. But $\Diamond q$ is not true in x_2 because there is no world accessible from x_2.

(b) Holds. $\Box p$ holds vacuously in x_2 (because there are no worlds accessible from x_2), and $\neg p$ holds in x_2. Since the antecedent and the consequent are true in x_2, $\Box p \to \neg p$ is true in x_2.

(c) Holds. Since p is true in x_3 and x_3 is accessible from itself, $\Diamond p$ is true in x_3, and so is $\Diamond \Diamond p$.

(d) Holds. $p \vee q$ is true in all worlds accessible from x_4, namely x_1 and x_2 (because p is true in x_1 and q is true in x_2), so $\Box (p \vee q)$ is true in x_4.

(ii) Some possibilities are $\Box p \vee \Diamond q$ or $q \vee \Diamond q$. Any other non-valid formula that is true in all worlds, is acceptable.

The student must explain why the formula is true in each world, i.e. in x_1, x_2, x_3 and x_4, and why it is not valid.

Question 3.2

(i) $\phi \rightarrow \Box \phi$ should not be valid: Even if ϕ is true, the agent may very well not believe that it is true.

(ii) $\Box \phi \rightarrow \Box \Box \phi$ should be valid. If the agent believes ϕ, he believes that he believes it.

Question 3.3

(i) If I will always be rich then sometimes I will be unhappy.

(ii) $\neg \Diamond p \wedge \Diamond q$

Question 3.4

(iii) Agent 1 knows p and he knows that agent 2 does not know q

(iv) $\neg K_1 \neg q \vee \neg K_2 \neg q \vee \neg K_3 \neg q \vee \neg K_4 \neg q \vee \neg K_5 \neg q$

Question 3.5

$\Box (p \rightarrow q) \vdash \neg \Box \neg p \rightarrow \neg \Box \neg q$

1	$\Box (p \rightarrow q)$	premise
2	$\neg \Box \neg p$	assumption
3	$\Box \neg q$	assumption
4	$p \rightarrow q \Box e\ 1$	
5	p	assumption
6	q	$\rightarrow e$ 4, 5
7	$\neg q$	$\Box e$ 3
8	$\bot$	$\neg e$ 6, 7
9	$\neg p$	$\neg i$ 5 - 8
10	$\Box \neg p$	$\Box i$ 4 - 9
11	$\bot$	$\neg e$ 2, 10
12	$\neg \Box \neg q$	$\neg i$ 3 - 11
13	$\neg \Box \neg p \rightarrow \neg \Box \neg q$	$\rightarrow i$ 2 - 12