Adapted from Liebenberg and Vlok 2000, The interpretation of maps, aerial photographs and satellite images.

How to calculate the scale of a vertical aerial photograph?

In practice there are three different situations in which you might have to calculate the scale of an aerial photograph, namely:

Situation A: you might have a topographical map with a known scale of the photographed area

Situation B: you might know the photographed area well and know a specific ground distance or be able to measure it on the ground

Situation C: you might not know the photographed area at all, and might not have a topographical map of it either

How would we proceed in each of these cases? Let's work out an example of each. Figure 1 shows the different parameters required to calculate the scale of an aerial photograph.

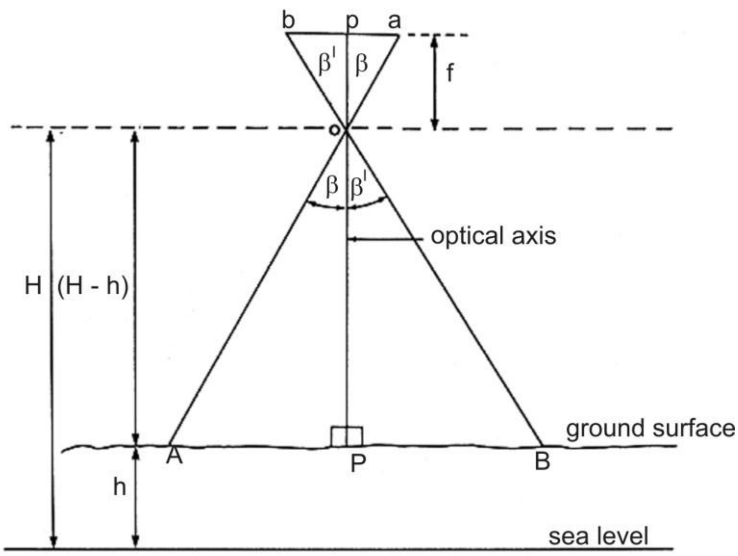

Figure 1: The different parameters for the calculation of the scale of a vertical aerial photograph

Situation A: A topographical map of the photographed area is available

You have a vertical aerial photograph with an unknown scale, but you also have a 1:50 000 topographical map of the same area.

1 We choose two phenomena or points which are clearly recognisable on both the photograph and the map. We then measure the straight-line distance between these two points in millimetres on both the map and the photograph. Suppose the photo distance is 50,4 mm and the map distance 42,8 mm.

$$\text{Scale of photograph} = \frac{\text{photo distance in mm}}{\text{map distance in mm} \times \text{scale factor of map}}$$

$$= \frac{50{,}4}{42{,}8 \times 50\ 000}$$

$$= \frac{50{,}4}{2\ 140\ 000}$$

$$= \frac{1}{42\ 460}$$

$$= 1{:}42\ 460$$

2 Repeat this procedure three or four times for various points on the aerial photograph and map and calculate the average scale. This will prevent you from basing your answer on an incorrect measurement and will also be more reliable than a single reading.

Situation B: You are unfamiliar with the area and do not have a map of it

This is the situation that you will encounter in most cases. Because you have no additional information, you are compelled to use the information in the margin of the photograph. The formula for calculating the scale of the photograph is as follows:

$$\text{Scale of aerial photograph} = \frac{1}{\text{scale}} = \frac{f}{H-h}$$

where

- S = scale factor of the photograph
- f = focal length of the lens
- H = absolute flying height above sea level
- h = average height above sea level of photographed area

Although we can explain how this formula can be mathematically derived from figure 2, you do not need to be able to demonstrate this. All you require is the individual values of f, H and h. The question that arises is: where does one get these values?

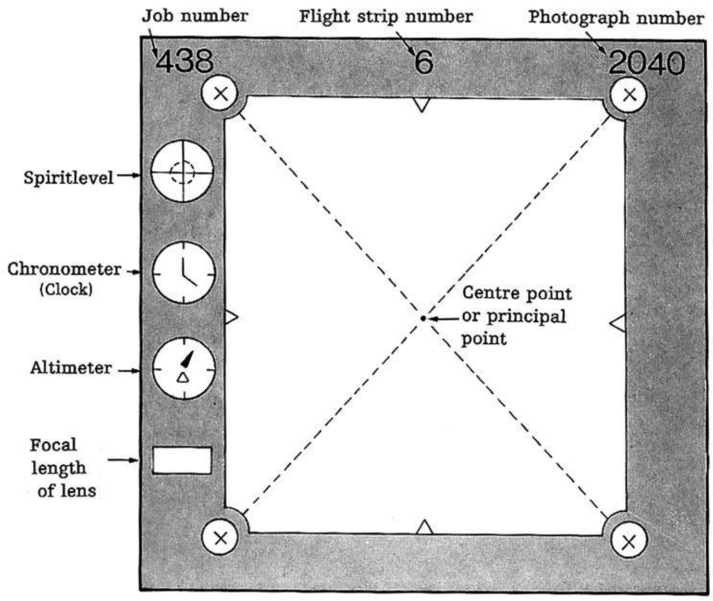

Figure 2: Information that appears in the margin of an aerial photograph

The focal length or (f) appears in the margin of the aerial photograph. In the case of the vertical aerial photograph of Mowbray in appendix C for example, the focal length is given in the margin as 153,14 mm.

■ The height above sea level of the area on the photograph (h) is usually obtained from a topographical map of the area, or it is measured in the field. If a large-scale map is used, the correct procedure is to read the height of 5 to 6 points on the contour map and calculate the average. The important fact is nevertheless that this height (the symbol h in the formula) should be known and cannot be read off the photograph. If you have to calculate the photographic scale, either this value should be directly given or you should obtain it yourself.

■ The flying height above sea level (H) is read off the altimeter on the aerial photograph.

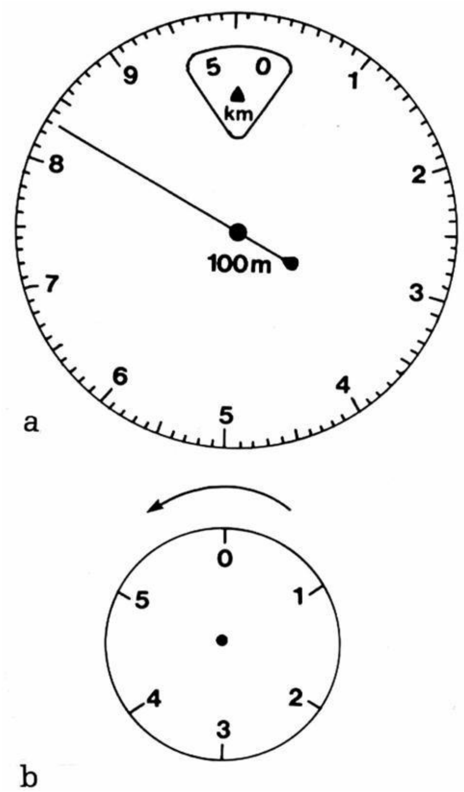

Figure 3: (a) The altimeter as it appears on the aerial photograph; (b) The smaller dial that is visible in the triangle on the larger dial

The usual type of altimeter seen on South African aerial photographs is illustrated in figure 3. As you can see, it consists of two parts, namely a large white dial (a) with subdivisions from 1 to 10 (the 10 is invisible), and a small black dial (b). The small dial is placed beneath the larger dial, in such a way that only a small section of it is visible in the triangular window between the 9 and the 1 on the larger dial.

The long hand on the large dial registers the absolute flying height above sea level in units of 100 m. The altitude indicated by the long hand in figure 3 is therefore 830 m. But what about the thousands? They are registered in the small black triangle between the 9 and the 1. As we said previously, all you can see in the small triangular window is a section of the small dial which is graduated from 0 to 6 in figure (b). On this dial the 0 takes the place of the 6. When the kilometre hand (1 000 m = 1 km) shows between 5 and 0, it means that the aircraft flew at a height of between 5 000 and 6 000 m above sea level. In figure 6.18 the long hand of the large dial registers the exact height as 5 830 m. If the kilometre hand points to a position between 0 and 1, this means that the aircraft either flew below 1 000 m, or between 6 000 m and 7 000 m above sea level. In the case of coastal terrain, the former is quite possible. If, however, the photographed area lies inland, an altitude of less than 1 000 m would be highly improbable.

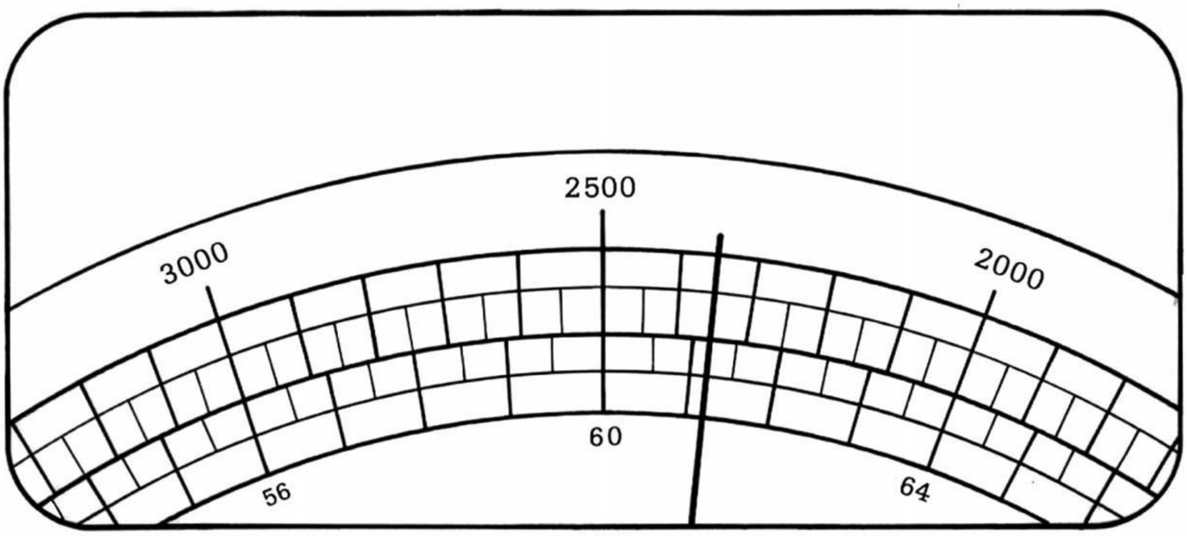

Figure 4: Alternative type of altimeter which is also frequently found on South African aerial photographs

Another type of altimeter which also appears on South African aerial photographs is shown in figure 4. The section shown on the figure is all that appears on the photograph. The altitude reading is taken on the outer part of the circular dial. As you can see, it registers the height above sea level in metres. The inner scale indicates atmospheric pressure above sea level and may be ignored for the purposes of calculating the scale. In figure 4 the height above sea level which is registered on the altimeter is about 2 350 m. (The scale is read from right to left with each subdivision on the outer scale representing 100 m.) This kind of altimeter is also to be seen on the two photographs of Mowbray in appendix C. Can you read the aircraft's height above sea level when these photographs were taken from this altimeter?

It is important that you should understand that the altimeter always shows the aircraft's height above sea level (H). To calculate the scale, subtract the height above sea level of the terrain in the photograph (h) from this height. The answer (H-h) is the flying height above ground level when the photograph was taken.

Having explained how all the values required to calculate the photographic scale are obtained, let us proceed to explain how you should apply the formula.

Example 1

Suppose you have a number of aerial photographs of an area which lies at an average of 700 m above sea level. The aircraft was equipped with a camera with a 88,9 mm wide-angled lens and was flying at a height of 4 200 m when the photographs were taken. How would you calculate the scale of these photographs?

1 Substitute the relevant values into the formula:

$$\frac{1}{\text{Scale factor}} = \frac{f}{H-h}$$

$$= \frac{0{,}0889 \text{ m}}{4\ 200-700 \text{ m}}$$

$$= \frac{0{,}0889}{3\ 500}$$

$$= \frac{1}{39\ 370}$$

2 Round off the photographic scale to the nearest thousand. The scale is therefore 1:39 000.

As regards the above calculation, we would like to share two hints with you:

1 Because the focal length is specified in millimetres, and the height above sea level in metres, you are forced to work either in millimetres or in metres throughout. If f is reduced to metres, its value is no longer 88,9 mm, but 0,0889 m.

2 Remember that 1/S is a fraction or ratio. In this fraction the numerator is 1 (one). To reduce this fraction, 0,0889/3 500, to a fraction in which the numerator is 1, you divide the numerator (0,0889) by the same number (0,0889), and then also divide the denominator by 0,0889. In this case

3 500/0,0889 = 3 9 370.

Example 2

Suppose you have some vertical aerial photographs on which the altimeter reading is 6 780 m. The focal length of the airphoto camera was 152,36 mm. The average height above sea level of the area photographed is 1 750 m. What is the scale of the photographs?

$$\frac{1}{\text{Scale factor}} = \frac{f}{H-h}$$

$$= \frac{0{,}15236 \text{ m}}{6\ 780-1\ 750 \text{ m}}$$

$$= \frac{0{,}15236}{5030}$$

$$= \frac{1}{33\ 014}$$

2 The scale of the photographs is 1:33 014. Approximated to the nearest thousand, it is 1 :33 000.

As we have already said, the interpreter of aerial photographs is often faced with having to calculate the scale of an aerial photograph merely with the aid of the data on the photograph and a formula.

How high should an aircraft fly if photographs on a specific scale are required?

It frequently happens that aerial photographs that are required for a specific purpose have to have a specific scale. The scale is specified beforehand. By modifying the above formula slightly, we can calculate the flying height provided the focal length and the desired scale are known.

$$\text{If} \quad \frac{1}{\text{scale factor}} = \frac{f}{H-h}$$

Then H - h = scale factor x f

Therefore the flying height above sea level (H) = (scale factor x f) + h and the height of the aircraft above the ground = h = H - (scale factor x f)

Has an aerial photograph an absolute scale and is the scale the same at all points on the photograph?

The answer to both questions is no as is reflected by the fact that we rounded off the scale to the nearest thousand in both the worked examples.

Unless the area being photographed is a completely flat landscape (something that does not exist in reality), all landscapes show variation in relief. The effect of this is evident from figure 5. Hilltops that are closer to the camera lens simply cast a larger image on the negative than the bottoms of valleys that are farther away. As a result any aerial photograph merely has an approximate scale.

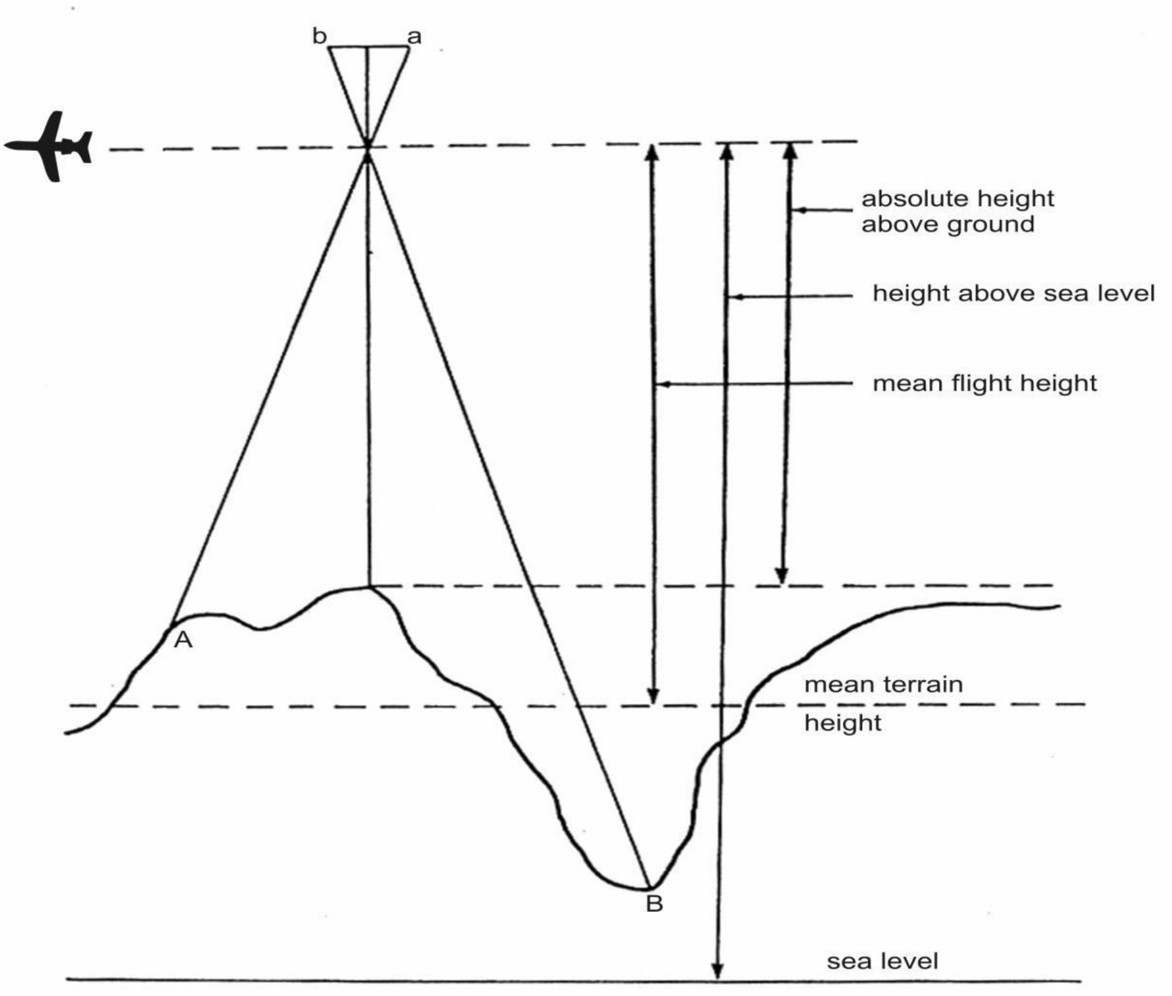

Figure 5: The effect of variation in relief on the approximate scale of a vertical aerial photograph

It is because aerial photographs only have an approximate scale that we use the average height above sea level in the case of hilly and mountainous areas. Because an average terrain height is used, the natural consequence is that (H - h) may be regarded as the average flying height above the terrain.

What is the relationship between the average flying height and the scale of the photograph?

If you have a good grasp of the geometrical principles of aerial photography, you should be able to answer this question with ease. The key naturally lies in the formula by means of which the scale is calculated.