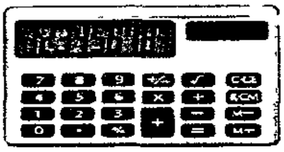
UNIVERSITY EXAMINATIONS



UNIVERSITEITSEKSAMENS

UNISA university of south africa

DSC2605

May/June 2015

LINEAR MATHEMATICAL PROGRAMMING

Duration 2 Hours

80 Marks

EXAMINERS
FIRST PROF EG JONES
SECOND DR JE SINGLETON

Use of a non-programmable pocket calculator is permissible

Closed book examination

This examination question paper remains the property of the University of South Africa and may not be removed from the examination venue.

This paper consists of 5 pages and a sheet of graph paper

INSTRUCTIONS

Answer all the questions

Show all workings.

Marks will be allocated for intermediate steps and not for final answers only

Question 1 [10]

Remove the graph paper attached to this examination paper and use it to answer this question Write your student number and the module code on the graph paper and, after you have answered the question, place it inside your answer book

Consider the following LP model

$$\begin{aligned} &\text{Maximise } Z = 70x_1 + 30x_2 \\ &\text{subject to} \\ &\qquad 5x_1 + 6x_2 \leq 90 \\ &\qquad 3x_1 + 2x_2 \leq 30 \\ &\qquad x_1 + x_2 \geq 10 \\ &\text{and} \qquad x_1, x_2 \geq 0 \end{aligned}$$

(1 1) Represent the constraints on a graph (represent x_1 on the horizontal axis and x_2 on the vertical axis) Indicate the feasible region clearly on your graph (7)

(1 2) Use the corner-point method to solve the LP model Write down your findings in detail (3)

Question 2 [4]

Determine the inverse of $F = \begin{bmatrix} 3 & 5 \\ 7 & 8 \end{bmatrix}$

Question 3 [8]

Let $A = \begin{bmatrix} 1 & 3 & 5 \\ 2 & -1 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 4 & -3 \\ 13 & 8 \end{bmatrix}$ and $C = \begin{bmatrix} 4 & -6 & 3 \\ 0 & 2 & 6 \\ -1 & 7 & 5 \end{bmatrix}$

Where possible, compute the matrix represented by each of the following expressions State clearly when an operation is not defined, and explain why

(3 1) $CA + B$ (2)

(3 2) $(AC)^T$ (3)

(3 3) $-B$ (1)

(3 4) $|C|$ (2)

Question 4 [10]

A South African company manufactures two types of office chairs, namely a business type (A) and an executive type (B) The production process at the company is divided into three distinct phases These phases are carpentry, finishing touches and packaging
The production of chair type (A) requires 7 hours of carpentry, 3 hours of finishing touches and 2 hours of packaging The production of chair type (B) requires 4 hours of carpentry, 2 hours of finishing touches and 3 hours of packaging
Due to the limited availability of skilled labour as well as machines and tools, 100 hours of carpentry, 30 hours of finishing touches and 20 hours of packaging are available each day
The profit for type (A) is R200 and for type (B) is R500 Management wants to determine how many chairs of type (A) and type (B) should be produced each day in order to maximise the total profit

Formulate this problem as a linear programming model Define the variables clearly
DO NOT solve your model

Question 5 [6]

Let $D = \begin{bmatrix} 3a - b & b + 3c \\ 2b + c & -a + 4b \end{bmatrix}$ and $E = \begin{bmatrix} 15 & 9 \\ 8 & 6 \end{bmatrix}$

What are the values of a, b and c in matrix D above if $D - E = 0$?

Question 6 [12]

A railway company wishes to purchase a new fleet of railway carriages The fleet must have the capacity to transport at least 2 800 tons of coal and at least 1 300 tons of lumber at any given moment Furthermore, at least a quarter of the carriages must be able to transport coal The company can buy three types of carriages models A, B and C

- One carriage of model A costs 5,8 million rand and can transport 21 tons of coal
- One carriage of model B costs 3,6 million rand and can transport 5 tons of lumber
- One carriage of model C costs 6,1 million rand and can transport either 17 tons of coal or 9 tons of lumber (not both simultaneously)

What is the least cost for which the company can buy a fleet with the desired capacity?

Formulate this problem as a linear programming model Define the variables clearly
DO NOT solve your model

Question 7 [15]

The following systems of equations were obtained by applying the simplex method to different maximisation problems (x_1, x_2, x_3 are decision variables, s_1, s_2, s_3 are slack variables and Z indicates the value of the objective function)

In each case, state whether the given solution is optimal or not.

- If the solution is optimal, write down the complete solution and identify any special kind of solution or constraint Justify your answer
- If the solution is not optimal, determine the entering and leaving variables for the next iteration of the simplex method Justify your answer
- If there is no feasible solution, or if the solution is unbounded, state why

(7 1)

$$\begin{aligned} 2x_1 + x_2 \qquad + 2s_1 \qquad &= 1\,400 \\ x_1 \qquad + x_3 \qquad + s_2 \qquad &= 600 \\ 3x_1 \qquad + 2x_3 - s_1 \qquad + s_3 &= 650 \\ Z - 2x_1 \qquad - 8x_3 + 6s_1 \qquad &= 4\,200 \end{aligned}$$

(7 2)

$$\begin{aligned} 0{,}5x_1 + x_2 \qquad + 0{,}5s_1 \qquad &= 350 \\ x_1 \qquad + x_3 \qquad + s_2 \qquad &= 600 \\ 0{,}5x_1 \qquad - 0{,}5s_1 - s_2 + s_3 &= 50 \\ Z + 1{,}5x_1 \qquad + 1{,}5s_1 + 2s_2 \qquad &= 2\,250 \end{aligned}$$

(7 3)

$$\begin{aligned} -\ 5x_1 + x_2 \qquad + 3s_2 \qquad &= 30 \\ -\ 2x_1 \qquad - s_2 + s_3 &= 30 \\ -\ x_1 \qquad + s_1 + 20s_2 \qquad &= 20 \\ Z - 85x_1 \qquad + 20s_2 \qquad &= 600 \end{aligned}$$

Question 8 [15]

Consider the following LP model

$$\begin{aligned} &\text{Maximise } Z = 2x_1 - x_2 + x_3 \\ &\text{subject to} \\ &\qquad 3x_1 + x_2 + x_3 \leq 60 \\ &\qquad x_1 - x_2 + 2x_3 \leq 10 \\ &\qquad x_1 + x_2 - x_3 \leq 20 \\ &\text{and} \qquad x_1, x_2, x_3 \geq 0 \end{aligned}$$

(8 1) Solve this linear programming model using the simplex method (12)

(8 2) Give the optimal solution in full (3)

TOTAL [80]

[illegible]