UNIVERSITY EXAMINATIONS

UNIVERSITEITSEKSAMENS

UNISA university of south africa

DSC2605

October/November 2013

DEPARTMENT OF DECISION SCIENCES
LINEAR MATHEMATICAL PROGRAMMING

Duration 2 Hours

80 Marks

EXAMINERS .
FIRST DR S BELABBES
SECOND PROF MP MULAUDZI

Programmable pocket calculator is permissible.

Closed book examination

This examination question paper remains the property of the University of South Africa and may not be removed from the examination venue

This paper consists of **four** pages and a sheet of graph paper

INSTRUCTIONS

Answer all the questions.

Show all workings.

Marks will be allocated for intermediate steps and not for final answers only

[TURN OVER]

Question 1 [14]

Remove the graph paper attached to this examination paper and use it to answer this question Write your student number and the module code on the graph paper and, after you have answered the question, place it inside your answer book

Consider the following LP model.

$$\begin{aligned} &\text{Minimise } z = 3x_1 + 2x_2 \\ &\text{subject to} \\ &\qquad x_1 + 2x_2 \leq 12 \\ &\qquad 2x_1 + 3x_2 = 12 \\ &\qquad 2x_1 + x_2 \geq 8 \\ &\text{and} \qquad x_1, x_2 \geq 0. \end{aligned}$$

(a) Represent the constraints on a graph. Show the solution set of each constraint clearly on your graph. Show the feasible region of the LP model clearly on your graph (7)

(b) Solve this LP model Write down your findings in detail. (2)

(c) For each constraint, indicate whether it is binding or not Explain your answer (3)

(d) If the objective function is changed to Maximise PROFIT $= 4x_1 - x_2$, find the optimal solution and give the solution in full. (2)

Question 2 [5]

Prove that $A = \begin{bmatrix} 3 & -5 & 1 \\ 2 & 1 & 4 \\ 5 & 6 & 7 \end{bmatrix}$ is an inverse of $B = \frac{1}{74}\begin{bmatrix} 17 & -41 & 21 \\ -6 & -16 & 10 \\ -7 & 43 & -13 \end{bmatrix}$.

Question 3 [11]

Let $A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -2 \\ 2 & 4 \end{bmatrix}$ and $C = \begin{bmatrix} 2 & -4 & 5 \\ 0 & 1 & 4 \\ 3 & 2 & 1 \end{bmatrix}$

Where possible, compute the matrix or value represented by each of the following expressions State clearly when an operation is not defined, and explain why.

(a) $AB - 2A$ (3)

(b) B^{-1} (4)

(c) $|C|$ (3)

(d) $|A|$ (1)

Question 4 [10]

Sun-Juice Company sells bags of oranges and cartons of orange juice Sun-Juice grades oranges on a scale of 1 (poor) to 10 (excellent) Sun-Juice now has a stock of 50 000 kg of grade-9 oranges and 60 000 kg of grade-6 oranges. The average quality of oranges sold in bags must be at least grade 7, and the average quality of the oranges used to produce orange juice must be at least grade 8 Each kilogram of oranges used for juice yields a revenue of R15 and incurs a variable cost (consisting of labour costs, variable overhead costs, inventory costs, and so on) of R10,50 Each kilogram of oranges sold in bags yields a revenue of R9 and incurs a variable cost of R6

Formulate a linear programming model to help Sun-Juice maximise its profit Define the variables clearly. **DO NOT** solve your model

Question 5 [10]

Use the Gauss-Jordan method to solve the following system of equations

$$\begin{aligned} x + y + z &= 3 \\ 2x + 3y + 7z &= 0 \\ x + 3y - 2z &= 17 \end{aligned}$$

Question 6 [8]

A furniture company manufactures tables and chairs A table requires 40 square units of wood, and a chair requires 30 square units of wood Wood may be purchased at a cost of R10 per square unit, and 40 000 square units of wood are available for purchase It takes two hours of skilled labour to manufacture an unfinished table or an unfinished chair Three more hours of skilled labour will turn an unfinished table into a finished table, and two more hours of skilled labour will turn an unfinished chair into a finished chair A total of 6 000 hours of skilled labour are available (and have already been paid for) All furniture produced can be sold at the following unit prices

- unfinished table R700
- finished table R1400
- unfinished chair R600
- finished chair: R1100

Formulate a linear programming model that will maximise the contribution to profit from manufacturing tables and chairs Define the variables clearly **DO NOT** solve your model

Question 7 [7]

A cattle farmer is trying to decide what to feed his cows He is considering using a combination of two feeds available from local suppliers Feed type A costs R40 per kg and feed type B costs R80 per kg. He would like to feed the cows at a minimum cost while also making sure each cow receives an adequate supply of calories and vitamins on a daily basis Feed type A supplies 800 calories per kg and 140 units of vitamins per kg Feed type B supplies 1 000 calories per kg and 70 units of vitamins per kg Each cow requires at least 8 000 calories per day and at least 700 units of vitamins per day. A further constraint is that no more than one third of the diet (by weight) can consist of feed type A, since it contains an ingredient that is toxic if consumed in too large a quantity.

Formulate a linear programming model for this problem Define the variables clearly **DO NOT** solve your model.

Question 8 [15]

Consider the following LP model

$$\begin{aligned}
&\text{Minimise } z = 4x_1 + 4x_2 + x_3 \\
&\text{subject to} \\
&\qquad x_1 + x_2 + x_3 \leq 2 \\
&\qquad 2x_1 + x_2 \leq 3 \\
&\qquad 2x_1 + x_2 + 3x_3 \geq 3 \\
&\text{and} \qquad x_1, x_2, x_3 \geq 0
\end{aligned}$$

Keep the model as a **minimisation problem** and answer the following questions

(a) Write this LP model in the augmented form. (2)

(b) Rewrite the objective function in the form required to start the Big M method (2)

(c) Write down the entering variable for the first iteration of the Big M method Explain your answer (1)

(d) Write down the leaving variable for the first iteration of the Big M method Explain your answer (2)

(e) Do one iteration of the Big M method (4)

(f) Is this the optimal solution? Explain your answer (1)

(g) Give the solution at this stage in full (3)

TOTAL [80]

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